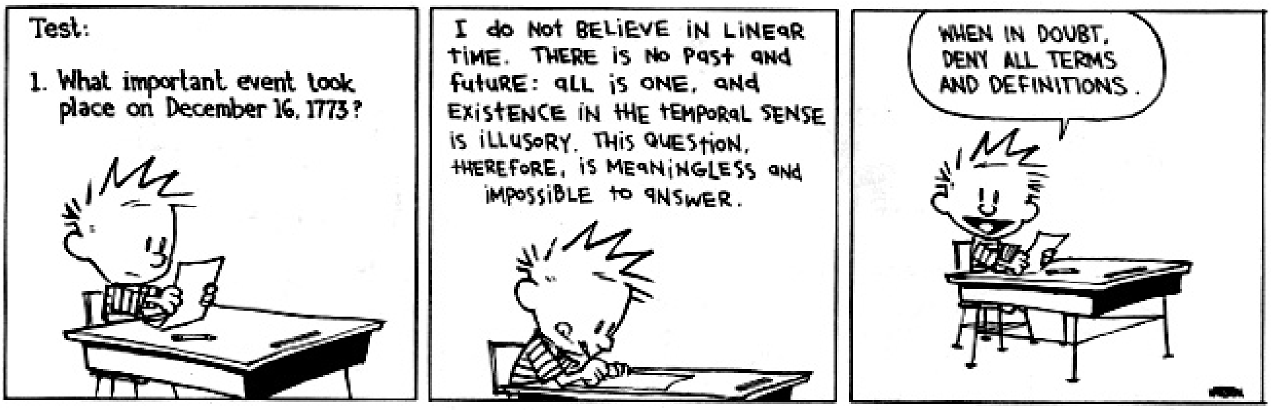
E14 Statics Midterm 2008.   NAME: ______________________________

I certify that I upheld the Stanford Honor code during this exam ____________________



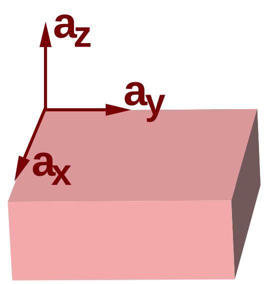
- Print your name and sign the honor code statement
- You may use your course notes, homework, books, etc.
- Computers and/or other electronic devices are prohibited (other than a **simple** calculator)
- Write your answers on this handout
- Where space is provided, show your work to get full credit
- If necessary, attach extra pages for scratch work

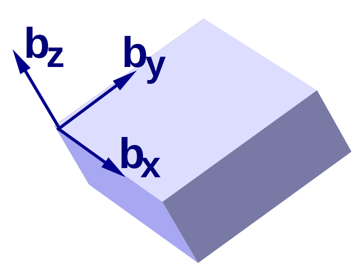
Page	Value	Score
1	5	
2	16	
3	28	
4	15	
5	12	
6	24	
Total	100	

M.1 (5 pts.) Calculating angles between unit vectors.

The following rotation table ${}^{a}R^{b}$ relates right-handed, orthogonal, unit vectors $\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$ and $\mathbf{b}_x, \mathbf{b}_y, \mathbf{b}_z$. Calculate the angle between $\mathbf{a}_x$ and $\mathbf{b}_z$ to four (or more) significant digits.

${}^{a}R^{b}$	$\mathbf{b}_x$	$\mathbf{b}_y$	$\mathbf{b}_z$
$\mathbf{a}_x$	0.9622502	-0.08418598	0.258819
$\mathbf{a}_y$	0.1700841	0.9284017	-0.3303661
$\mathbf{a}_z$	-0.2124758	0.3619158	0.9076734





$\angle(\mathbf{a}_x, \mathbf{b}_z) = \qquad ^\circ$

M.2 (5 pts.) Position concepts: What objects have a position vector?

