E209A LECTURE 9

GOALS OF THIS LECTURE:

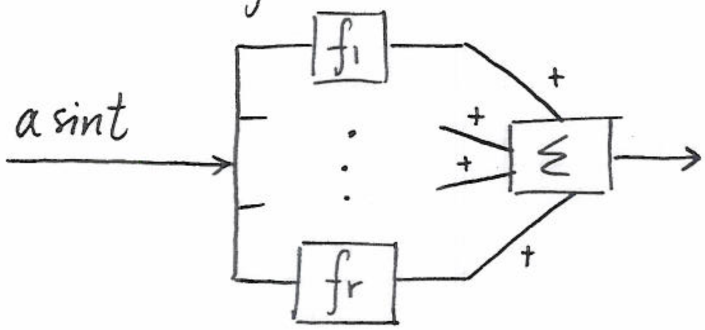
- to show how describing functions may be used in stability analysis of closed loop systems.

REFS

SASTRY § 4.1
KHALIL § 7.2

Using Describing Functions in Stability Studies

WARNING: block diagram manipulations for nonlinear systems.



first harmonic terms from each f_i add to give first harmonic from f.

nonlinear systems f_i connected in ||

$$\Rightarrow N_f(a) = N_{f_1}(a) + N_{f_2}(a) + \cdots + N_{f_r}(a)$$

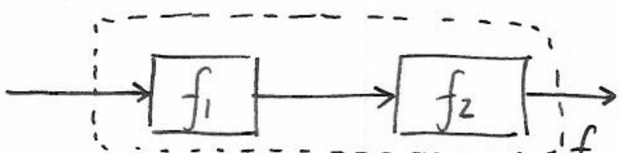
Application: if we can decompose a given f as

$$f(x) = \sum_i f_i(x)$$

for f_i having known N_{f_i}, then

$$N_f(a) = \sum N_{f_i}(a) \quad \text{(easy to find)}$$

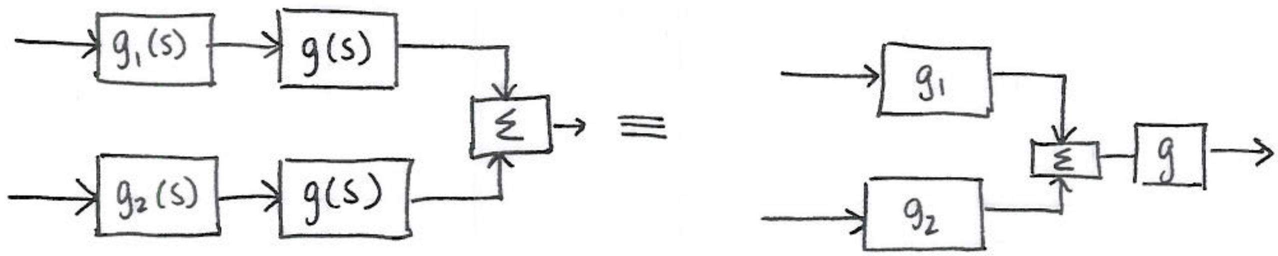
HOWEVER:



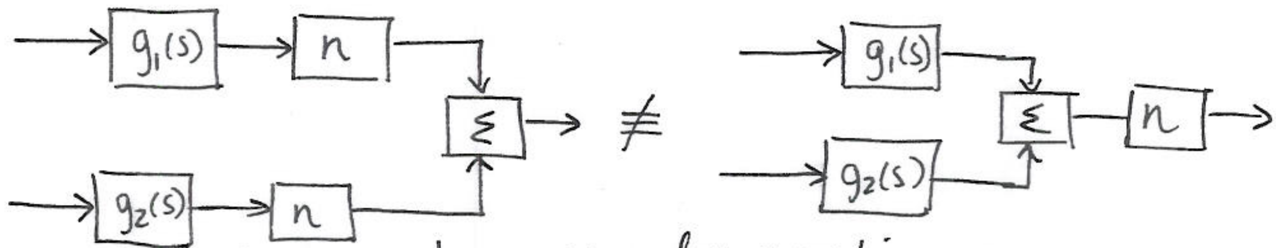
WARNING: $N_f(a) \neq N_{f_1}(a) \cdot N_{f_2}(a)$
in general

and similarly for several functions f_i in series.

