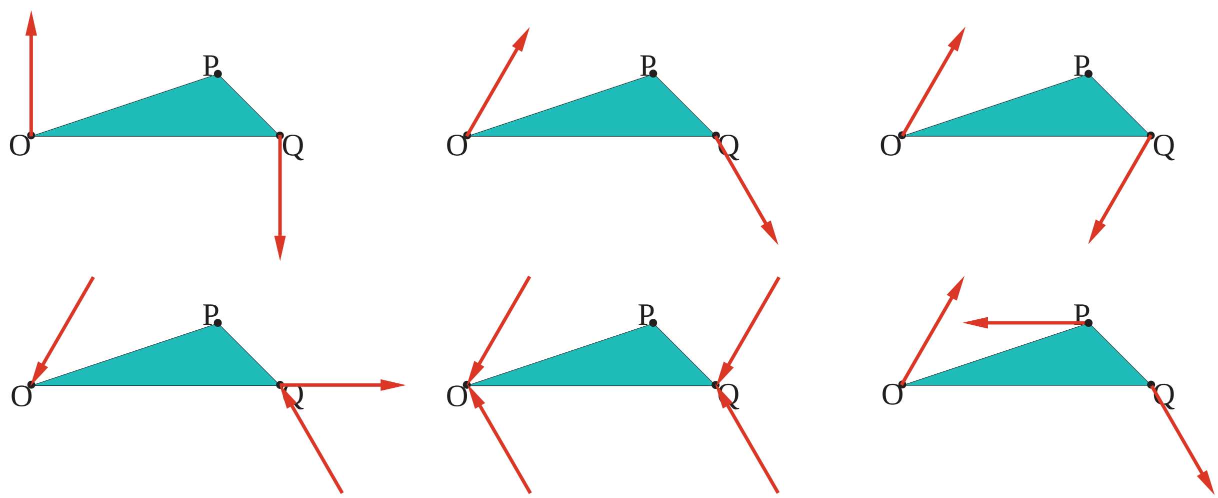
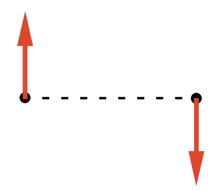
13.1 Moments of forces about various points

Consider the following sets of forces. Circle the set(s) in which the following are all equal:
Note: All forces have the same magnitude. Forces that are not horizontal or vertical are 30° from vertical.

- Moment of the set around point O
- Moment of the set around point P
- Moment of the set around point Q

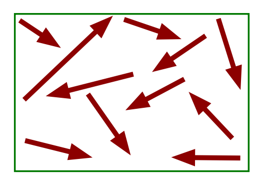


13.2 Resultants, moments, and torques



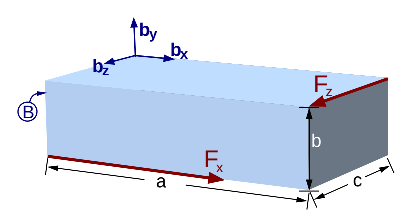
Prove that the couple of forces shown to the right cannot be replaced by just its resultant, no matter where the resultant is applied.

13.3 Optional**: Resultants, moments, and torques



Prove that a set S of ***coplanar*** forces (other than a couple) can be replaced by its resultant as long as the resultant's line of action lies along a certain line (called the ***central axis*** of S).

13.4 Resultants, moments, and torques



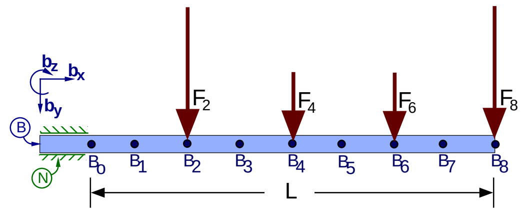
Prove that the set of two forces $\mathbf{F}_x$ and $\mathbf{F}_z$ shown to the right cannot be replaced by its resultant, no matter where the resultant is applied.

13.5 Cantilever beam with end load: Shear force and bending moment diagram.

The following figure shows points B_i (i=0, ..., 8) uniformly distributed along a relatively light (***massless***) rigid horizontal beam B of length $L = 8$ m. The beam is loaded with vertically downward forces of magnitude F_i ($i = 2, 4, 6, 8$) at points B_i ($i = 2, 4, 6, 8$), respectively, with $F_2 = F_8 = 2000$ N and $F_4 = F_6 = 1000$ N.

