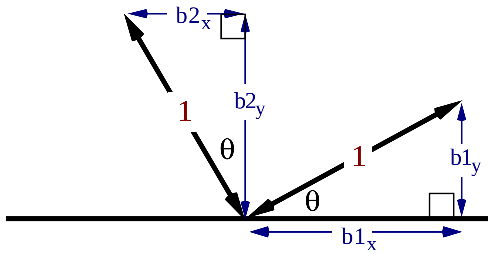
2.1 Sine and cosine review.

Trigonometry plays a central role in kinematics, particularly in the formation of ***rotation matrices***. Referring to the figure below, express $b1_x$, $b1_y$, $b2_x$, and $b2_y$ in terms of $\sin(\theta)$ and $\cos(\theta)$.



$b1_x =$ $\qquad$ $b1_y =$

$b2_x =$ $\qquad$ $b2_y =$

2.2 Right-handed, orthogonal, unitary basis.

Draw a right-handed orthogonal (mutually perpendicular) basis consisting of the unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$.

2.3 Perpendicular vectors.

The vectors $\mathbf{v}_1 = x\,\mathbf{a}_x + 2\,\mathbf{a}_y + 3\,\mathbf{a}_z$ and $\mathbf{v}_2 = 4\,\mathbf{a}_x + 5\,\mathbf{a}_y + 6\,\mathbf{a}_z$ are expressed in terms of orthogonal unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$.
Find the value of x so $\mathbf{v}_1$ and $\mathbf{v}_2$ are perpendicular.
Result:

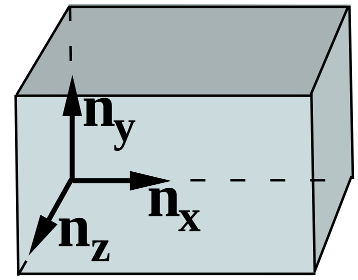
$$x =$$

2.4 Column matrices and vectors.

The column matrix $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is identical to the vector $\mathbf{a}_x + 2\,\mathbf{a}_y + 3\,\mathbf{a}_z$. **True/False**.

2.5 Calculating vector dot products with bases.

The figure to the right shows a right-handed (dextral) set of orthogonal unit vectors $\mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{y}$, $\mathbf{n}_\mathrm{z}$. The vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are defined as:



$$\begin{aligned}\mathbf{u} &= 2\,\mathbf{n}_\mathrm{x} + 3\,\mathbf{n}_\mathrm{y} + 4\,\mathbf{n}_\mathrm{z}\\ \mathbf{v} &= x\,\mathbf{n}_\mathrm{x} + y\,\mathbf{n}_\mathrm{y} + z\,\mathbf{n}_\mathrm{z}\\ \mathbf{w} &= 5\,\mathbf{n}_\mathrm{x} - 6\,\mathbf{n}_\mathrm{y} + 7\,\mathbf{n}_\mathrm{z}\end{aligned}$$

(a) Use the distributive law for dot products to write $\mathbf{u}\cdot\mathbf{v}$ in terms of $\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{y}$, $\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{z}$, etc.
Result:

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} \;=\;& 2\,x\,\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{x} \;+\; 2\,y\,\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{y} \;+\; 2\,z\,\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{z}\\ +&\qquad\qquad\qquad +\qquad\qquad\qquad +\\ +&\qquad\qquad\qquad +\qquad\qquad\qquad +\end{aligned}$$

(b) Use the definition of the dot product to calculate $\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{y}$, etc.
Result:

$\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{x} =$ $\qquad$ $\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{y} =$ $\qquad$ $\mathbf{n}_\mathrm{x}\cdot\mathbf{n}_\mathrm{z} =$

$\mathbf{n}_\mathrm{y}\cdot\mathbf{n}_\mathrm{x} =$ $\qquad$ $\mathbf{n}_\mathrm{y}\cdot\mathbf{n}_\mathrm{y} =$ $\qquad$ $\mathbf{n}_\mathrm{y}\cdot\mathbf{n}_\mathrm{z} =$

$\mathbf{n}_\mathrm{z}\cdot\mathbf{n}_\mathrm{x} =$ $\qquad$ $\mathbf{n}_\mathrm{z}\cdot\mathbf{n}_\mathrm{y} =$ $\qquad$ $\mathbf{n}_\mathrm{z}\cdot\mathbf{n}_\mathrm{z} =$