Also: for linear systems.



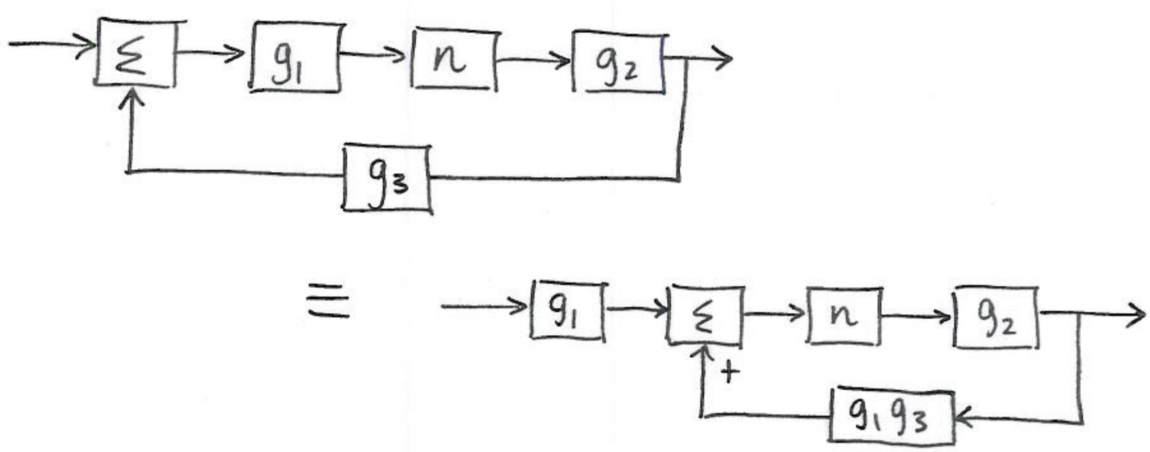
but for nonlinear systems



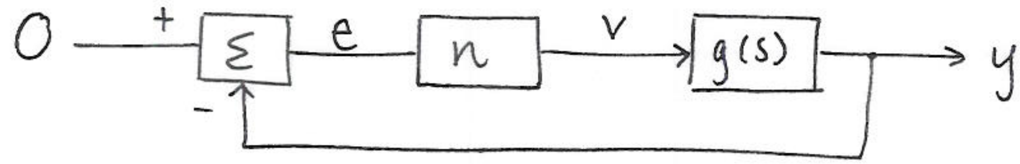
because, for nonlinear n, superposition is not valid.

For nonlinear systems, manipulations are OK iff inputs to nonlinearities remain unaltered.

eg.



Many nonlinear systems can be written as:



where n is a SVSS or DVSS function.

Concentrate on error e: → since if we know what happens to e, we can determine what happens elsewhere in the system, easily.

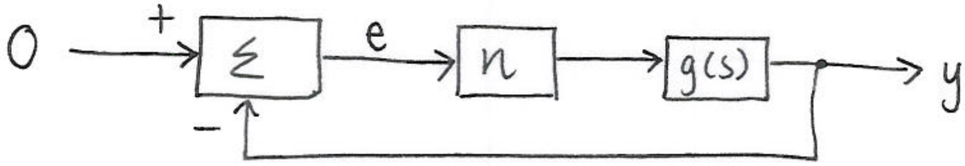
- will it oscillate?
- what amplitude?
- what ω?

<u>Assume</u> $e \cong$ sustained sinusoidal oscillation

$$\Longrightarrow e(t) \cong a \sin(\omega t + \theta) \text{ for some } \underbrace{a, \omega}_{\text{to be estimated}}, \theta$$

Then
$$\begin{aligned} v(t) &= n(e(t)) \\ &\cong n(a \sin(\omega t + \theta)) \\ &= a|N(a)| \sin(\omega t + \theta + \phi(a)) \\ &\quad + \text{higher order terms} \end{aligned}$$

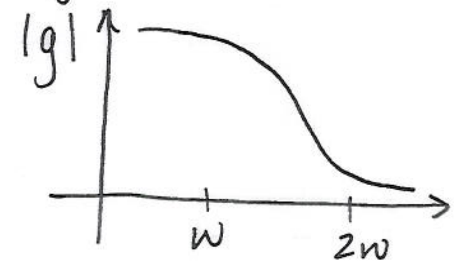
where $N(a)$ is the transfer function between the input to n and the 1st harmonic in the output of n, and $\phi(a)$ is the phase angle associated with $N(a)$.