The beam is cantilevered to ground (a Newtonian reference frame N) which means that:

- Point B_0 of B cannot ***translate*** relative to N
- Beam B cannot ***rotate*** relative to N



Right-handed orthogonal unit vectors $\mathbf{b}_x, \mathbf{b}_y, \mathbf{b}_z$ are fixed in B with $\mathbf{b}_x$ horizontally right and $\mathbf{b}_y$ vertically downward.

At point B_i of the beam, the beam can be "sliced" into two sections namely the beam-section containing point B_i and the beam to the left of B_i and the beam-section to the right of B_i. The cut is made to create a planar cross-section normal to $\mathbf{b}_x$.

Since B_i cannot ***translate*** relative to the right-beam-section and since the left-beam-section cannot ***rotate*** relative to the right-beam section, one knows the set $\bar{S}$ of forces exerted on B_i's left-beam-section by its right-beam-section can be replaced by an equivalent set consisting of:

- A force $\mathbf{F}^{B_i}$ applied to B_i that is equal to the resultant of $\bar{S}$.
  Note: The vertical component of $\mathbf{F}^{B_i}$ is called a ***shear force***.
  Note: The horizontal component of $\mathbf{F}^{B_i}$ is called an ***axial force***.
- A couple whose torque $\mathbf{M}^{\bar{S}/B_i}$ is equal to the moment of $\bar{S}$ about B_i.
  Note: The $\mathbf{b}_z$ component of $\mathbf{M}^{\bar{S}/B_i}$ is called a ***bending moment***.

(a) Determine $\mathbf{F}^{B_0}$, the resultant of the set $\bar{S}$ of forces exerted on B_0's left-beam-section by its right-beam-section, and determine $\mathbf{M}^{\bar{S}/B_0}$, the moment of $\bar{S}$ about B_0.
Result:

$$\mathbf{F}^{B_0} = (F_2 + F_4 + F_6 + F_8)\,\mathbf{b}_y$$
$$\mathbf{M}^{\bar{S}/B_0} = \left(\frac{L}{4}\,F_2 + \frac{L}{2}\,F_4 + \frac{3\,L}{4}\,F_6 + L\,F_8\right)\mathbf{b}_z$$

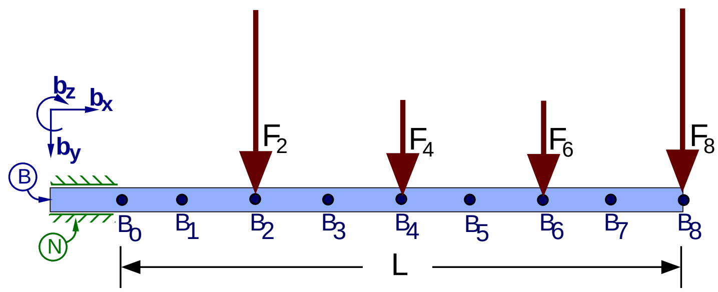
(b) Determine $\mathbf{F}^{B_1}$, the resultant of the set $\bar{S}$ of forces exerted on B_1's left-beam-section by its right-beam-section, and determine $\mathbf{M}^{\bar{S}/B_1}$, the moment of $\bar{S}$ about B_1.
Result:

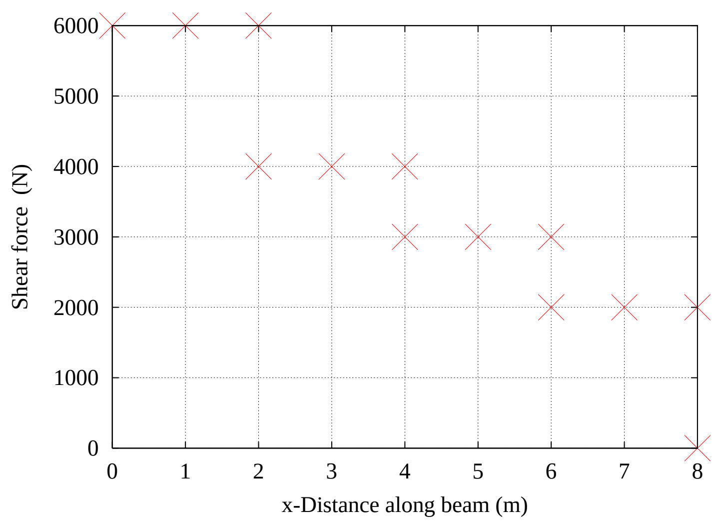
$$\mathbf{F}^{B_1} = (F_2 + F_4 + F_6 + F_8)\,\mathbf{b}_y$$
$$\mathbf{M}^{\bar{S}/B_1} = \left(\frac{L}{8}\,F_2 + \frac{3\,L}{8}\,F_4 + \frac{5\,L}{8}\,F_6 + \frac{7\,L}{8}\,F_8\right)\mathbf{b}_z$$