(c) In view of your previous two results, calculate $\mathbf{u}\cdot\mathbf{v}$.
Result:

$$\mathbf{u}\cdot\mathbf{v} =$$

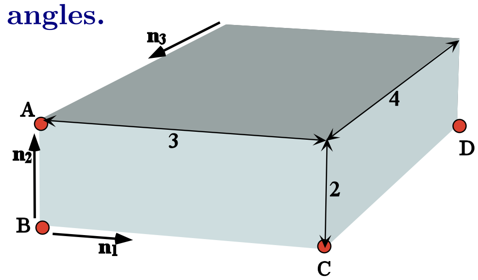
(d) As shown in Section 2.10.3, the dot product $\mathbf{u}\cdot\mathbf{v}$ is relatively easy to calculate when $\mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{y}$, $\mathbf{n}_\mathrm{z}$ are ***orthogonal unit*** vectors. When two arbitrary vectors $\mathbf{a}$ and $\mathbf{b}$ are expressed in terms of ***orthogonal unit*** vectors as shown below, the dot product $\mathbf{a}\cdot\mathbf{b}$ can be calculated as

$$\begin{aligned}\mathbf{a} &= a_x\,\mathbf{n}_\mathrm{x} + a_y\,\mathbf{n}_\mathrm{y} + a_z\,\mathbf{n}_\mathrm{z}\\ \mathbf{b} &= b_x\,\mathbf{n}_\mathrm{x} + b_y\,\mathbf{n}_\mathrm{y} + b_z\,\mathbf{n}_\mathrm{z}\\ \mathbf{a}\cdot\mathbf{b} &= a_x\,b_x + a_y\,b_y + a_z\,b_z\end{aligned}$$

In view of this short-cut, calculate $\mathbf{u}\cdot\mathbf{v}$, $\mathbf{u}\cdot\mathbf{w}$, and $\mathbf{v}\cdot\mathbf{w}$.
Result:

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= 2\,x + 3\,y + 4\,z\\ \mathbf{u}\cdot\mathbf{w} &=\\ \mathbf{v}\cdot\mathbf{w} &=\end{aligned}$$

2.6 Definition of a dot product and its use for calculating angles.



The figure to the right shows a rectangular parallelepiped (block) of sides 2, 3, and 4. Unit vectors $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ are directed along the sides of the block as shown. The points A, B, C and D are located at corners of the block.

(a) Express $\mathbf{r}^{C/A}$, the position vector of C from A, in terms of $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$.
Result:

$$\mathbf{r}^{C/A} \;=$$

(b) Find a numerical value for $\mathbf{r}^{C/A} \cdot \mathbf{r}^{C/A}$.
Next, use equation (2.3) to calculate the magnitude of $\mathbf{r}^{C/A}$ (the distance from A to C).
Result:

$$\mathbf{r}^{C/A} \cdot \mathbf{r}^{C/A} \;= \qquad\qquad \left|\mathbf{r}^{\mathbf{C/A}}\right| \;=$$

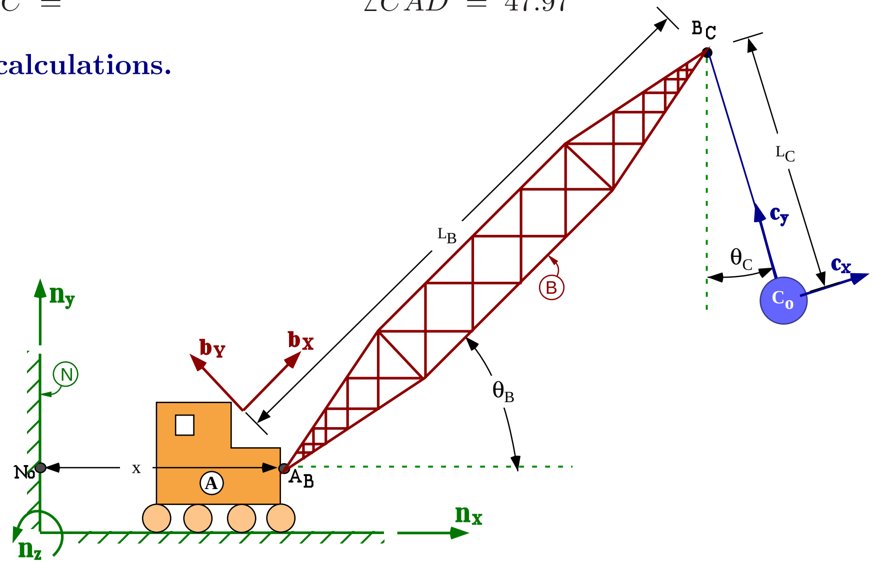
(c) Using equation (2.1), calculate the unit vector $\mathbf{u}$ directed from A to C in terms of $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$.
Next, find the unit vector $\mathbf{v}$ directed from A to D in terms of $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$.
Result:

$$\mathbf{u} \;=\; \frac{3\,\mathbf{n}_1 \;-\; 2\,\mathbf{n}_2}{\sqrt{13}} \qquad\qquad \mathbf{v} \;=$$

(d) Calculate $\angle BAC$, the angle between line AB and line AC.
Next, calculate $\angle CAD$, the angle between line AC and line AD.
Result:

$$\angle BAC \;= \qquad ^\circ \qquad\qquad \angle CAD \;=\; 47.97^\circ$$

2.7 Dot products and distance calculations.



The figure to the right shows a crane whose cab A supports a boom B that swings a wrecking ball C_o. To prevent the wrecking ball from accidently destroying nearby cars, the distance between the nearest car, point N_o, and the tip of the boom, point BC, must be controlled.

(a) Express the position vector of BC from N_o in terms of x, L_B, and the unit vectors $\mathbf{n}_\mathrm{x}$, and $\mathbf{b}_\mathrm{x}$.
Result:

$$\mathbf{r}^{BC/N_o} \;= \qquad + $$

(b) Using the distributive property for dot-multiplication of vectors, i.e.,

$$(\mathbf{a}+\mathbf{b}) \cdot (\mathbf{c}+\mathbf{d}) \;=\; \mathbf{a}\cdot\mathbf{c} \;+\; \mathbf{a}\cdot\mathbf{d} \;+\; \mathbf{b}\cdot\mathbf{c} \;+\; \mathbf{b}\cdot\mathbf{d}$$

express $\mathbf{r}^{BC/N_o} \cdot \mathbf{r}^{BC/N_o}$ in terms of x, L_B, and $\mathbf{n}_\mathrm{x} \cdot \mathbf{b}_\mathrm{x}$.
Result:

$$\mathbf{r}^{BC/N_o} \cdot \mathbf{r}^{BC/N_o} \;=$$

(c) Using the ***definition*** of the dot-product in equation (2.2), calculate $\mathbf{n}_x \cdot \mathbf{b}_x$.
Result:

$$\mathbf{n}_x \cdot \mathbf{b}_x \;=$$

(d) Using your previous two results, rewrite $\mathbf{r}^{BC/N_o} \cdot \mathbf{r}^{BC/N_o}$ in terms of x, L_B, and θ_B.
Result:

$$\mathbf{r}^{BC/N_o} \cdot \mathbf{r}^{BC/N_o} \;=$$

(e) Using equation (2.3) to calculate the magnitude of $\mathbf{r}^{BC/N_o}$, express the distance from N_o to BC in terms of x, L_B, and θ_B, and calculate its value when x=20, L_B=10, and θ_B=30°.
Result:

$$\left|\mathbf{r}^{\mathbf{BC/N_o}}\right| \;=\; \qquad\qquad\qquad\qquad \;=\; 29.1$$

(f) Two colleagues are confused by your use of ***mixed-bases*** vectors (i.e., $\mathbf{r}^{BC/N_o} = x\,\mathbf{n}_x + L_B\,\mathbf{b}_x$), and ask you to verify the position vector of B from N_o can be expressed in the ***uniform-basis*** as shown below. Use this uniform-basis expression to verify your previous result for $\left|\mathbf{r}^{\mathbf{BC/N_o}}\right|$. Note: This uniform-basis approach necessitates the simplifying trigonometric identity $\sin^2(\theta_B) + \cos^2(\theta_B) = 1$.
Result:

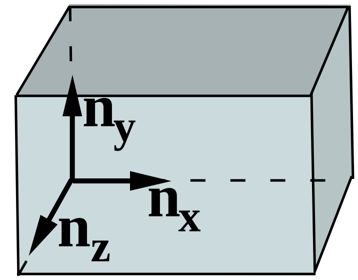
$$\mathbf{r}^{BC/N_o} \;=\; [x + L_B\,\cos(\theta_B)]\,\mathbf{n}_x \;+\; L_B\,\sin(\theta_B)\,\mathbf{n}_y$$

(g) Optional**: Calculate the distance from N_o to C_o in terms of x, L_B, L_C, θ_B, and θ_C.
Result:

$$\left|\mathbf{r}^{\mathbf{C_o/N_o}}\right| \;=$$

2.8 Calculating vector cross products with bases.

The figure to the right shows a right-handed set of orthogonal unit vectors $\mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{y}$, $\mathbf{n}_\mathrm{z}$. The vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are defined as:



$$\begin{aligned} \mathbf{u} &= 2\,\mathbf{n}_\mathrm{x} + 3\,\mathbf{n}_\mathrm{y} + 4\,\mathbf{n}_\mathrm{z} \\ \mathbf{v} &= x\,\mathbf{n}_\mathrm{x} + y\,\mathbf{n}_\mathrm{y} + z\,\mathbf{n}_\mathrm{z} \\ \mathbf{w} &= 5\,\mathbf{n}_\mathrm{x} - 6\,\mathbf{n}_\mathrm{y} + 7\,\mathbf{n}_\mathrm{z} \end{aligned}$$

(a) Use the distributive law for cross products to write $\mathbf{u} \times \mathbf{v}$ in terms of $\mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{y}$, etc.
Result:

$$\begin{aligned} \mathbf{u} \times \mathbf{v} \;=\;& 2\,x\,\mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{x} \;+\; 2\,y\,\mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{y} \;+\; 2\,z\,\mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{z} \\ &+ \qquad\qquad + \qquad\qquad + \\ &+ \qquad\qquad + \qquad\qquad + \end{aligned}$$

(b) Use the definition of the cross product to calculate $\mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{y}$, etc.
Result:

$$\begin{array}{lll} \mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{x} = \mathbf{0} & \mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{y} = \mathbf{n}_\mathrm{z} & \mathbf{n}_\mathrm{x} \times \mathbf{n}_\mathrm{z} = \text{-}\mathbf{n}_\mathrm{y} \\ \mathbf{n}_\mathrm{y} \times \mathbf{n}_\mathrm{x} = & \mathbf{n}_\mathrm{y} \times \mathbf{n}_\mathrm{y} = & \mathbf{n}_\mathrm{y} \times \mathbf{n}_\mathrm{z} = \\ \mathbf{n}_\mathrm{z} \times \mathbf{n}_\mathrm{x} = & \mathbf{n}_\mathrm{z} \times \mathbf{n}_\mathrm{y} = & \mathbf{n}_\mathrm{z} \times \mathbf{n}_\mathrm{z} = \end{array}$$

(c) In view of your previous two results, calculate $\mathbf{u} \times \mathbf{v}$.
Result:

$$\mathbf{u} \times \mathbf{v} =$$

(d) Using the determinant method for calculating the cross product proved in Homework 2.9, calculate $\mathbf{u} \times \mathbf{v}$, $\mathbf{u} \times \mathbf{w}$, and $\mathbf{v} \times \mathbf{w}$.
Result:

$$\begin{aligned} \mathbf{u} \times \mathbf{v} &= (3\,z - 4\,y)\,\mathbf{n}_\mathrm{x} + (4\,x - 2\,z)\,\mathbf{n}_\mathrm{y} + (2\,y - 3\,x)\,\mathbf{n}_\mathrm{z} \\ \mathbf{u} \times \mathbf{w} &= \\ \mathbf{v} \times \mathbf{w} &= \end{aligned}$$