$e(t) \cong a \sin(wt + \theta)$

$v(t) \cong a\,|N(a)| \sin(wt + \theta + \phi(a))$
+ higher harmonics.

<u>Assume</u>: $g(s)$ attenuates higher harmonics, in that g looks like:



Then $y(t) \cong$ result of g operating on 1st harmonic in v

$$\cong |g(jw)|\, a\, |N(a)| \sin(wt + \theta + \phi(a) + \psi(w))$$

phase angle associated to $g(jw)$

So $\quad e(t) = -y(t)$

$$\Rightarrow \quad a \sin(wt+\theta) \cong -|g(jw)|\, a\, |N(a)| \sin(wt + \theta + \phi(a) + \psi(w))$$

assume this is actually =

$$\therefore \quad a \sin(wt + \theta) = -|g(jw)|\, a\, |N(a)| \cdot \sin(wt + \theta + \phi(a) + \psi(w))$$

In phaser form:

$$a e^{j(wt+\theta)} = -|g(jw)|\, a\, |N(a)|\, e^{j(wt + \theta + \phi(a) + \psi(w))}$$
$$= -a\, |g(jw)| e^{j\psi(w)} \cdot |N(a)| e^{j\phi(a)} e^{j(wt+\theta)}$$

i.e. $$1 = -\underbrace{|g(jw)|e^{j\psi(w)}}_{g(jw)} \cdot \underbrace{|N(a)|e^{j\phi(a)}}_{N(a)}$$

harmonic balance equation

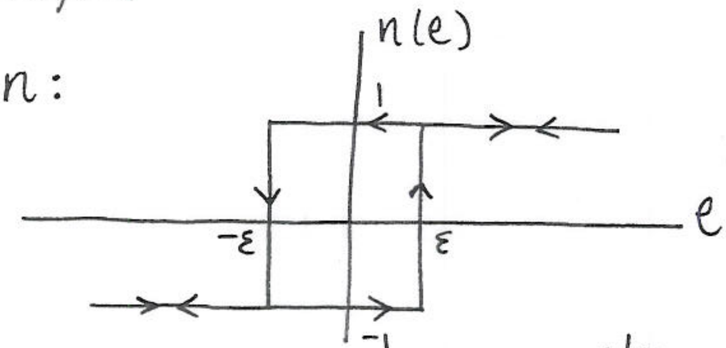
ie. $$\boxed{g(jw) = -\frac{1}{N(a)}}$$ relationship between w and a

Assumptions made:

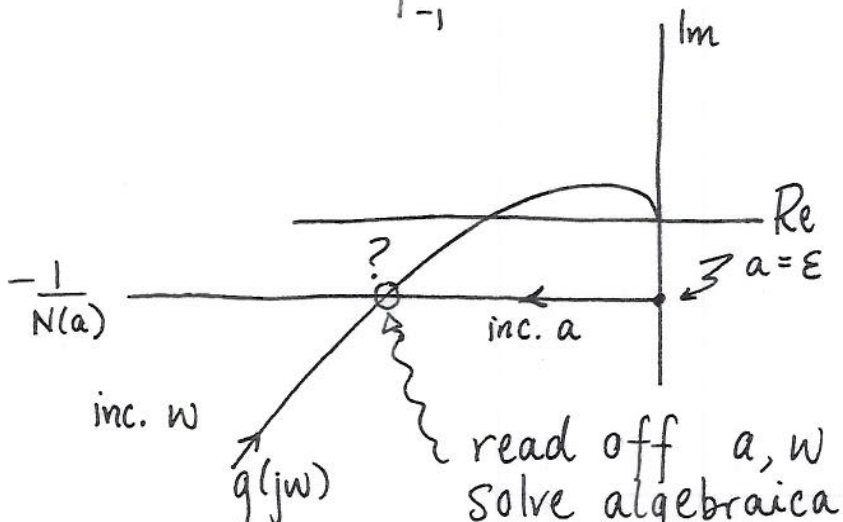
- $e(t) \cong a \sin(wt + \theta)$
- g attenuates higher frequencies
- describing function is exact.