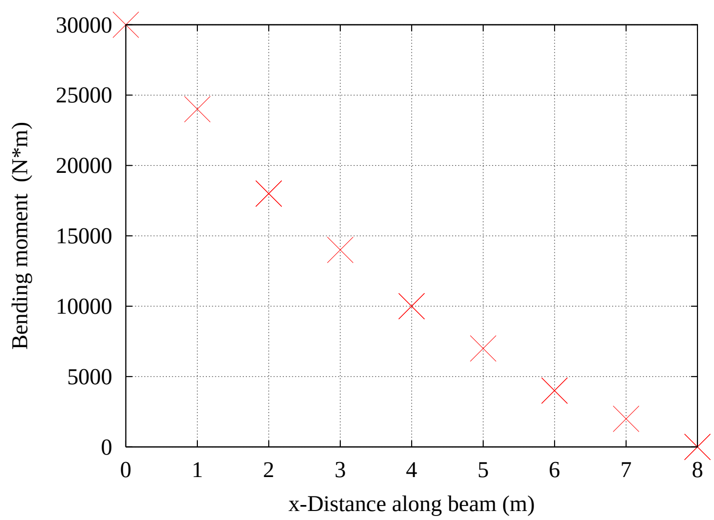
(c) Complete the following table with numerical values. For the points marked with

Name of point P	Resultant of set $\bar{S}$ of forces exerted on P's left-beam-section by its right-beam-section (N)	Moment of $\bar{S}$ about P (N m)
B_0	6000	30000
B_1	6000	24000
B_2	4000	18000
B_3		
B_4		
B_5		
B_6		
B_7		
B_8	0	0

(d) Data from the previous table is marked with an **X** on the following "***shear-force diagram***" and "***bending-moment diagram***". Complete these diagrams by properly connecting the data so that shear-force diagram looks like "steps" whereas the data in the bending-moment diagrams is connected with lines.



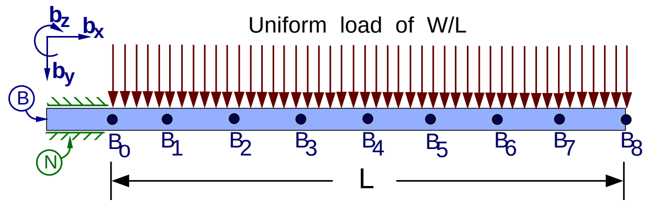




13.6 Cantilever beam with uniform load: Shear force and bending moment diagram.

The following figure shows points B_i (i=0, ..., 8) uniformly distributed along a relatively heavy (***massive***) rigid horizontal beam B of length $L = 8$ m. The beam is uniformly loaded vertically downward due to its own weight. The beam's weight per unit length is $\frac{W}{L}\,\frac{\text{N}}{\text{m}}$ where $W = 800$ kg.

The beam is cantilevered to ground (a Newtonian reference frame N), Right-handed orthogonal unit vectors $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ are fixed in B with $\mathbf{b}_x$ horizontally right and $\mathbf{b}_y$ vertically downward.



At point B_i of the beam, the beam can be "sliced" into two sections namely the beam-section containing point B_i and the beam to the left of B_i and the beam-section to the right of B_i. The cut is made to create a planar cross-section normal to $\mathbf{b}_x$.

(a) Determine $\mathbf{F}^{B_0}$, the resultant of the set $\bar{S}$ of forces exerted on B_0's left-beam-section by its right-beam-section, and determine $\mathbf{M}^{\bar{S}/B_0}$, the moment of $\bar{S}$ about B_0. Repeat for point B_4.
Result:

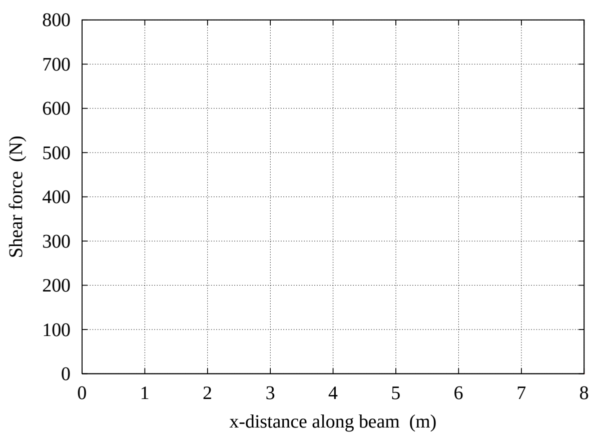
$$\mathbf{F}^{B_0} = W\,\mathbf{b}_y \qquad \mathbf{M}^{\bar{S}/B_0} = \frac{W\,L}{2}\,\mathbf{b}_z$$

