

SEPT 28, 2009

ASSIGNMENT

CONTINUE READING FROM LAST TIME  
HW1 DUE WEDNESDAY

TODAY

ONE DIMENSIONAL PULSE DESIGN EXAMPLES  
K-SPACE INTERPRETATION  
EXCITATION AND RECEPTION

NEXT TIME

TWO DIMENSIONAL PULSES

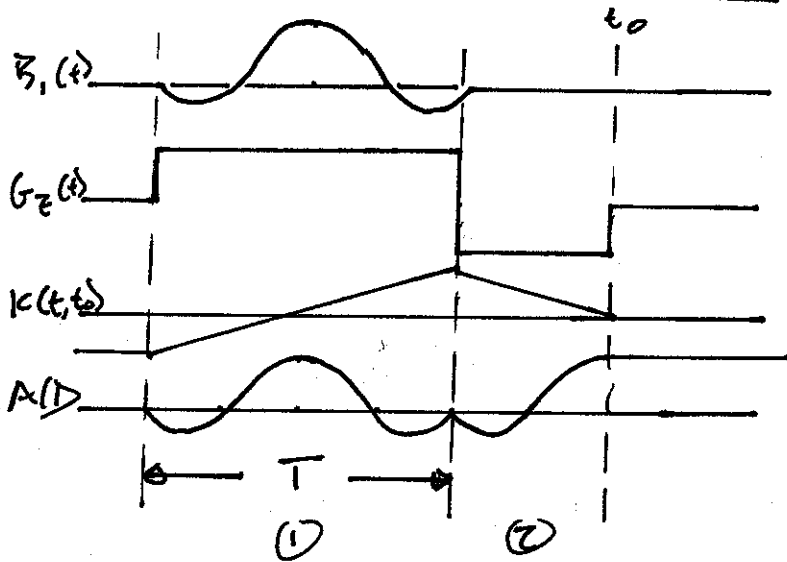
# SMALL-TIP-ANGLE SOLUTION

$$M_{xy}(E, t) = i m_0 \int_{-\infty}^t \delta B_1(\tau) e^{-i \int_{\tau}^t \gamma G(s) ds} d\tau$$

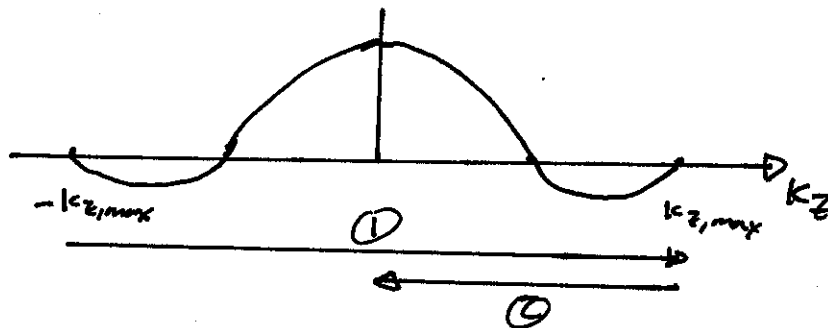
$$E(\tau, t) = -\frac{\gamma}{2\pi} \int_{\tau}^t G(s) ds$$

$$M_{xy}(E, t) = i m_0 \int_{-\infty}^t \delta B_1(\tau) e^{i 2\pi E(\tau, t) \cdot \tau} d\tau$$

## ONE-DIMENSIONAL EXAMPLE



$$E(t, t_0) = -\frac{\gamma}{2\pi} \int_t^{t_0} G(s) ds$$

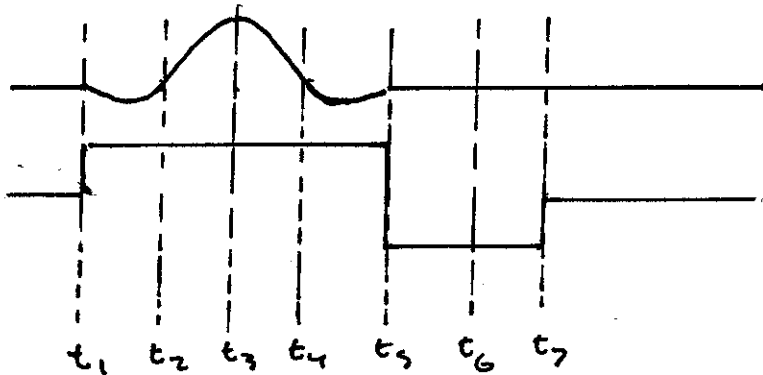


$$k_{z,max} = \frac{T}{2} \frac{\gamma}{2\pi} G_z$$

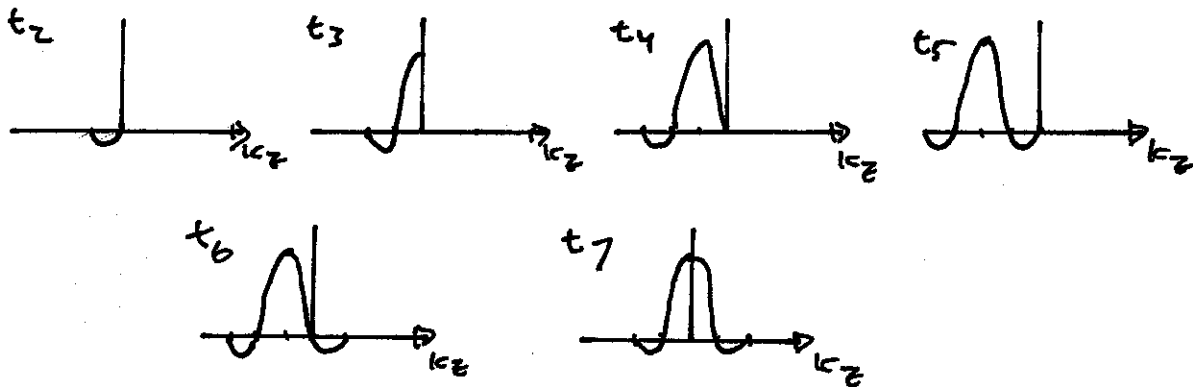
THIS GIVES MAGNETIZATION AT  $t = t_0$ , THE END OF THE PULSE.

LOOKS LIKE YOU SCAN ACROSS  $k$ -SPACE, THEN RETURN TO ORIGIN...

# EVOLUTION OF MAGNETIZATION DURING PULSE



CONSIDER  $t_2, t_3, \dots, t_7$   
THE END OF A PULSE  
WHAT  $k_z$ -SPACE WEIGHTING  
IS PRODUCED?



RF GOES IN AT DC ( $k_z = 0$ )!

GRADIENTS MOVE PREVIOUSLY APPLIED WEIGHTING AROUND.

THINK OF THE RF AS "WRITING" AN ANALOG WAVEFORM IN  $k_z$ -SPACE.

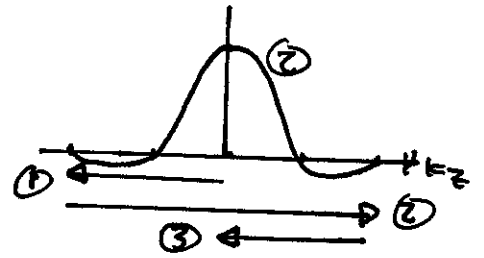
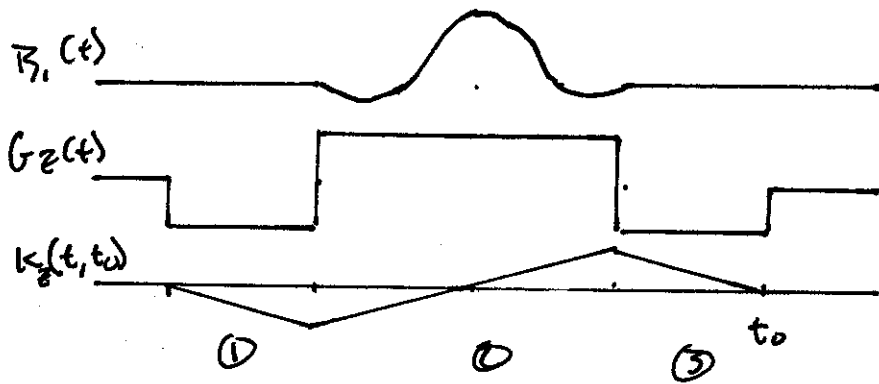
SAME IDEA APPLIES TO RECEPTION

READOUT GRADIENTS SHIFT OBJECT'S FOURIER TRANSFORM  
(MULT BY COMPLEX EXPONENTIAL IN IMAGE SPACE  
 $\Rightarrow$  SHIFT IN  $k_z$ -SPACE)

RF COIL IS NON-SELECTIVE

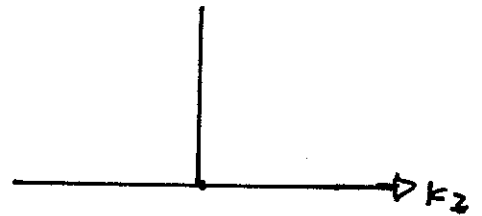
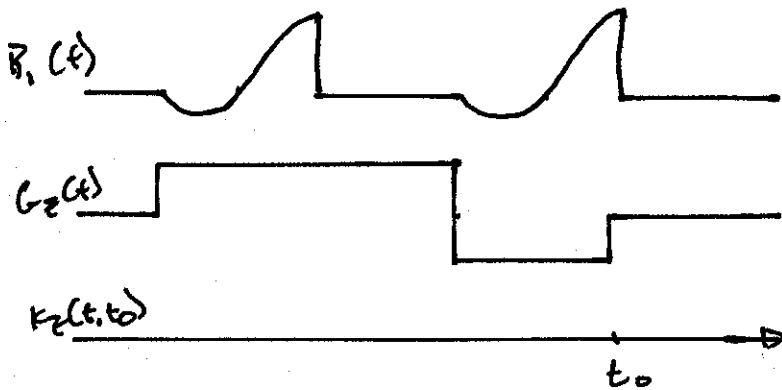
RECEIVE WHATEVER SIGNAL IS AT  $k_z = 0$   
MORE ON THIS LATER...

# OTHER 1D EXAMPLES

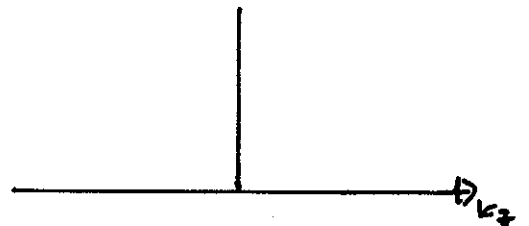
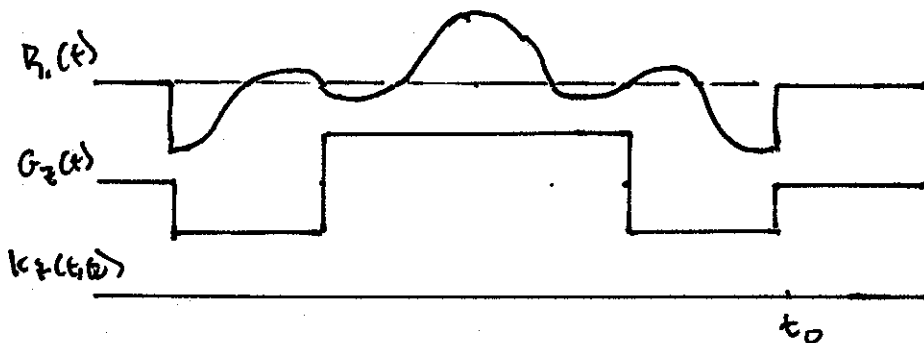


STARTS AND ENDS AT  $k_2=0$ .  
 IMPORTANT SPECIAL CASE LATER ON.