<u>example</u>:

n:



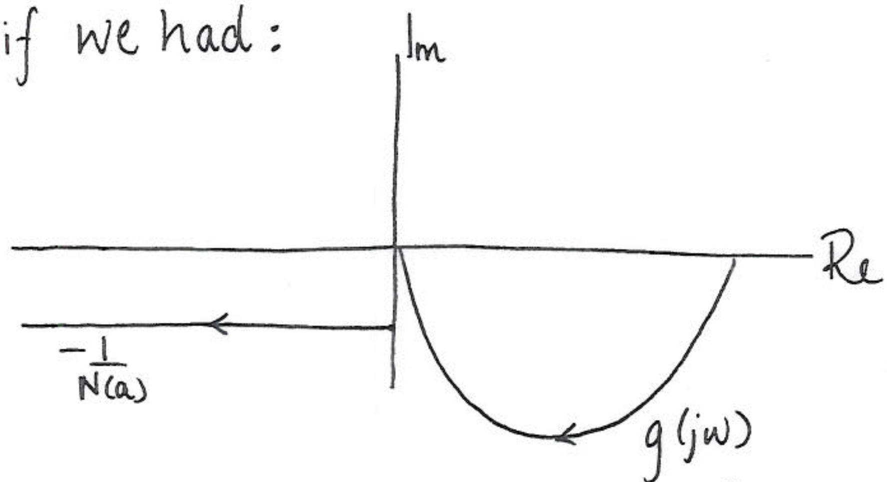
Assume:
$$g(s) = \frac{40}{s(s+2)(s+8)}$$



read off a, w or solve algebraically

Suggests: if intersection takes place, then oscillations may occur in e with the w, a associated with the intersection point.

But if we had:



Then $g(j\omega) = -\frac{1}{N(a)}$ not true for any ω, a, which suggests no oscillation (because if $e(t) \cong a \sin(\omega t + \theta)$ and assumptions all valid, then $g(j\omega) = -\frac{1}{N(a)}$ must be true for some ω, a.

<u>NOTE</u>: predictions not foolproof in that, owing to the approximations involved in the analysis:

- predicted oscillations might not happen
- predicted no-oscillations might be false

Nonetheless, this is often a useful tool.

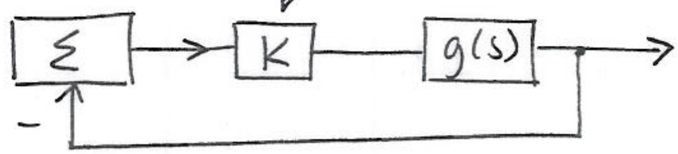
Q: Can we predict whether oscillations will
- decay
- be sustained
- explode

?

A: Yes.... using an extension of Nyquist.

NYQUIST :

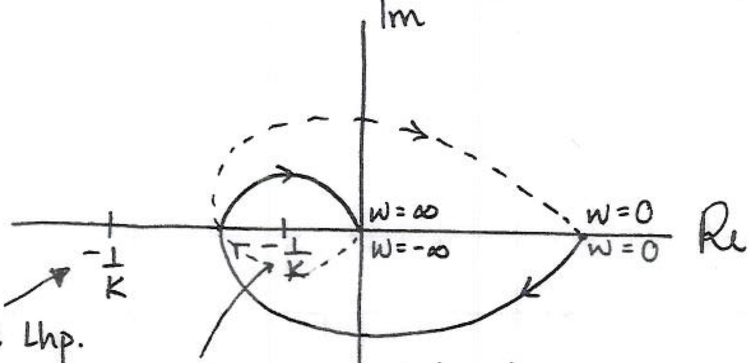
constant gain



Then: closed loop poles are in open left half plane iff complete $g(jw)$-locus encircles $[-\frac{1}{K} + j0]$ P times (anticlockwise)

\# OL poles in open RHP

eg. $P = 0$

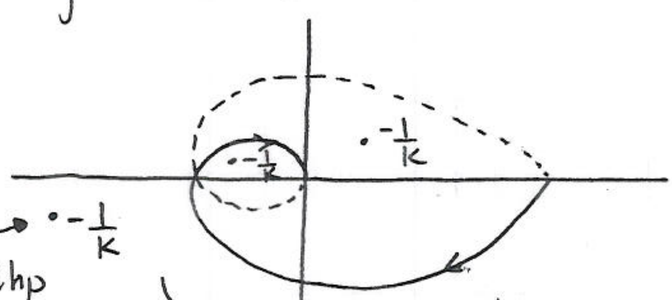


$-\frac{1}{K}$ here: all poles in open lhp.