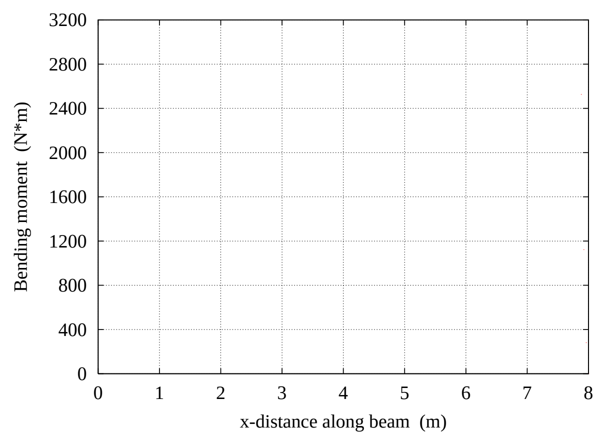
$$\mathbf{F}^{B_4} = \frac{W}{2}\,\mathbf{b}_y \qquad \mathbf{M}^{\bar{S}/B_4} = \frac{W\,L}{8}\,\mathbf{b}_z$$

(b) Consider a point $\mathbf{F}^{B_i}$ located a distance x to the right of B_0. Determine $\mathbf{F}^{B_i}$, the resultant of the set $\bar{S}$ of forces exerted on B_i's left-beam-section by its right-beam-section, and determine $\mathbf{M}^{\bar{S}/B_4}$, the moment of $\bar{S}$ about B_4. Express these results in terms of W, L, and x.
Result:

$$\mathbf{F}^{B_i} = W\left(\square - \square\right)\mathbf{b}_y \qquad \mathbf{M}^{\bar{S}/B_i} = \frac{W}{2\,L}\left(\square - \square\right)^2\mathbf{b}_z$$

(c) Draw the "***shear-force diagram***" and "***bending-moment diagram***".





13.7 Problems from Sheri Sheppard's textbook.

Problem 9.4.3
Problem 9.4.4

13.8 Mechanical advantage of pulley systems.

This problem investigates the mechanical advantage of several pulley systems.

The following figure shows several pulley systems that are attached to a rigid ceiling and support a heavy engine block. The pulleys and rope are relatively light (massless), and the pulley are frictionless (hence the tension is the same on either side of the pulley). For each system, draw a suitable number of free-body diagram(s) to determine each pulley system's mechanical advantage. Ensure each free-body diagram clearly

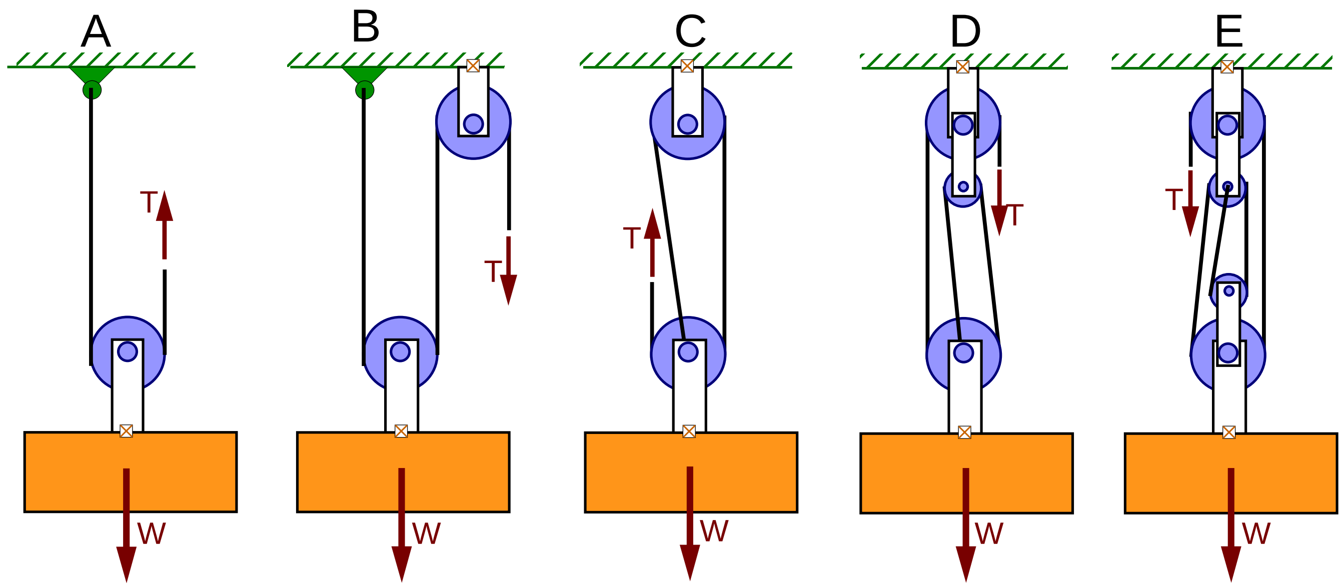
- Identifies the system being analyzed
- Identifies the system's external **contact** and **distance** forces