## WHAT DOES THIS DO?



## HOW ABOUT THIS ONE?



# MULTIPLE EXCITATIONS

MOST ACQUISITION METHODS REQUIRE SEVERAL REPETITIONS TO MAKE AN IMAGE

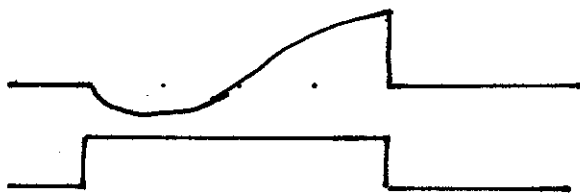
128 PHASE ENCODES

16 SPIRAL INTERLEAVES

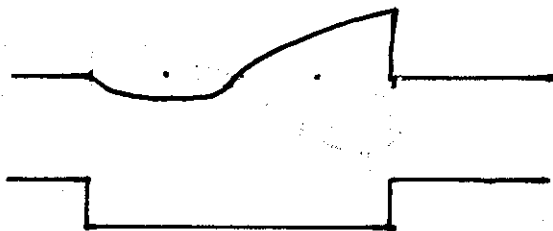
DATA IS COMBINED TO RECONSTRUCT AN IMAGE

SAME IDEA WORKS FOR EXCITATION

## SIMPLE 1D EXAMPLE



FIRST REPETITION

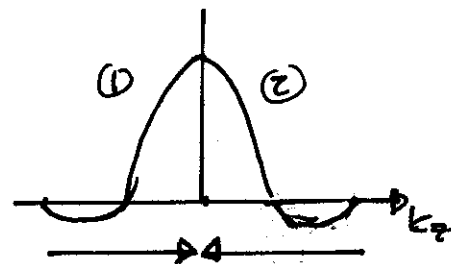
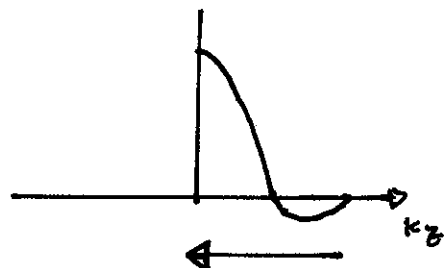
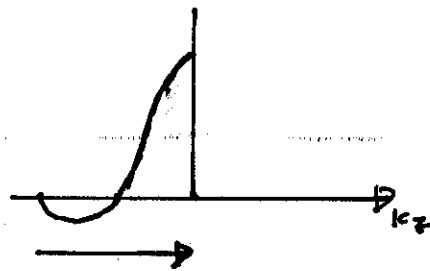


SECOND REPETITION

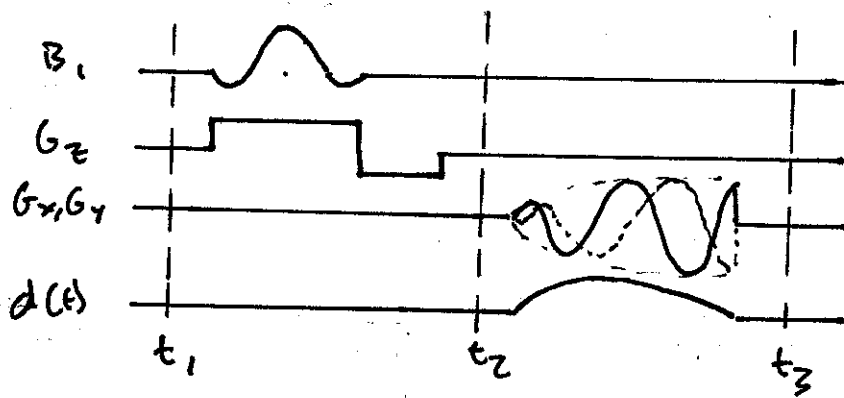
SUM THE DATA FROM TWO ACQUISITIONS.

SAME PROFILE AS SLICE SELECT PULSE.

ZERO ECHO TIME.



## EXCITATION AND RECEPTION



DENSITY, AMPLITUDE  
(K-SPACE)

## MAGNETIZATION AT TIME $t_2$

$$M_{xy}(\underline{r}, t_2) = i M_0(\underline{r}) \int_{t_1}^{t_2} \delta B_1(t) e^{-i \int_{t_1}^t \delta G_z(s) \cdot \underline{r} ds} dt$$

## SIGNAL FOR $t > t_2$

$$S(t) = \int_{\underline{R}} M_{xy}(\underline{r}, t_2) e^{-i \int_{t_2}^t \delta G_z(s) \cdot \underline{r} ds} d\underline{r}$$

## RECONSTRUCTED SIGNAL AT $\underline{r} = \underline{r}_0$

$$\hat{M}_{xy}(\underline{r}_0) = \int_{t_2}^{t_3} S(t) \underbrace{d(t)}_{\text{DENSITY}} e^{i \int_{t_2}^t \delta G_z(s) \cdot \underline{r}_0 ds} \underbrace{dt}_{\text{CONJUGATE PHASE KERNEL}}$$

## SUBSTITUTE FOR $S(t)$

$$\hat{M}_{xy}(r_0) = \int_{t_2}^{t_3} \left[ \int_{\underline{R}} m_{xy}(r, t_2) e^{-i \int_{t_2}^t \delta G(s) \cdot r ds} d\underline{r} \right] \left[ d(t) e^{i \int_{t_2}^t \delta G(s) \cdot r_0 ds} \right] dt$$

$$= \int_{\underline{R}} m_{xy}(r, t_2) \underbrace{\left[ \int_{t_2}^{t_3} d(t) e^{-i \int_{t_2}^t \delta G(s) \cdot (r-r_0) ds} dt \right]}_{\text{ACQUISITION IMPULSE RESPONSE}} d\underline{r}$$

SUBSTITUTE FOR  $m_{xy}(r, t_2)$

$$\hat{M}_{xy}(r_0) = \int_{\underline{R}} i m_0(r) \left[ \int_{t_1}^{t_2} \delta B_1(\tau) e^{-i \int_{\tau}^{t_2} \delta G(s) \cdot r ds} d\tau \right] \left[ \int_{t_2}^{t_3} d(t) e^{-i \int_{t_2}^t \delta G(s) \cdot (r-r_0) ds} dt \right] d\underline{r}$$

K-SPACE DEFINITION

$$\underline{k}(t_1, t_2) = -\frac{\gamma}{2\pi} \int_{t_1}^{t_2} G(s) ds = \frac{\gamma}{2\pi} \int_{t_2}^{t_1} G(s) ds \quad \left( \begin{array}{l} \text{IF } t_2 = 0, \\ \text{THIS IS CONVENTIONAL} \\ \cdot k(t) \end{array} \right)$$

SUBSTITUTE FOR  $\underline{k}(\cdot, \cdot)$

$$\hat{M}_{xy}(r_0) = \int_{\underline{R}} i m_0(r) \left[ \int_{t_1}^{t_2} \delta B_1(\tau) e^{i 2\pi \underline{k}(\tau, t_2) \cdot r} d\tau \right] \left[ \int_{t_2}^{t_3} d(t) e^{-i 2\pi \underline{k}(t, t_2) \cdot (r-r_0)} dt \right] d\underline{r}$$

## EXCITATION AND RECEPTION ARE SYMMETRIC

EXCITATION AND RECEPTION + RECONSTRUCTION ARE ALMOST COMPLETELY SYMMETRIC!

THE RF WEIGHTING  $\gamma B_1(t)$  CORRESPONDS EXACTLY TO  $d(t)$ , THE DENSITY WEIGHTING AND K-SPACE APPOINTMENT USED IN RECONSTRUCTION  
SAME K-SPACE TRAJECTORIES CAN BE USED FOR EITHER ONE.

### DIFFERENCES

ON EXCITATION, THE VOXEL POSITION IS SET

ON RECEPTION, THE VOXEL POSITION CAN BE MOVED (THIS IS IMAGE RECONSTRUCTION)

ONE IS FORWARD, ONE IS INVERSE TRANSFORM  
NOT FUNDAMENTAL

ARTIFACT OF CHOOSING  $t_2$  AS REFERENCE TIME

GENERALLY, RF PULSE REVERSES ACQUISITION

PRACTICAL CONSIDERATIONS THE SAME (DEPHASING,  $T_2$ )

## SHIFTING THE SLICE

SO FAR, SLICE HAS BEEN AT ISOCENTER

$$M_{xy}(z, t) = i m_0 \int_{-\omega}^t \delta B_z(\tau) e^{-i \left( \int_{\tau}^t \delta G_z(s) ds \right) \cdot z} d\tau$$

SHIFT TO  $z_0$  BY APPLYING

$$B_z(\tau) e^{i \left( \int_{\tau}^t \delta G_z(s) ds \right) \cdot z_0} = B_z(\tau) e^{-i 2\pi \mathbf{k}(\tau, t) \cdot \mathbf{r}_0}$$

THEN

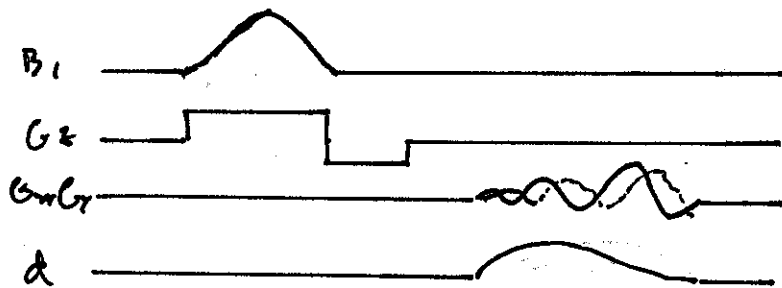
$$\begin{aligned} & i m_0 \int_{-\omega}^t \delta B_z(\tau) e^{i \left( \int_{\tau}^t \delta G_z(s) ds \right) \cdot z_0} e^{-i \left( \int_{\tau}^t \delta G_z(s) ds \right) \cdot z} d\tau \\ &= i m_0 \int_{-\omega}^t \delta B_z(\tau) e^{-i \left( \int_{\tau}^t \delta G_z(s) ds \right) (z - z_0)} d\tau \\ &= M_{xy}(z - z_0, t) \end{aligned}$$

FOR A CONSTANT GRADIENT IN ONE DIMENSION

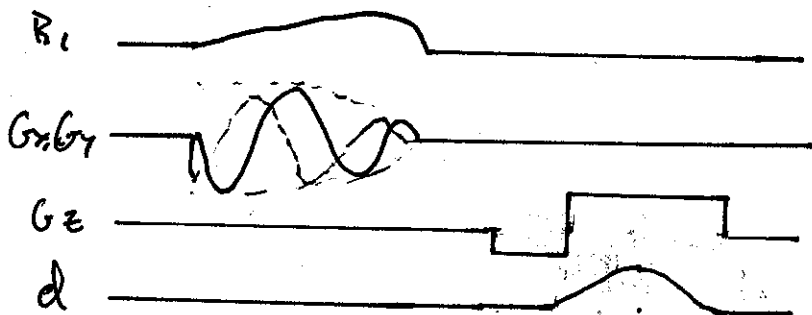
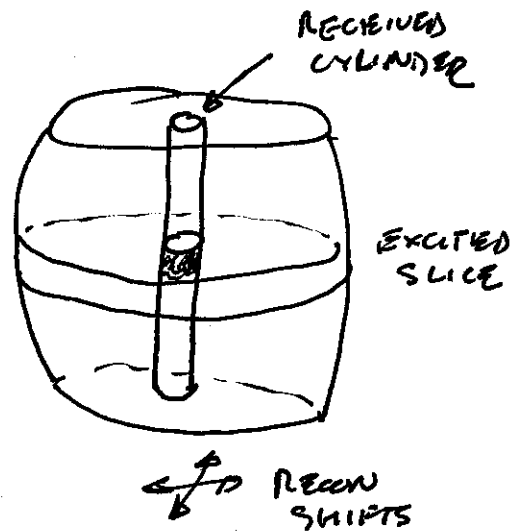
$$\begin{aligned} e^{i \int_{\tau}^t \delta G_z(s) ds \cdot z_0} &= e^{i \delta G_z (t - \tau) z_0} \\ &= e^{-i \delta G_z \tau z_0} e^{i \delta G_z t z_0} \\ &= \underbrace{e^{-i (\delta G_z z_0) \tau}}_{\substack{\text{FREQUENCY} \\ \text{SHIFT} \\ \delta G_z z_0}} \underbrace{e^{i (\delta G_z z_0) t}}_{\substack{\text{CONSTANT} \\ \text{PHASE}}} \end{aligned}$$

ANY IMAGING METHOD CAN BE FLIPPED  
AROUND TO MAKE AN RF PULSE!

EXAMPLE: 2D SPIRAL



SLICE SELECT EXCITE  
 SPIRAL RECEIVE



SPIRAL EXCITE  
 SLICE SELECT RECEIVE

