Homework 1. Chapters 2.
Basis independent vectors and their properties

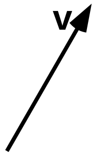
1.1 What is a vector.

Two properties (attributes) of a vector are ______________ and _____________.

1.2 The zero vector.

The zero vector **0** has a magnitude of **0/1/2/∞** and **no/two/all/any-one** direction(s).
All zero vectors are equal. **True/False**.

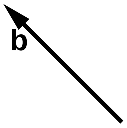
1.3 Drawing vectors: Magnitude, orientation, and sense.



The figure to the right shows a vector **v**.
Draw and label **v** and five more vectors, namely:

- A vector **a** that is equal to **v**
- A vector **b** with the same magnitude and orientation, but different sense
- A vector **c** with the same direction, but different magnitude
- A vector **d** with the same magnitude, but different orientation
- A vector **e** with different magnitude and different direction

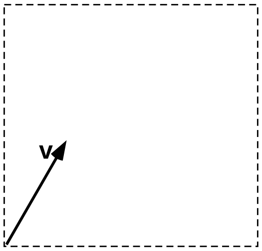
1.4 Negating a vector.



Complete the figure to the right by drawing the vector -**b**.
Negating the vector **b** changes the vector's (circle all that apply)

magnitude direction orientation sense

1.5 Multiplying a vector by a scalar.



Complete the figure to the right by drawing the vectors 2 **v** and -2 **v**.
Multiplying the vector **v** by 2 changes the vector's (circle all that apply)

magnitude direction orientation sense

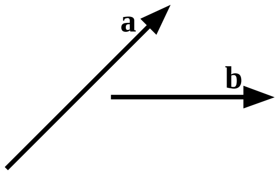
Multiplying the vector **v** by -2 changes the vector's (circle all that apply)

magnitude direction orientation sense

Multiplying the vector **v** by a real scalar can change the vector's ***magnitude*** **True/False**
Multiplying the vector **v** by a real scalar can change the vector's ***direction*** **True/False**
Multiplying the vector **v** by a real scalar can change the vector's ***sense*** **True/False**
Multiplying the vector **v** by a real scalar can change the vector's ***orientation*** **True/False**

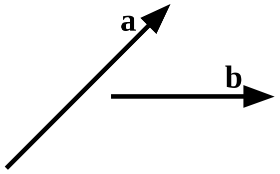
1.6 Graphical vector addition.

Complete the following figure by graphically adding the vectors $\mathbf{a}+\mathbf{b}$ and then $\mathbf{b}+\mathbf{a}$.



1.7 Graphical vector subtraction.

Complete the following figure by drawing the vectors $\mathbf{a}-\mathbf{b}$ and then $\mathbf{b}-\mathbf{a}$.



1.8 Graphical representation of a vector dot-product.

Write the ***definition*** of the dot-product between a vector **a** and a vector **b**.
Include a ***sketch*** with ***each symbol*** in your definition clearly labeled.
Result:

$$\mathbf{a}\cdot\mathbf{b} \triangleq$$

1.9 Graphical representation of a vector cross-product.

Write the ***definition*** of the cross-product between a vector **a** and a vector **b**.
Include a ***sketch*** with ***each symbol*** in your definition labeled and described.
Result:

$$\mathbf{a}\times\mathbf{b} \triangleq$$

where

1.10 Properties of vector dot-products and cross-products.

The following questions apply to a vector **a** and a vector **b**:

When **a** is parallel to **b**:	$\mathbf{a}\cdot\mathbf{b}=0$	**True/False**	$\mathbf{a}\times\mathbf{b}=\mathbf{0}$	**True/False**
When **a** is perpendicular to **b**:	$\mathbf{a}\cdot\mathbf{b}=0$	**True/False**	$\mathbf{a}\times\mathbf{b}=\mathbf{0}$	**True/False**
For arbitrary vectors **a** and **b**:	$\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$	**True/False**	$\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{a}$	**True/False**

1.11 Properties of scalar triple products?

For arbitrary non-zero vectors **a**, **b**, **c**: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$ **Never/Sometimes/Always**

A property of the ***scalar triple product*** is $\mathbf{a}\cdot\mathbf{b}\times\mathbf{a} = 0$. **True/False**.

1.12 Property of vector triple cross-product

Complete the following equation: $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \mathbf{b}(\qquad) - \mathbf{c}(\qquad)$

1.13 Vector exponentiation and $\mathbf{v}^2$.

Write the definition of $\mathbf{v}^2$ in terms of $|\mathbf{v}|$. Use the definition of the dot-product to show how $\mathbf{v} \cdot \mathbf{v}$ can be expressed in terms of $|\mathbf{v}|$. Provide a way to calculate $\mathbf{v}^2$ with a vector dot-product.

Result:

$$\mathbf{v}^2 \triangleq \qquad\qquad\qquad \mathbf{v} \cdot \mathbf{v} = \qquad\qquad\qquad \mathbf{v}^2 = \quad \cdot$$

1.14 Forming a unit vector.

Form the ***unit*** vector $\mathbf{u}$ having the same direction as $c\,\mathbf{a}_{\mathrm{x}}$, where $\mathbf{a}_{\mathrm{x}}$ is a unit vector and c is a real number.

Result:

$$\mathbf{u} =$$

1.15 Coefficient of u in cross products

The ***cross-product*** of vectors $\mathbf{a}$ and $\mathbf{b}$ can be written in terms of a real scalar s as $\mathbf{a} \times \mathbf{b} = s\,\mathbf{u}$ where $\mathbf{u}$ is a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ in a direction dictated by the **right-hand rule**. A colleague tells you that the coefficient s of the unit vector $\mathbf{u}$ is inherently non-negative.

