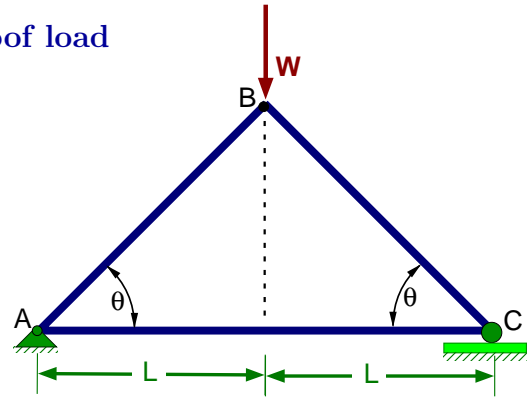


### 14.2.5 Example: Static analysis of a truss with roof load

The figure to the right shows a roof load of weight  $W$  that is supported at point  $B$  of a planar *truss*.<sup>a</sup> The truss is attached to ground at point  $A$  by a pin-joint and point  $C$  by a smooth (frictionless) pin-roller joint. The roof load and truss are in **static equilibrium**.



<sup>a</sup>A *truss* has *two-force members* and is considered massless relative to the loads it carries. As shown by Homework 12.??, forces applied to the end of each member must be parallel to the line connecting the member's end-points.

The steps for analyzing this structure are:<sup>1</sup>

1. Identify **a** system  $S$ .
2. Identify the relevant **external contact** and **distance** forces on  $S$  (or their *equivalent* force systems).
3. Introduce points, unit vectors, scalar symbols (e.g., force or torque measures), etc. (as necessary).  
Note: Use Newton's law of action/reaction to reduce the introduction of unknown force/torque measures.
4. Use laws of static or dynamic equilibrium, e.g, for statics:  $\mathbf{F}^S = \mathbf{0}$  and/or  $\mathbf{M}^{S/P} = \mathbf{0}$ .  
Form scalar equations via vector dot-products.
5. If there are an insufficient number of equations for a determinate system to answer the questions "what are the forces" and "where is it", identify **another** system  $S$  and repeat.

The 2<sup>nd</sup> column of the following table shows the magnitude of the force in each member as a function of  $L$ ,  $\theta$  and  $W$ . The 3<sup>rd</sup> column shows whether each member is in **compression** (forces on the member try to shorten it) or **tension** (forces on the member try to elongate it) when  $W$  is positive. The 4<sup>th</sup> column shows the minimum and maximum load in each member for values of  $\theta$  between  $0^\circ$  and  $90^\circ$ .

Note: Forces in member BC can be determined by *symmetry*.

Member	Force magnitude	Compression or tension	Minimum load	Maximum load
AB	$\frac{W}{2 \sin(\theta)}$	Compression	$\frac{W}{2}$ at $\theta = 90^\circ$	$\infty$ at $\theta = 0^\circ$
AC	$\frac{W \cos(\theta)}{2 \sin(\theta)}$	Tension	0 at $\theta = 90^\circ$	$\infty$ at $\theta = 0^\circ$
BC	$\frac{W}{2 \sin(\theta)}$	Compression	$\frac{W}{2}$ at $\theta = 90^\circ$	$\infty$ at $\theta = 0^\circ$

One *incorrect* conclusion from this analysis is  $\theta \approx 90^\circ$  is structurally superior to smaller values of  $\theta$ . Since members AB and BC are in **compression** and a member's chance of **buckling** increases significantly as its length increases,<sup>2</sup> one must trade-off decreasing axial compressive loads with an increased chance of **buckling**. When buckling dominates a design, additional members (sometimes carrying no-load) are added near the center of each member to decrease the possibility of buckling.

<sup>1</sup>The Autolev analysis for this truss structure is provided in Section 14.4.