$-\frac{1}{K}$ here: at least one closed loop pole in closed rhp

Actually, true for all $k \in \mathbb{C}$:

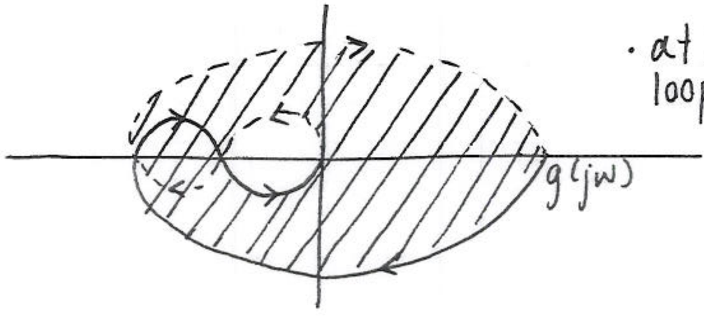
eg. $p = 0$



$-\frac{1}{K}$ here: all poles in open lhp

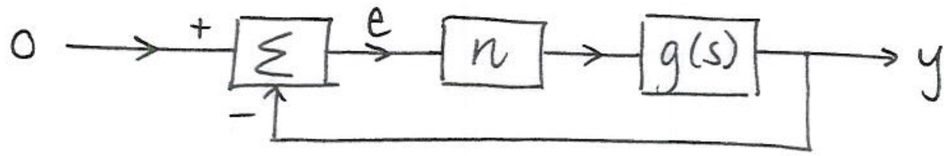
$-\frac{1}{K}$ here: at least one closed loop pole in closed rhp.

eg. $p = 0$

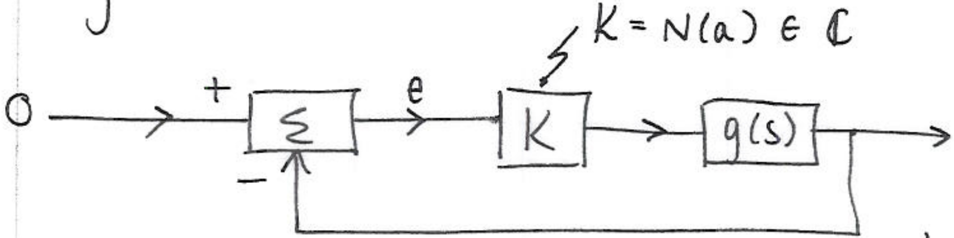


· at least one closed loop pole in closed rhp if $-\frac{1}{K} \in$ ///.

Application to:



For $e(t) \cong a\sin(\omega t+\theta)$ view $\uparrow$ as approximated by:

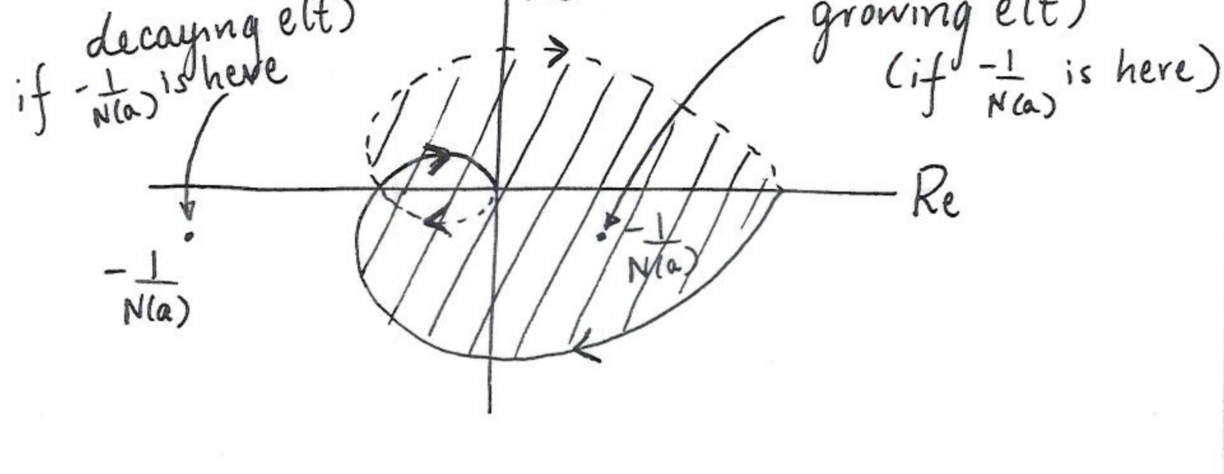


because the complex number $N(a)$ is the transfer function between the input sinusoid to n and the first harmonic in the output of n.

Also, <u>assume</u> $N(a)$ works for $e(t)$ of the form $a e^{\sigma t}\sin(\omega t+\theta)$ [as well as the case we just did - for $e(t)$ of the form $a\sin(\omega t+\theta)$]

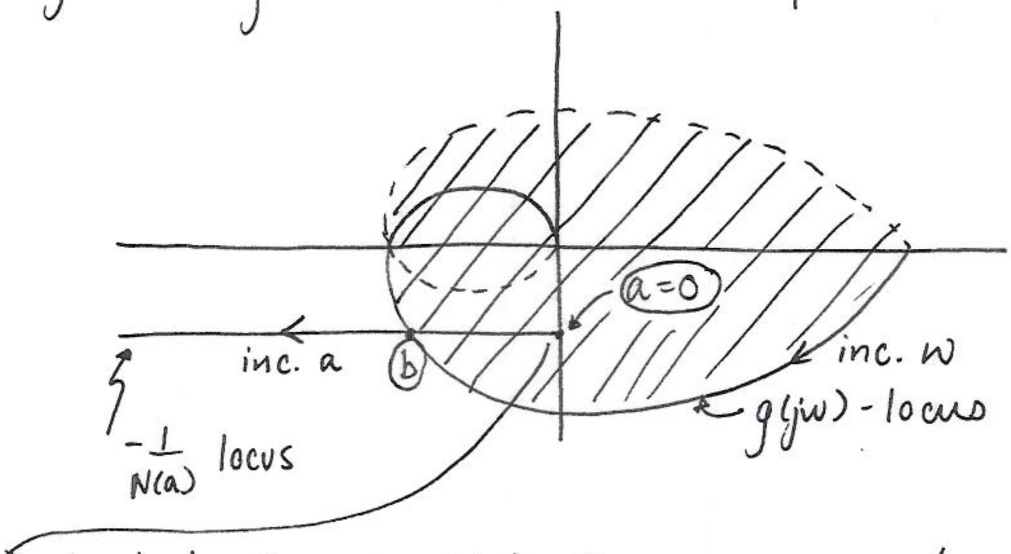
Then, growth or decay of $e(t)$ is predictable from position of $-\frac{1}{K} = -\frac{1}{N(a)}$ with respect to $g(j\omega)$-locus.

e.g.



Hence can predict $e(t)$ behavior:

eg. Say $e(0) = 0$ and $p = 0$



start here as $e(0)=0$; corresponds to $a=0$
initially oscillation grows as $-\frac{1}{N(a)} \in$ ////

≈> a increases

≈> $-\frac{1}{N(a)}$ moves to b, but stops at b as $e(t)$ and hence a decreases on left of b #

≈> sustained oscillation with a, ω values those at b.

≈> means "implies, more or less"

\# Since:

$-\frac{1}{N(a)} \in$ /// ⇒ growing amplitude a of assumed sinusoid e

$-\frac{1}{N(a)} \in$ unshaded area ⇒ decaying amplitude a of assumed sinusoid e.

WARNING:

describing function analysis based on many approximations

$\Rightarrow$ predictions not necessarily correct

Can redo theory for:

$$e(t) = \sum_{i=1}^{r} \alpha_i \sin(\omega_i t + \theta_i)$$

$\Downarrow$

multiple-input describing functions

$\Downarrow$

for which, can be shown: if a closed loop oscillation exists, it can be predicted for some finite r,

and for which:

error bounds can be obtained.