Your colleague is **correct/wrong**.

1.16 Optional**:Proof of magnitude of vector cross product property.

Letting $\boldsymbol{\lambda}$ be a ***unit vector*** and $\mathbf{v}$ be ***any vector***, prove[1] $|\mathbf{v} \times \boldsymbol{\lambda}|^2 = \mathbf{v} \cdot \mathbf{v} - (\mathbf{v} \cdot \boldsymbol{\lambda})^2$.

1.17 Vector operations and units.

Shown below are various vector operations (e.g., scalar multiplication, addition, dot-product, etc.). Circle those operations that are **defined** for a position vector $\mathbf{a}$ (with **units** of m) and a velocity vector $\mathbf{b}$ (with **units** of $\frac{\text{m}}{\text{sec}}$).

$$-\mathbf{a} \qquad 5\,\mathbf{a} \qquad \mathbf{a}/5 \qquad \mathbf{a} + \mathbf{b} \qquad \mathbf{a} \cdot \mathbf{b} \qquad \mathbf{a} \times \mathbf{b}$$

1.18 Using vector identities to simplify expressions.

One reason to treat vectors as ***basis-independent*** quantities is that vector expressions can be simplified *without* resolving the vectors into orthogonal "**x**, **y**, **z**" or "**i**, **j**, **k**" components.

The first row in the table below shows a vector expression which is simplified using various properties of vector dot-products and cross-products (please verify the answer). Simplify the other vector expressions in the table and express your results in terms of the arbitrary vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ and the dot-product and cross-product operators.

Vector expression	Simplified vector expression
$(3\,\mathbf{u} - 2\,\mathbf{v}) \times (\mathbf{u} + \mathbf{v})$	$5\,\mathbf{u} \times \mathbf{v}$
$(3\,\mathbf{u} - 2\,\mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$	$3\,\mathbf{u}^2 - 2\,\mathbf{v}^2 + \mathbf{u} \cdot \mathbf{v}$
$(\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$	
$(3\,\mathbf{u} - 2\,\mathbf{v}) \times (\mathbf{u} + \mathbf{v}) \cdot (2\,\mathbf{u} - 7\,\mathbf{v})$	
$(\mathbf{u} + \mathbf{v}) \times (\mathbf{v} + 2\,\mathbf{w}) \cdot (\mathbf{w} + 2\,\mathbf{u})$	

---

[1] One way to prove this is to write $(\mathbf{v} \times \boldsymbol{\lambda})^2 = (\mathbf{v} \times \boldsymbol{\lambda}) \cdot (\mathbf{v} \times \boldsymbol{\lambda}) = \mathbf{v} \cdot [\boldsymbol{\lambda} \times (\mathbf{v} \times \boldsymbol{\lambda})]$, and then use the vector triple cross-product property in Section 2.11. Alternately, it is helpful to write $\mathbf{v} = \mathbf{v}_{\perp}\,\boldsymbol{\lambda}_{\perp} + \mathbf{v}_{||}\,\boldsymbol{\lambda}$ where $\mathbf{v}_{\perp}\,\boldsymbol{\lambda}_{\perp}$ is the component of $\mathbf{v}$ that is perpendicular to $\boldsymbol{\lambda}$ and $\mathbf{v}_{||}\,\boldsymbol{\lambda}$ is the component of $\mathbf{v}$ that is parallel to $\boldsymbol{\lambda}$.

1.19 Changing a vector equation to scalar equations.

(a) Draw three orthogonal unit vectors $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{r}$.

(b) Use a vector operation $(+,\ -,\ *,\ \cdot,\ \text{or}\ \times)$ to transform the following **vector** equation into a **scalar** equation and subsequently solve the scalar equation to find $x = 2$.

$$(2x - 4)\ \mathbf{p} \ = \ \mathbf{0}$$

(c) Show ***every*** vector operation (e.g., $+,\ -,\ *,\ \cdot$, or $\times$) that transforms the following **vector** equation into **three scalar** equations and subsequently solve the scalar equation to find $x = 2$, $y = 3$, and $z = 4$.

$$(2\,x - 4)\ \mathbf{p}\ +\ (3\,y - 9)\ \mathbf{q}\ +\ (4\,z - 16)\ \mathbf{r}\ =\ \mathbf{0}$$

(d) A vector equation is written in terms of the unitary ***non-orthogonal non-coplanar*** unit vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ as

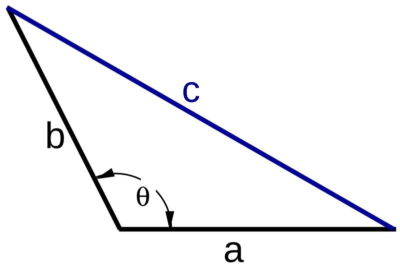
$$(2\,x - 4)\ \mathbf{a}_1\ +\ (3\,y - 9)\ \mathbf{a}_2\ +\ (4\,z - 16)\ \mathbf{a}_3\ =\ \mathbf{0}$$

In general, the solution for Solve

1.20 Proof of the law of cosines with vectors.

Vectors are very useful for geometry. Use **vectors** and **vector operations** to prove the ***law of cosines*** for a triangle with sides of length a, b, and c and an angle θ opposite the side of length c.[a]

$$c^2 \ = \ a^2 \ + \ b^2 \ - \ 2\,a\,b\,\cos(\theta)$$



---

[a] Hint: Form vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c} = \mathbf{b} - \mathbf{a}$. Then form $\mathbf{c}^2$.