$\mathbf{r}^{S/O}$, the position vector of an object S from a point O is to be determined.
In general and **without ambiguity**, S could be a (circle all appropriate objects):

Scalar	Complex number	Quaternion	Center of a circle
Vector	Point	Reference Frame	Mass center of a set of particles
Dyadic	Set of Points	Rigid Body	Mass center of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
Orthogonal unit basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

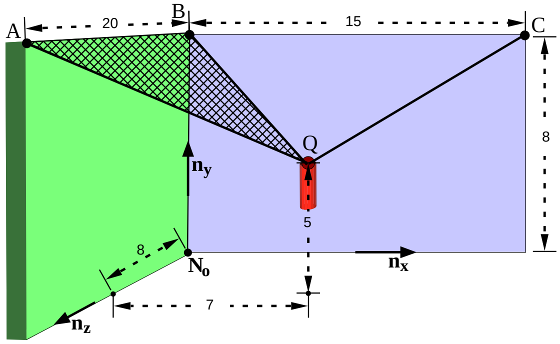
M.3 (5 pts.) Force, moment, and torque concepts.

The resultant of a set of forces is a force.	**True/False**
The resultant of a set of forces has units of force.	**True/False**
All torques are moments.	**True/False**
All moments are torques.	**True/False**
In the SI system, the units of force are:	___________

M.4 (6 pts.) Sum of all forces in the universe.

Consider the system S consisting of all matter, forces, energy, life, etc. (approximately $1 \text{ x } 10^{80}$ atoms).
According to Newton's laws, the resultant of all forces on S is $\mathbf{0}$, i.e., $\mathbf{F}^S = \mathbf{0}$. **True/False**.
According to Newton's laws, the moment of all forces on S about any point is $\mathbf{0}$, i.e., $\mathbf{M}^S = \mathbf{0}$. **True/False**
According to Newton's laws, the universe is in **static equilibrium**. **True/False**.
Newton's $\mathbf{1}^{st}/\mathbf{2}^{nd}/\mathbf{3}^{rd}$ law explains my answer? (Circle one)
Why:

M.5 (28 pts.) Microphone geometry: Length, surface area, and mass (orthogonal walls).
A microphone Q is attached to three pegs A, B, and C by three cables. Knowing the peg locations and microphone location from point N_o, answer the following questions with two or more significant digits.



Quantity	Value
Distance from A to B	20 m
Distance from B to C	15 m
Distance from N_o to B	8 m
Q's measure from N_o along back-wall	5 m
Q's height above N_o	6 m
Q's measure from N_o along left-wall	7 m

$$\mathbf{r}^{Q/N_o} = 5\,\mathbf{n}_\text{x} + 6\,\mathbf{n}_\text{y} + 7\,\mathbf{n}_\text{z}$$

- **(10 pts.)** Determine L_B (the length of the cable joining B and Q).

$$L_B = \underline{\qquad} \text{ m}$$

- **(14 pts.)** Determine the mass m of a thin cloth (density 0.1 kg/m^2) whose surface area covers the triangle formed by points A, B, and Q.

$$m = \underline{\quad} \text{ kg}$$

- **(4 pts.)** Determine a unit vector $\mathbf{u}$ perpendicular to the triangle formed by A, B, and Q.

$$\mathbf{u} =$$

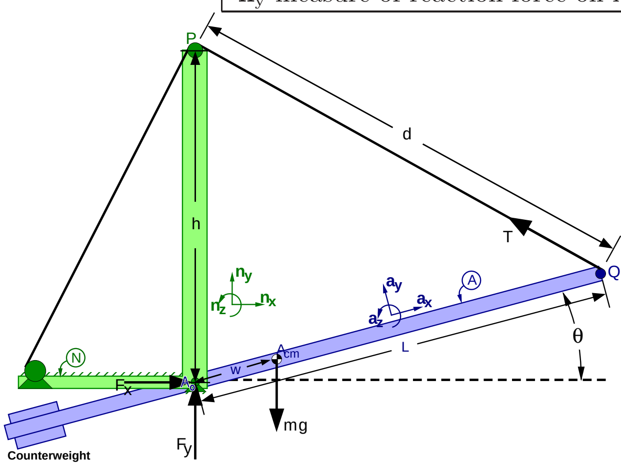
M.6 (53 pts.) **Static equilibrium of a draw-bridge.**