2.9 Cross products and determinants.

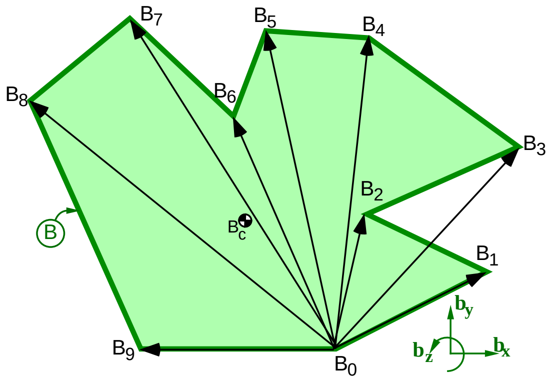
Given **right-handed orthogonal unit** vectors $\mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{y}$, $\mathbf{n}_\mathrm{z}$ and two arbitrary vectors $\mathbf{a}$ and $\mathbf{b}$ that are expressed in terms of $\mathbf{n}_\mathrm{x}$, $\mathbf{n}_\mathrm{y}$, $\mathbf{n}_\mathrm{z}$ as shown to the right, prove that calculating $\mathbf{a} \times \mathbf{b}$ with the distributive property of the cross product happens to be equal to the determinant of the matrix shown to the right.

$$\begin{aligned} \mathbf{a} &= a_x\,\mathbf{n}_\mathrm{x} + a_y\,\mathbf{n}_\mathrm{y} + a_z\,\mathbf{n}_\mathrm{z} \\ \mathbf{b} &= b_x\,\mathbf{n}_\mathrm{x} + b_y\,\mathbf{n}_\mathrm{y} + b_z\,\mathbf{n}_\mathrm{z} \end{aligned}$$

$$\mathbf{a} \times \mathbf{b} = \det \begin{bmatrix} \mathbf{n}_\mathrm{x} & \mathbf{n}_\mathrm{y} & \mathbf{n}_\mathrm{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$

2.10 Cross products and area calculations.

One reason that triangles are important is that complex **planar objects** can be decomposed into triangles. For example, the polygon B in the figure below can be decomposed into triangles. Knowing the area of two-dimensional objects is helpful in various professions. For example, area measurements are necessary in calculating the acreage and costs associated with building and farming. Knowing the mass properties of a polygon is helpful in determining the motion of two-dimensional objects.



$$
\begin{aligned}
\mathbf{r}^{B_1/B_0} &= 2.0\,\mathbf{b}_\text{x} + 2.0\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_2/B_0} &= 0.5\,\mathbf{b}_\text{x} + 2.5\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_3/B_0} &= 3.0\,\mathbf{b}_\text{x} + 4.0\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_4/B_0} &= 0.2\,\mathbf{b}_\text{x} + 6.0\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_5/B_0} &= -0.5\,\mathbf{b}_\text{x} + 7.0\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_6/B_0} &= -1.0\,\mathbf{b}_\text{x} + 5.0\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_7/B_0} &= -2.0\,\mathbf{b}_\text{x} + 7.0\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_8/B_0} &= -4.0\,\mathbf{b}_\text{x} + 5.0\,\mathbf{b}_\text{y}\\
\mathbf{r}^{B_9/B_0} &= -2.0\,\mathbf{b}_\text{x} + 0.0\,\mathbf{b}_\text{y}
\end{aligned}
$$

One way to calculate the area of an arbitrary polygon B such as the one shown above is to:

- Label a vertex B_0 and number the remaining vertices sequentially in a counter-clockwise fashion.
- Form $\mathbf{r}^{B_i/B_0}$, the position vector of vertex B_i $(i = 1, 2, \ldots)$ from vertex B_0
- Calculate $\mathbf{A}_1$, the "vector-area" of the triangle defined by vertices B_0, B_1, and B_2. Similarly, calculate $\mathbf{A}_2$, $\mathbf{A}_3$, $\ldots$ $\mathbf{A}_8$, the vector-areas of the triangles defined by vertices $B_0B_2B_3$, $B_0B_3B_4$, $\ldots B_0\,B_8\,B_9$, respectively. The formula for the vector-area of a triangle is

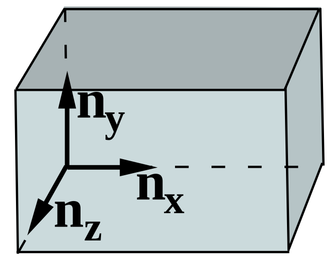
$$
\begin{aligned}
\mathbf{A}_1 &= 1/2 * \mathbf{r}^{B_1/B_0} \times \mathbf{r}^{B_2/B_0} &&= 2\,\mathbf{b}_\text{z}\\
\mathbf{A}_2 &= 1/2 * \mathbf{r}^{B_2/B_0} \times \mathbf{r}^{B_3/B_0} &&=\\
\mathbf{A}_3 &= \ldots &&= 8.6\,\mathbf{b}_\text{z}\\
\mathbf{A}_4 &= \ldots &&=\\
\mathbf{A}_5 &= \ldots &&= 2.25\,\mathbf{b}_\text{z}\\
\mathbf{A}_6 &= \ldots &&= 1.5\,\mathbf{b}_\text{z}\\
\mathbf{A}_7 &= \ldots &&= 9\,\mathbf{b}_\text{z}\\
\mathbf{A}_8 &= 1/2 * \mathbf{r}^{B_8/B_0} \times \mathbf{r}^{B_9/B_0} &&=
\end{aligned}
$$

- Calculate $\mathbf{A} = \sum_{i=1}^{8} \mathbf{A}_\text{i} =$
- The polygon's area is the magnitude of $\mathbf{A}$, i.e., $Area = 27.8$.

Fill in the previous blanks and determine the polygon's area. Compute cross products with the distributive property $(\mathbf{a}+\mathbf{b}) \times (\mathbf{c}+\mathbf{d}) = \mathbf{a} \times \mathbf{c} + \mathbf{a} \times \mathbf{d} + \mathbf{b} \times \mathbf{c} + \mathbf{b} \times \mathbf{d}$ and its **definition with the right-hand rule** (do not use determinants or look up special formulas in a book). Also, use the fact that $\mathbf{b}_\text{x}$, $\mathbf{b}_\text{y}$, $\mathbf{b}_\text{z}$ are orthogonal unit vectors.

2.11 Scalar triple product with bases.

The figure to the right shows a right-handed set of orthogonal unit vectors $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$. The vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are defined as:



$$\mathbf{u} = 2\,\mathbf{n}_x + 3\,\mathbf{n}_y + 4\,\mathbf{n}_z$$
$$\mathbf{v} = x\,\mathbf{n}_x + y\,\mathbf{n}_y + z\,\mathbf{n}_z$$
$$\mathbf{w} = 5\,\mathbf{n}_x - 6\,\mathbf{n}_y + 7\,\mathbf{n}_z$$

Calculate $\mathbf{u} \times \mathbf{v} \cdot \mathbf{u}$, $\mathbf{u} \times \mathbf{v} \cdot \mathbf{w}$, and $\mathbf{u} \cdot \mathbf{v} \times \mathbf{w}$.

Note: Although the order of operations in $\mathbf{u} \times \mathbf{v} \cdot \mathbf{u}$ is unambiguous, parentheses may clarify your work.

Result:

$$\mathbf{u} \times \mathbf{v} \cdot \mathbf{u} =$$
$$\mathbf{u} \times \mathbf{v} \cdot \mathbf{w} =$$
$$\mathbf{u} \cdot \mathbf{v} \times \mathbf{w} = 27\,z - 45\,x - 6\,y$$

In view of your last two results, $\mathbf{u} \times \mathbf{v} \cdot \mathbf{w}$ is **equal/not equal** (circle one) to $\mathbf{u} \cdot \mathbf{v} \times \mathbf{w}$.
It **is/is not** OK to switch the $\cdot$ and $\times$ in the scalar triple product.

2.12 Optional: Scalar triple products and determinants.**

$$\mathbf{a} = a_x\,\mathbf{n}_x + a_y\,\mathbf{n}_y + a_z\,\mathbf{n}_z$$
$$\mathbf{b} = b_x\,\mathbf{n}_x + b_y\,\mathbf{n}_y + b_z\,\mathbf{n}_z$$
$$\mathbf{c} = c_x\,\mathbf{n}_x + c_y\,\mathbf{n}_y + c_z\,\mathbf{n}_z$$

Given **right-handed orthogonal unit** vectors $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ and three arbitrary vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ that are expressed in terms of $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ as shown to the right, prove that calculating $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ happens to be equal to the determinant of the matrix shown to the right.

$$\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

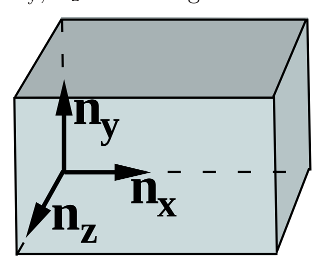
2.13 Constructing unit vectors.

Form the unit vector $\mathbf{u}$ having the same direction as each vector in the table below.

Note: Ensure your answer to the last question agrees with your first two answers, i.e., if $c = 3$ or $c = -3$.

Vector	Unit vector
$3\,\mathbf{n}_x$	$\mathbf{n}_x$
$-3\,\mathbf{n}_x$	
$3\,\mathbf{n}_x - 4\,\mathbf{n}_y$	
$3\,\mathbf{n}_x - 4\,\mathbf{n}_y + 12\,\mathbf{n}_z$	
$c\,\mathbf{n}_x$	

Note: $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ are orthogonal unit vectors.

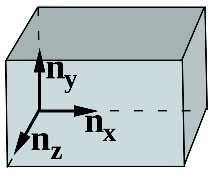


2.14 Getting started with Autolev.

Go to http://www.stanford.edu/class/me331b and click on `Getting Started`. Follow the directions up to `Vector Operations`. Print out and **submit** your `firstDemo.al` file with your homework. Continue through `Vector Operations` and also **submit** your `vectorDemo.al`.

2.15 Vector operations with Autolev.

The figure below shows a right-handed set of orthogonal unit vectors $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$.
Note: To declare x, y, and z as variables, type `Variable x, y, z`



$$\mathbf{u} = 2\,\mathbf{n}_x + 3\,\mathbf{n}_y + 4\,\mathbf{n}_z$$
$$\mathbf{v} = x\,\mathbf{n}_x + y\,\mathbf{n}_y + z\,\mathbf{n}_z$$
$$\mathbf{w} = 5\,\mathbf{n}_x - 6\,\mathbf{n}_y + 7\,\mathbf{n}_z$$

Use the Autolev commands `Cross`, `Dot`, and `UnitVector` to calculate the following quantities. Submit your work via the printed Autolev files `vectorOperations.al` and `vectorOperations.all`.
Note: **Assign** each output quantity in Autolev to a scalar or vector name, e.g., type:
`a = Dot(u>,v>);`   `b = Dot(u>,w>);`   `c = Dot(v>,w>);`   `d> = Cross(u>,v>)`

$\mathbf{u} \cdot \mathbf{v}$ = `Dot( u>, v> )` = $2\,x + 3\,y + 4\,z$

$\mathbf{u} \cdot \mathbf{w}$ =

$\mathbf{v} \cdot \mathbf{w}$ =

$\mathbf{u} \times \mathbf{v}$ = `Cross(u>,v>)` = $(3z - 4y)\,\mathbf{n}_x + (4x - 2z)\,\mathbf{n}_y + (2y - 3x)\,\mathbf{n}_z$

$\mathbf{u} \times \mathbf{w}$ =

$\mathbf{v} \times \mathbf{w}$ =

$\mathbf{u} \times \mathbf{v} \cdot \mathbf{u}$ = `Dot( Cross(u>,v>), u> )` = 0

$\mathbf{u} \times \mathbf{v} \cdot \mathbf{w}$ =

$\mathbf{u} \cdot \mathbf{v} \times \mathbf{w}$ =

`UnitVector` $(3\,\mathbf{n}_x)$ = `UnitVector( 3*Nx> )` = $\mathbf{n}_x$

`UnitVector` $(-\mathbf{n}_x)$ =

`UnitVector` $(3\,\mathbf{n}_x - 4\,\mathbf{n}_y)$ =

`UnitVector` $(3\mathbf{n}_x - 4\mathbf{n}_y + 12\mathbf{n}_z)$ =

`UnitVector` $(x\,\mathbf{n}_x)$ =

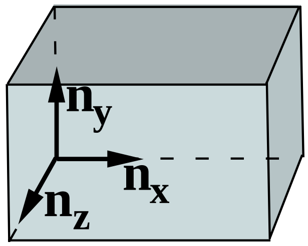
Doing vector operations with Autolev is **easier/harder** than doing it by hand.

2.16 More vector operations with Autolev.

Dot-products and cross-products are fundamental vector operations and are useful for kinematics (motion), mass-distribution calculations, kinetics (forces), statics, and dynamics. Use the Autolev commands `Dot`, `Cross`, `Magnitude`, `UnitVector`, and `AngleBetweenVectors` to perform the following operations and pass in the file `VectorOperationsB.al`.