System	Mechanical Advantage
A	2
B	
C	
D	
E	

Determine each pulley's mechanical advantage defined as

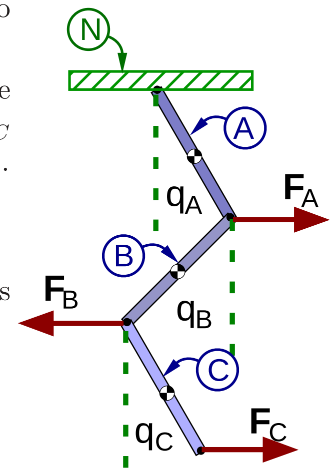
$$\text{Mechanical Advantage} \triangleq \frac{|\text{ Output force }|}{|\text{ Input force }|} = \frac{T}{W}$$

where T is the tension in the pulley's rope and W is the weight of the engine block.



13.9 Static equilibrium of a triple-pendulum.

A system S consists of 3 frictionless pin-connected rods A, B, and C, each of mass m and length L, suspended from a horizontal ceiling N. Horizontally-directed forces of magnitude F_A, F_B, and F_C are applied to ***distal*** (outboard) ends of rods A, B, and C, respectively as shown.



Knowing that S is in **static equilibrium**, find three equations that relate m, L, F_A, F_B, F_C, and g (the local gravitational constant) with q_A, q_B, and q_C (angles between the local vertical and the long-axes of rods A, B, and C, respectively).

To do this most efficiently,

- Use only **three** free-body-diagrams
- Use replacement and equivalent sets of forces to simplify your results
- Start with a free-body-diagram of rod C

Result:

$$(F_A + F_C - F_B)\cos(q_A) = 2.5\, m\, g\, \sin(q_A)$$

$$\boxed{} = \boxed{}$$

$$F_C \cos(q_C) = 0.5\, m\, g\, \sin(q_C)$$

Knowing $F_A = 30$ n, $F_B = 20$ n, $F_C = 10$ n, $m = 1$ kg, and $g = 9.8$ m/sec^2. determine q_A, q_B and q_C.

Result:

$q_A = \boxed{}^\circ \qquad q_B = 34.2^\circ \qquad q_C = \boxed{}^\circ$

The set of forces exerted by N on A across the light (massless) revolute joint (pin) is equivalent to a force $\mathbf{F}^{A/N}$ applied to the ***proximal*** (inboard) end of A. This force is written in terms of the unknown ***force variables*** F_x^A and F_y^A as shown below. Similarly forces exerted by A on B and B on C are replaced, resulting in the following expressions.

$$\mathbf{F}^{A/N} = F_x^A\, \mathbf{n}_\mathrm{x} + F_y^A\, \mathbf{n}_\mathrm{y}$$
$$\mathbf{F}^{B/A} = F_x^B\, \mathbf{n}_\mathrm{x} + F_y^B\, \mathbf{n}_\mathrm{y}$$
$$\mathbf{F}^{C/B} = F_x^C\, \mathbf{n}_\mathrm{x} + F_y^C\, \mathbf{n}_\mathrm{y}$$

Find these force variables in terms of m, g, F_A, F_B, and F_C.

Result:

$F_x^A = \boxed{} \qquad F_y^A = 3\, m\, g$

$F_x^B = F_B - F_C \qquad F_y^B = \boxed{}$

$F_x^C = \boxed{} \qquad F_y^C = m\, g$

It was easier for me to find the **angles/forces** (circle one).