<sup>2</sup>Euler's **critical buckling load**  $F_{\text{buckling}}$  for a long simply-supported column under an axial compressive force is  $F_{\text{buckling}} = \frac{c}{L_{\text{column}}^2}$  where  $L_{\text{column}}$  is the length of the column (member) and  $c$  is a constant that depends on the column's elastic modulus and area moment of inertia.

## 14.4 Static analysis of the truss of Section 14.2.5 with Autolev

```

(1) % File: TrussABCTopLoad.al (Truss analysis)
(2) % -----
(3) NewtonianFrame N % Ground
(4) RigidFrame S % Entire truss
(5) Point A(S), B(S), C(S) % Nodes on truss
(6) % -----
(7) % Constants and variables
(8) Constant L % Twice the distance between A and C
(9) Constant theta % Angle between AC and AB
(10) Constant W % Weight applied to node B
(11) Variable FAX % Nx> measure of external force on A
(12) Variable FAY % Ny> measure of external force on A
(13) Variable FCY % Ny> measure of external force on E
(14) Variable FAB % Force on A from AB member directed from A to B
(15) Variable FAC % Force on A from AC member directed from A to C
(16) % -----
(17) % Relevant external contact and distance forces on S
(18) A.AddForce( FAX*Nx> + FAY*Ny> )
-> (19) Force_A> = FAX*Nx> + FAY*Ny>

(20) B.AddForce( -W*Ny> )
-> (21) Force_B> = -W*Ny>

(22) C.AddForce( FCY*Ny> )
-> (23) Force_C> = FCY*Ny>

(24) % -----
(25) % Static analysis of entire system
(26) ResultantForceOnS> = S.GetResultantForce()
-> (27) ResultantForceOnS> = FAX*Nx> + (FAY+FCY-W)*Ny>

(28) MomentOfSAboutA> = Cross( L*Nx>, -W*Ny> ) + Cross( 2*L*Nx>, FCY*Ny> )
-> (29) MomentOfSAboutA> = -L*(W-2*FCY)*Nz>

(30) ZeroSystem[1] = Dot( ResultantForceOnS>, Nx> )
-> (31) ZeroSystem[1] = FAX

(32) ZeroSystem[2] = Dot( ResultantForceOnS>, Ny> )
-> (33) ZeroSystem[2] = FAY + FCY - W

(34) ZeroSystem[3] = Dot( MomentOfSAboutA>, Nz> )
-> (35) ZeroSystem[3] = -L*(W-2*FCY)

(36) Solve( ZeroSystem, FAX, FAY, FCY )
-> (37) FAX = 0
-> (38) FAY = 0.5*W
-> (39) FCY = 0.5*W

(40) % -----
(41) % Relevant external contact and distance forces on pin A
(42) UnitVectorFromAToB> = Vector( N, cos(theta), sin(theta), 0 )
-> (43) UnitVectorFromAToB> = cos(theta)*Nx> + sin(theta)*Ny>

(44) A.AddForce( B, FAB * UnitVectorFromAToB> )
-> (45) Force_A_B> = cos(theta)*FAB*Nx> + sin(theta)*FAB*Ny>

(46) A.AddForce( C, FAC * Nx> )
-> (47) Force_A_C> = FAC*Nx>

(48) % -----
(49) % Static analysis of node A
(50) ResultantForceOnA> = A.GetResultantForce()
-> (51) ResultantForceOnA> = (FAC+FAX+cos(theta)*FAB)*Nx> + (FAY+sin(theta)*FAB)*Ny>

(52) ZeroA = Dot( ResultantForceOnA>, [Nx>; Ny>] ) % Creates 2x1 matrix
-> (53) ZeroA = [FAC + FAX + cos(theta)*FAB; FAY + sin(theta)*FAB]

(54) AutoRhs ALL % Ensure results are explicit in W, L, and theta
(55) Solve( ZeroA, FAB, FAC )
-> (56) FAB = -0.5*W/sin(theta)
-> (57) FAC = 0.5*W*cos(theta)/sin(theta)

```