Note: **Assign** each output quantity in Autolev to a scalar or vector name, e.g., type:

```
a> = 10*v>;        b> = v> / 10;        c> = v> + w>;        d> = v> - w>
```



The figure to the left shows a right-handed set of orthogonal unit vectors $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$. Given below are two vectors $\mathbf{v}$ and $\mathbf{w}$.

$$\mathbf{v} = 2\,\mathbf{n}_x + 3\,\mathbf{n}_y + 4\,\mathbf{n}_z$$
$$\mathbf{w} = 5\,\mathbf{n}_x - 6\,\mathbf{n}_y + 7\,\mathbf{n}_z$$

$$10 * \mathbf{v} =$$

$$\mathbf{v}/10 =$$

$$\mathbf{v} + \mathbf{w} =$$

$$\mathbf{v} - \mathbf{w} =$$

$$\mathbf{v} \cdot \mathbf{w} =$$

$$\mathbf{v} \times \mathbf{w} =$$

$$\mathbf{w} \times \mathbf{v} =$$

$$|\mathbf{w}| =$$

$$|\mathbf{v}| =$$

$$\mathbf{v}^2 =$$

$$\mathbf{v}^3 =$$

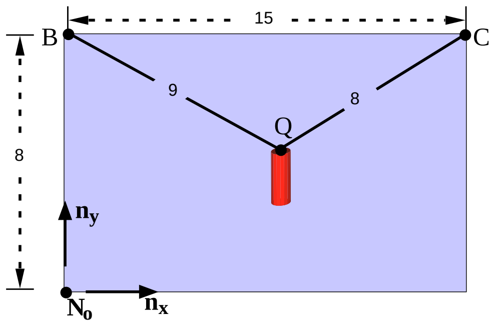
$$\texttt{UnitVector}(\mathbf{v}) = \frac{2\,\mathbf{n}_x + 3\,\mathbf{n}_y + 4\,\mathbf{n}_z}{\sqrt{29}}$$
$$= 0.3714\,\mathbf{n}_x + 0.5571\,\mathbf{n}_y + 0.7428\,\mathbf{n}_z$$

$$\angle(\mathbf{v}, \mathbf{w}) = \qquad \text{radians or} \qquad {}^\circ$$

2.17 Locating a microphone (2D). **Also see Homework 1.20.**

A microphone Q is attached to two pegs B and C by two cables. The point of this practical problem is to determine the distance between Q and point N_o knowing the peg locations, cable lengths, and the fact that B, C, Q, and N_o all lie in the same plane. Introduce whatever **identifiers** facilitate your work and try to do the problem first using Euclidean geometry - and then try vectors.

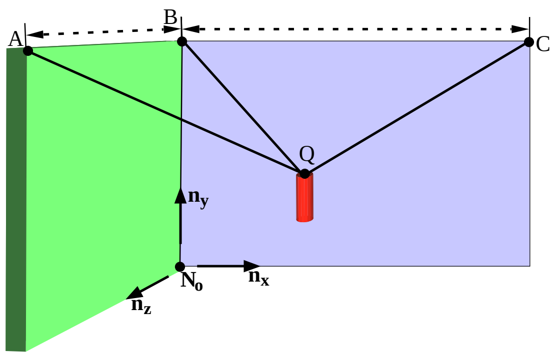
Note: There are at two "mathematical" answers to this problem, but one is above the ceiling and requires the cables to be in compression.



Quantity	Value
Distance from B to C	15 m
Distance from N_o to B	8 m
Length of cable joining B and Q	9 m
Length of cable joining C and Q	8 m
Distance between N_o and Q	9.01 m

2.18 Locating a microphone (3D).

A microphone Q is attached to three pegs A, B, and C by three cables. The point of this practical problem is to determine the distance between Q and point N_o knowing the peg locations, cable lengths, and the fact that the walls are perpendicular ("easy" problem with the right method).[1]



Quantity	Value
Distance from A to B	20 m
Distance from B to C	15 m
Distance from N_o to B	8 m
Length of cable joining A and Q	15 m
Length of cable joining B and Q	13 m
Length of cable joining C and Q	11 m
Distance between N_o and Q	13.3 m

[1] Hint: See Section 2.10.4 and introduce the measures x, y, and z so the position vector of Q from N_o is $x\,\mathbf{n}_x + y\,\mathbf{n}_y + z\,\mathbf{n}_z$. Note: Section 1.11.1 shows how to solve nonlinear algebraic equations. This problem can also be solved "by-hand".