The following figure shows a draw-bridge whose platform A (A includes the bridge's road-way and counter-weight) is supported by a frictionless revolute joint at point A_o and a light (massless) cable attached to point Q (the distal end of the platform). The cable runs over a pulley at point P and into a winch that is connected to ground N.

Right-handed sets of mutually perpendicular unit vectors $\mathbf{n}_i$ and $\mathbf{a}_i$ (i=x, y, z) are fixed in N and A, respectively, with $\mathbf{n}_x$ horizontally rights, $\mathbf{n}_y$ vertically upward, $\mathbf{a}_x$ directed from A_o to Q, and $\mathbf{n}_z = \mathbf{a}_z$ parallel to the revolute joint axis.

The following identifiers are useful in this analysis

Quantity	Identifier	Type
Distance between A_o and A_{cm}	w	constant
Distance between A_o and Q	L	constant
Distance between A_o and P	h	constant
Mass of A (includes roadway and counterweight)	m	constant
Local gravitational constant	g	constant
Angle between $\mathbf{n}_x$ and $\mathbf{a}_x$	θ	specified
Distance between P and Q	d	variable
Tension in cable	T	variable
$\mathbf{n}_x$ measure of reaction force on A at A_o	F_x	variable
$\mathbf{n}_y$ measure of reaction force on A at A_o	F_y	variable



${}^{a}R^{n}$	$\mathbf{n}_x$	$\mathbf{n}_y$	$\mathbf{n}_z$
$\mathbf{a}_x$			
$\mathbf{a}_y$			
$\mathbf{a}_z$			

(a) **(5 pts.)** Complete the previous ${}^{a}R^{n}$ rotation table.

(b) **(5 pts.)** Form the **shortest expression possible** for $\mathbf{r}^{P/Q}$, the position vector of P from Q.
Result:

$$\mathbf{r}^{P/Q} \quad =$$

(c) **(5 pts.)** Form an expression for the distance d between P and Q in terms of h, L, and θ.
Result:

$$d \;=$$

(d) **(4 pts.)** Verify the unit vector $\mathbf{u}$ directed from Q to P can be expressed in terms of L, d, h, and the unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ and $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ as shown below.
Note: Use the boxed expression for $\mathbf{u}$ shown below to simplify subsequent answers.

$$\boxed{\mathbf{u} \;=\; \frac{-L}{d}\,\mathbf{a}_x \;+\; \frac{h}{d}\,\mathbf{n}_y}$$

(e) **(8 pts.)** The resultant of all contact and distance forces on the road-way A is

$$\mathbf{F}^A \;=\; F_x\,\mathbf{n}_x \;+\; F_y\,\mathbf{n}_y \;+\; T\,\mathbf{u} \;-\; m\,g\,\mathbf{n}_y$$

Knowing the bridge is in **static equilibrium**, solve for F_x and F_y in terms of T, m, g, h, L, d, and θ.
Result:

$$F_x \;=$$

$$F_y \;=$$

(f) **(15 pts.)** Find the moment of all forces on A about A_o.
Result:

$$\mathbf{M}^{A/A_o} =$$

(g) **(2 pts.)** Use $\mathbf{M}^{A/A_o}$ to solve for T in terms of m, g, w, d, h, and L.
Result:

$$T =$$

(h) **(2 pts.)** The maximum tension in the cable occurs when $\theta = \quad ^\circ$.

(i) **(3 pts.)** The cable tension is **nonlinear** in (circle all applicable quantities)

w L h m g d θ

(j) **(2 pts.)** Many draw-bridges have a counterweight because:

Physical explanation:

Mathematical explanation: