

NOV 4, 2009

ASSIGNMENT

INITIAL PROJECT TOPIC

READS ADIABATIC EXCITATION AND INVERSION PULSES  
SECTIONS 6.1, 6.2 PAGES 177-199

TODAY

ADIABATIC INVERSION PULSES

NEXT TIME

ADIABATIC REFocusing PULSES

# ADIABATIC PULSES

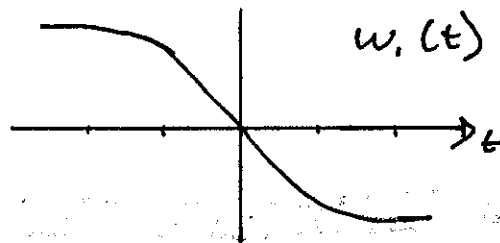
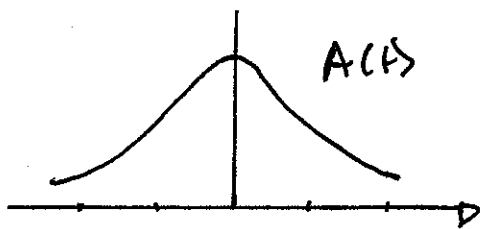
FREQUENCY MODULATED PULSES OF THE FORM

$$\beta_1(t) = A(t) e^{-i\omega_1(t)t}$$

WHERE

$A(t)$  - ENVELOPE

$\omega_1(t)$  - FREQUENCY SWEEP



MANY DIFFERENT OPTIONS. MOST FAMOUS

$$A(t) = A_0 \operatorname{sech}(\beta t)$$

$$\omega_1(t) = -\mu\beta \tanh(\beta t)$$

$\mu, \beta$  PARAMETERS. SILVER-MOULT PULSE, OR HYPERBOLIC SECANT.

ONE OF THE FEW ANALYTIC SOLUTIONS  
TO THE BLOCH EQUATION

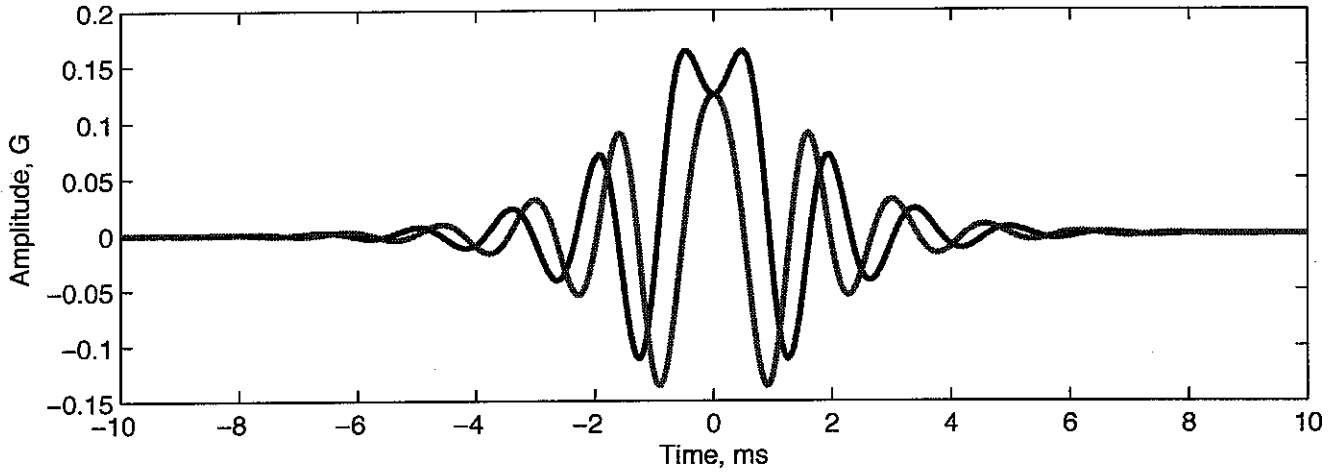
FIRST OF A FAMILY OF SOLUTIONS CALLED  
SOLUTIONS.

### REMARKABLE CHARACTERISTIC

A HYPERBOLIC SECANT PULSE PERFORMS  
A FREQUENCY (SLICE) SELECTIVE INVERSION  
FOR ANY RF AMPLITUDE ABOVE A  
THRESHOLD,

# Hyperbolic Secant

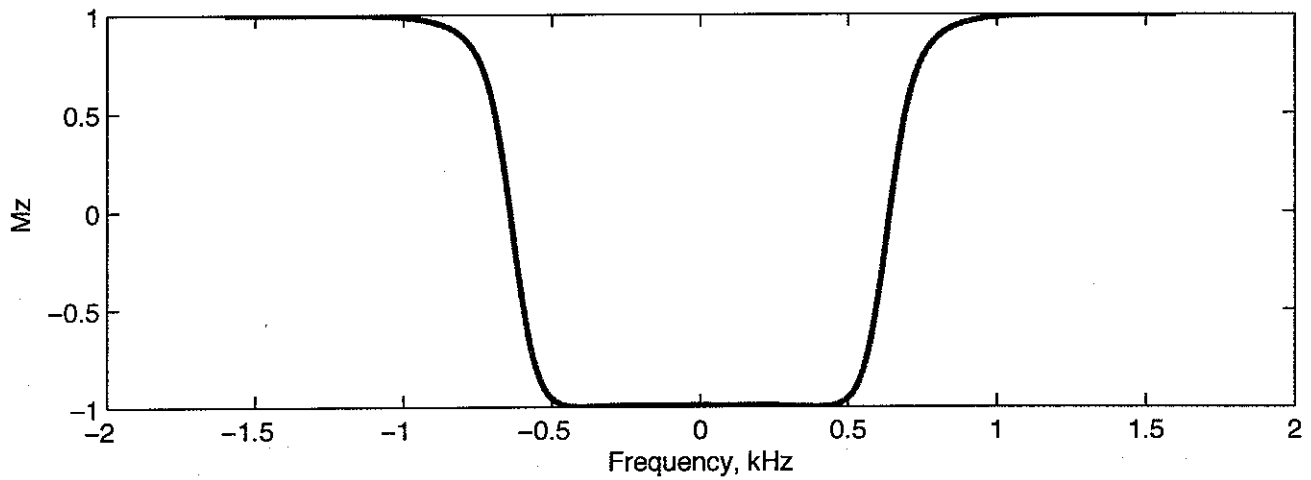
## RF Pulse



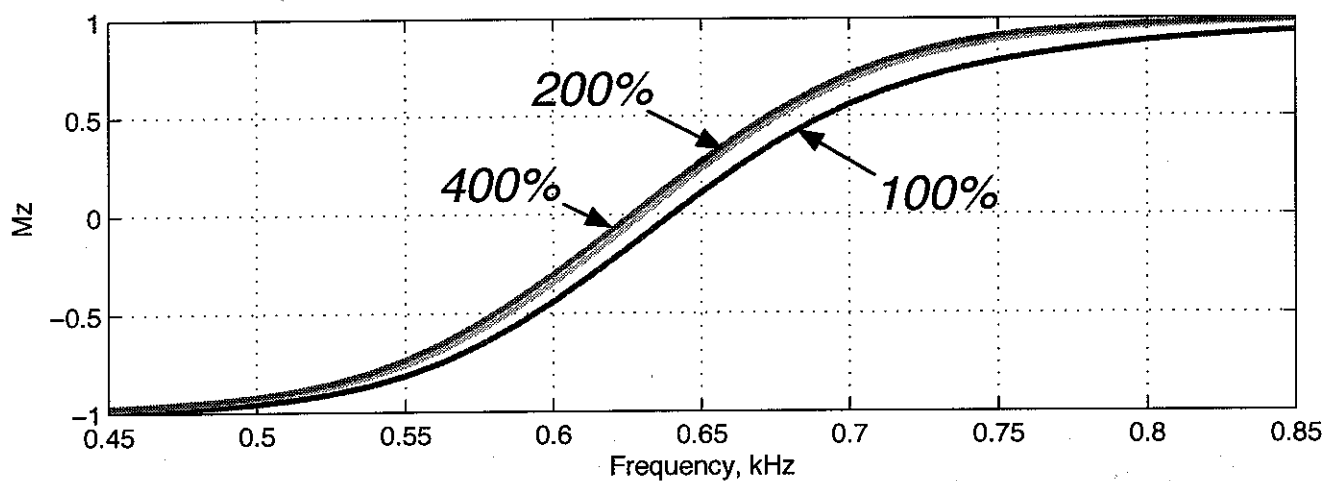
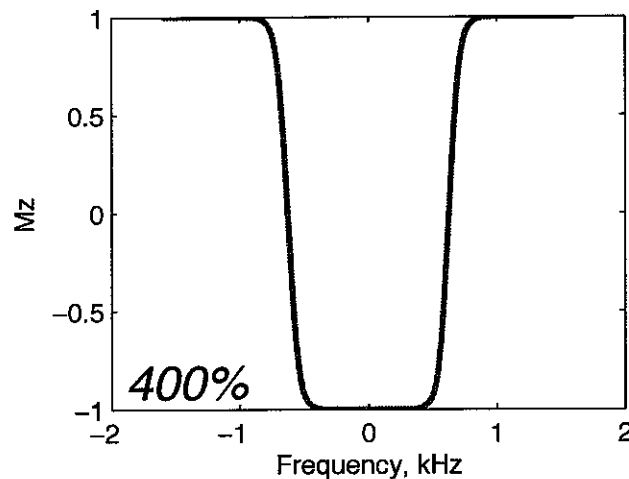
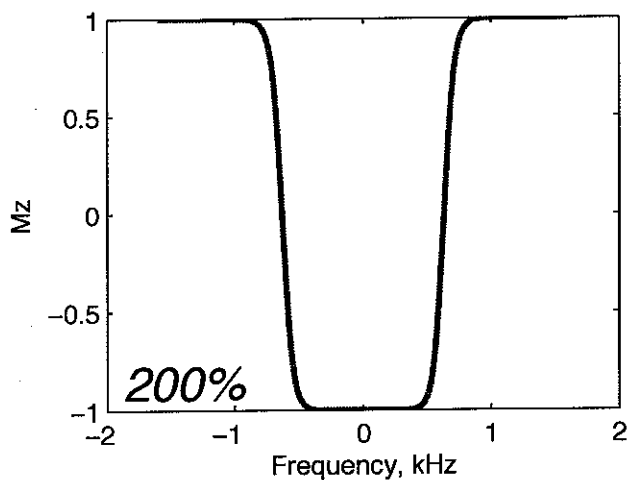
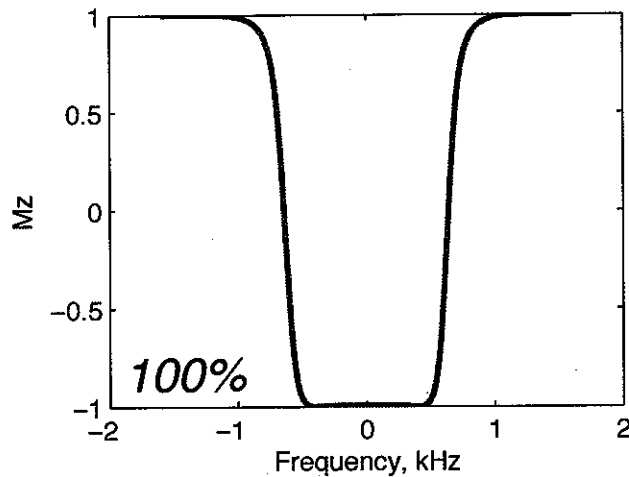
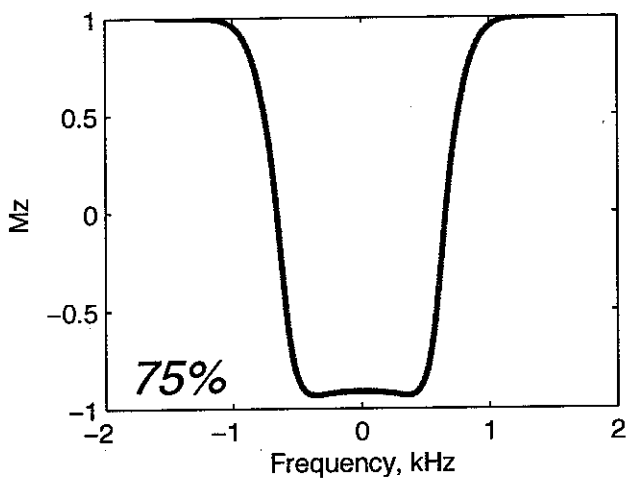
$$B_1(t) = A_0 \operatorname{sech}(\beta t) e^{-i\mu\beta \tanh(\beta t)t}$$

$$\beta = 800 \text{ rad/s}; \mu = 4.9$$

## Inversion Profile



# Inversion Profiles



## ROTATING REFERENCE FRAMES

BLOCH EQUATION IN LABORATORY FRAME

$$\left(\frac{d\mathbf{M}}{dt}\right) = -\gamma \underline{\mathbf{B}} \times \underline{\mathbf{M}}$$

WHERE

$$\underline{\mathbf{B}} = B_1 \cos \omega_0 t \hat{\mathbf{i}}_0 + B_1 \sin \omega_0 t \hat{\mathbf{j}}_0 + B_0 \hat{\mathbf{k}}_0$$

AND  $\hat{\mathbf{i}}_0, \hat{\mathbf{j}}_0, \hat{\mathbf{k}}_0$  ARE FIXED UNIT VECTORS IN LAB FRAME, AND  $B_1 = B_{1,x}$  FOR CONVENIENCE

WE WANT TO DESCRIBE THE MAGNETIZATION IN A FRAME ROTATING AT  $\omega_0$

$$\hat{\mathbf{i}} = \hat{\mathbf{i}}_0 \cos \omega_0 t + \hat{\mathbf{j}}_0 \sin \omega_0 t$$

$$\hat{\mathbf{j}} = -\hat{\mathbf{i}}_0 \sin \omega_0 t + \hat{\mathbf{j}}_0 \cos \omega_0 t$$

$$\hat{\mathbf{k}} = \hat{\mathbf{k}}_0$$

THEN

$$\underline{\mathbf{B}} = B_1 \hat{\mathbf{i}} + B_0 \hat{\mathbf{k}}$$

$$\underline{\mathbf{M}} = M_x \hat{\mathbf{i}} + M_y \hat{\mathbf{j}} + M_z \hat{\mathbf{k}}$$

BOTH  $(M_x, M_y, M_z)$  AND  $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$  TIME VARYING IN LAB FRAME.

Differentiating

$$\begin{aligned}\left(\frac{d}{dt} \underline{m}\right)_{\text{LAB}} &= \frac{\partial m_x}{\partial t} \hat{i} + \frac{\partial m_y}{\partial t} \hat{j} + \frac{\partial m_z}{\partial t} \hat{k} \\ &\quad + m_x \frac{\partial \hat{i}}{\partial t} + m_y \frac{\partial \hat{j}}{\partial t} + m_z \frac{\partial \hat{k}}{\partial t} \\ &= \left(\frac{\partial \underline{m}}{\partial t}\right)_{\text{ROT}} + m_x \frac{\partial \hat{i}}{\partial t} + m_y \frac{\partial \hat{j}}{\partial t} + m_z \frac{\partial \hat{k}}{\partial t}\end{aligned}$$

$\hat{i}, \hat{j}$  ARE ROTATING UNIT VECTORS WHICH WE CAN REPRESENT AS A CROSS PRODUCT WITH

$$\underline{\omega}_0 = \omega_0 \hat{k}$$

SO

$$\frac{\partial \hat{i}}{\partial t} = \underline{\omega}_0 \times \hat{i}; \quad \frac{\partial \hat{j}}{\partial t} = \underline{\omega}_0 \times \hat{j}; \quad \frac{\partial \hat{k}}{\partial t} = \underline{\omega}_0 \times \hat{k}$$

SUBSTITUTING

$$\begin{aligned}\left(\frac{d}{dt} \underline{m}\right)_{\text{LAB}} &= \left(\frac{\partial \underline{m}}{\partial t}\right)_{\text{ROT}} + \underline{\omega}_0 \times (m_x \hat{i} + m_y \hat{j} + m_z \hat{k}) \\ &= \left(\frac{\partial \underline{m}}{\partial t}\right)_{\text{ROT}} + \underline{\omega}_0 \times \underline{m}\end{aligned}$$

IN THE ROTATING FRAME

$$\begin{aligned}\left(\frac{\delta \underline{M}}{\delta t}\right)_{\text{ROT}} &= \frac{d}{dt} \underline{M} - \underline{\omega}_0 \times \underline{M} \\ &= -\gamma \underline{B} \times \underline{M} - \underline{\omega}_0 \times \underline{M} \\ &= (-\gamma \underline{B} - \underline{\omega}_0) \times \underline{M} \\ &= -\gamma \left( \underline{B} + \frac{\underline{\omega}_0}{\gamma} \right) \times \underline{M} \\ &\quad \underbrace{\hspace{1.5cm}}_{\underline{B}_{\text{eff}}}\end{aligned}$$

AT THE LARMOR FREQUENCY

$$\underline{\omega}_0 = -\gamma B_0 \hat{k}$$

AND

$$\begin{aligned}\underline{B} &= B_1 \cos \omega_0 t \hat{i}_0 + B_1 \sin \omega_0 t \hat{j}_0 + B_0 \hat{k}_0 \\ &= B_1 \hat{i} + B_0 \hat{k}\end{aligned}$$

SO

$$\begin{aligned}\underline{B}_{\text{eff}} &= B_1 \hat{i} + \cancel{B_0 \hat{k}} + \frac{-\cancel{\delta B_0}}{\delta} \hat{k} \\ &= B_1 \hat{i}\end{aligned}$$

(7)

THE RESULT IS THAT THE STATIC  $B_0$  FIELD HAS BEEN "DELETED" IN ROTATING FRAME!

### KEY IDEA

CHANGING REFERENCE FRAMES BY A FREQUENCY  $\omega$  CORRESPONDS TO ADDING A  $z$ -FIELD OF  $\frac{\omega}{\gamma}$ .

IN PARTICULAR IF I HAVE AN RF PULSE

$$B_1(t) = A(t) e^{i\omega_1(t)t}$$

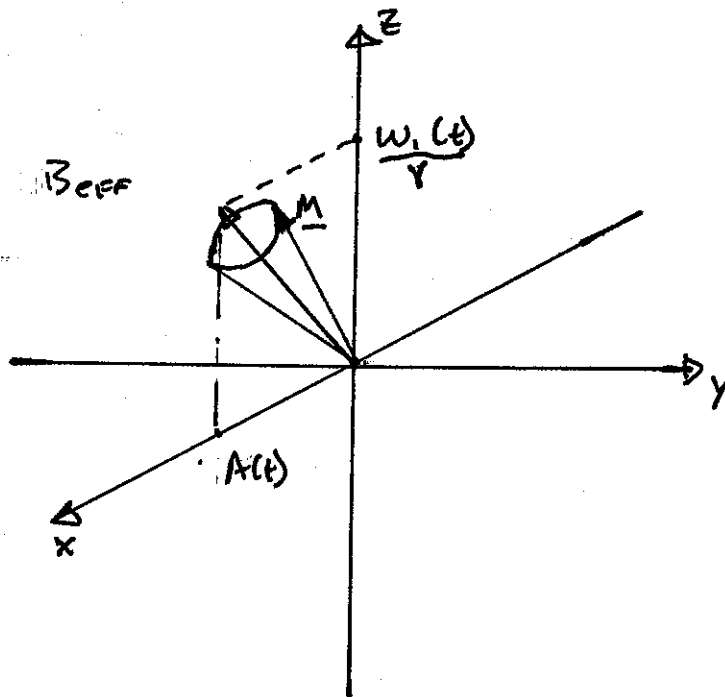
AT THE LARMOR FREQUENCY, THEN

$$\left(\underline{B}\right)_{\omega_0} = A(t) \cos \omega_1(t) \hat{x}_{\omega_0} + A(t) \sin \omega_1(t) \hat{y}_{\omega_0} + 0 \hat{z}$$

IF WE CHANGE TO THE ROTATING FRAME AT  $\omega_0 = \omega_1(t)$

$$\begin{aligned} \left(\underline{B}\right)_{\omega_0 = \omega_1(t)} &= A(t) \hat{x}_{\omega_0 = \omega_1(t)} + \frac{\omega_1(t)}{\gamma} \hat{z} \\ &= \left( A(t), 0, \frac{\omega_1(t)}{\gamma} \right) \end{aligned}$$

## MAGNETIZATION PLOT



MAGNETIZATION PRECESSES ABOUT  $\underline{B}_{eff}$

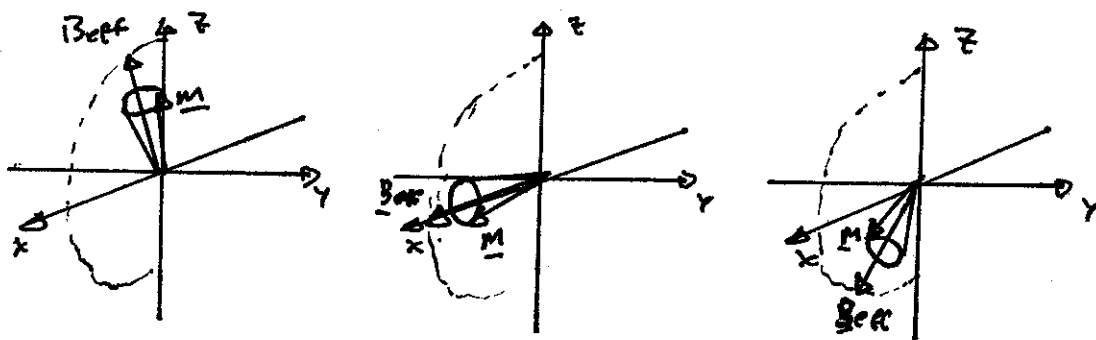
$$\underline{B}_{eff} = (A(t), 0, \frac{\omega_L(t)}{\gamma})$$

$$|\underline{B}_{eff}| = \sqrt{A^2(t) + \left(\frac{\omega_L(t)}{\gamma}\right)^2}$$

$$\angle \underline{B}_{eff} = \psi = \tan^{-1} \left( \frac{A(t)}{\omega_L(t)/\gamma} \right)$$

## BASIC IDEA OF ADIABATIC PULSES

GIVEN INITIAL  $\underline{M}$  ALONG  $z$  AXIS, SWEEP  $\underline{B}_{eff}$  FROM  $+z$  TO  $-z$  SLOWLY ENOUGH THAT  $\underline{M}$  FOLLOWS  $\underline{B}_{eff}$



$\underline{M}$  SHOULD PRECESS FASTER THAN THE ANGLE OF  $\underline{B}_{eff}$  CHANGES,  $\frac{d\psi}{dt}$

$$\left| \frac{d\psi}{dt} \right| \ll \gamma |\underline{B}_{eff}|$$

ADIABATIC CONDITION. DEFINE

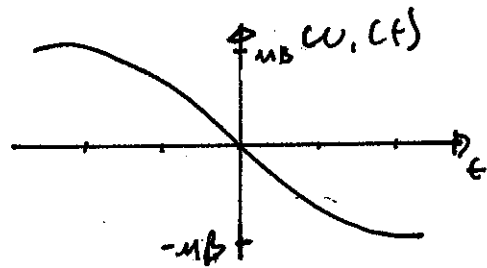
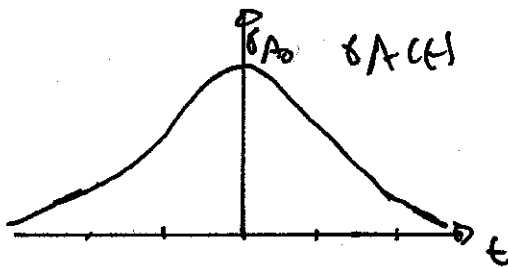
$$\eta = \frac{\gamma |\underline{B}_{eff}|}{\left| \frac{d\psi}{dt} \right|}$$

AS ADIABATIC FACTOR, LARGER THAN 1.

# SWEEP DIAGRAMS

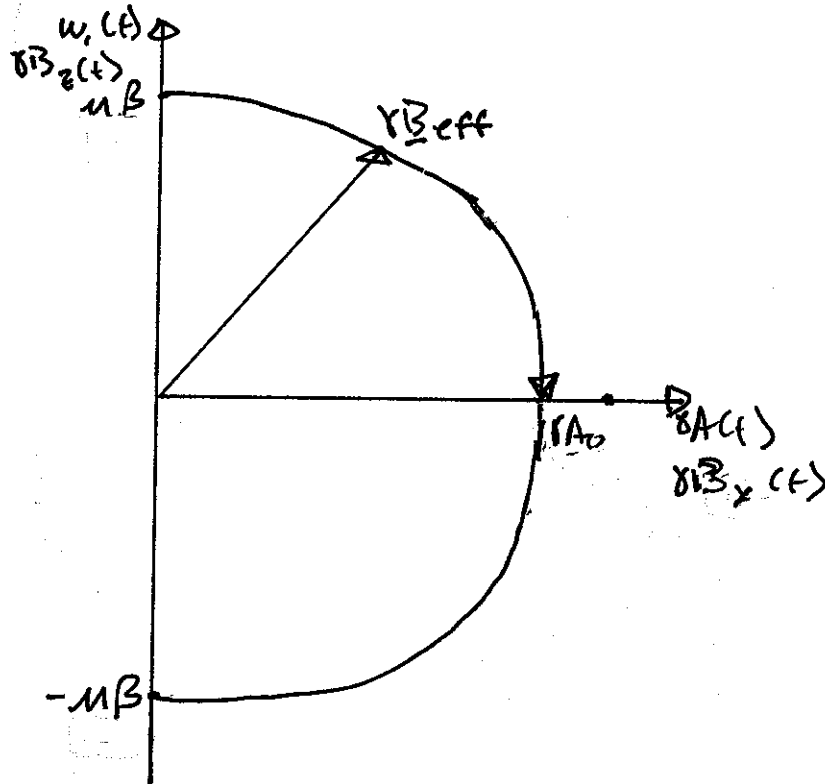
VERY USEFUL TOOL FOR UNDERSTANDING  
ADIBATIC PULSES. DUE TO CONJUG.

PLOT  $\delta A(t)$  vs  $w(t)$

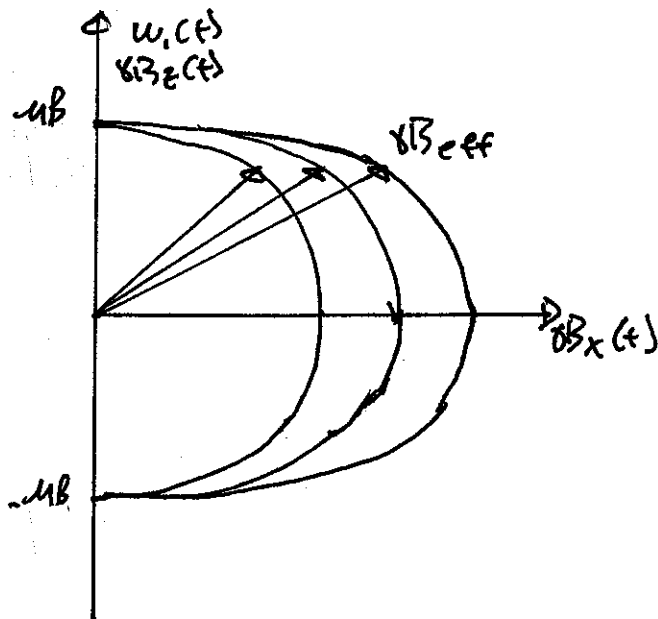


$$\delta A(t) = \delta A_0 \operatorname{sech}(\beta t)$$

$$\frac{1}{2} w(t) = -m\beta \operatorname{tanh}(\beta t)$$



ADIABATIC PULSE ARE INSENSITIVE TO  $\beta_1$  SCALING, PROVIDED ADIABATIC CONDITION IS MET



ALL PULSE INVERSIONS PROVIDED

$$\frac{\partial \psi}{\partial t} \ll \delta |B_{eff}|$$

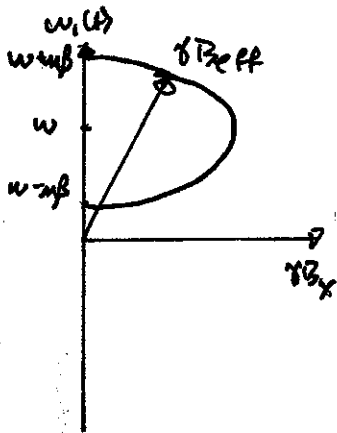
AS  $\beta_1$  INCREASES, CAN LOOSE ADIABATICITY AT THE BEGINNING AND END.

ADIABATICITY IMPROVES IN THE MIDDLE.

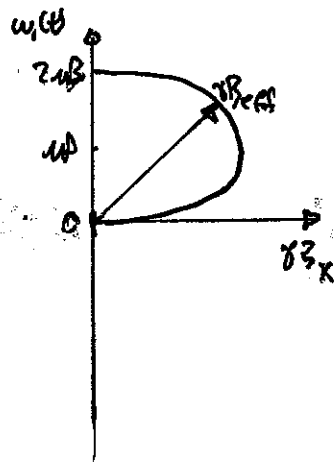
# ADIABATIC PULSES ARE SELECTIVE

## FREQUENCY OF SLICE SELECTIVE

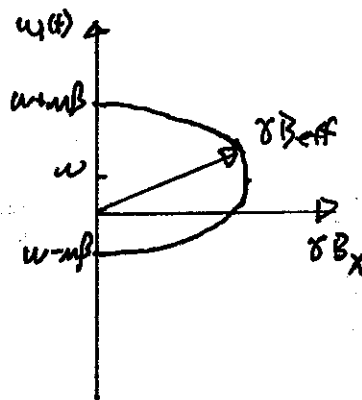
DETERMINED BY SWEEP BANDWIDTH



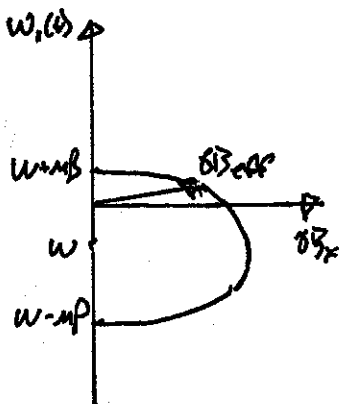
$w > \mu\beta$   
NOT INVERTED



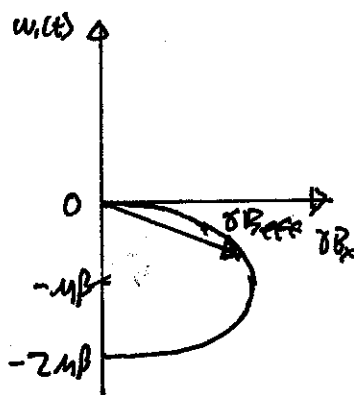
$w = \mu\beta$   
TRANSITION BAND  
(NOT ADIABATIC)



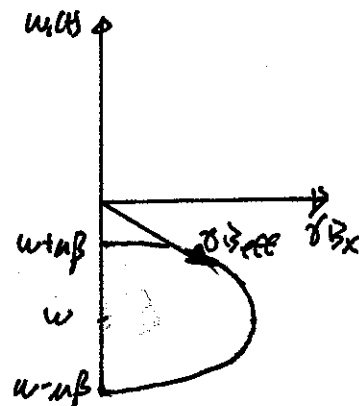
$-\mu\beta < w < \mu\beta$   
INVERTED



$-\mu\beta < w < \mu\beta$   
INVERTED



$w = -\mu\beta$   
TRANSITION BAND  
(NOT ADIABATIC)



$w < -\mu\beta$   
NOT INVERTED  
ANTI-PARALLEL

FOR  $\omega > \omega_B$  OR  $\omega < -\omega_B$ ,  $\underline{m}$  RETURNS  
TO  $+z$  AXIS

FOR  $-\omega_B < \omega < \omega_B$   $\underline{m}$  IS INVERTED

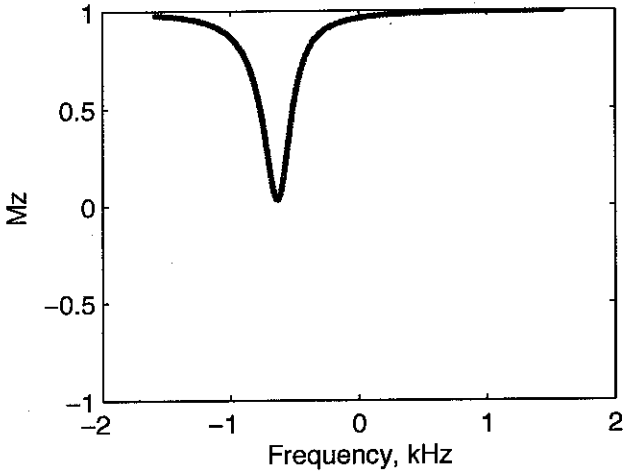
BANDWIDTH IS

$$\begin{aligned}\Delta f &= \frac{2\omega_B}{2\pi} \\ &= \underline{\frac{\omega_B}{\pi}} \quad \text{HZ}\end{aligned}$$

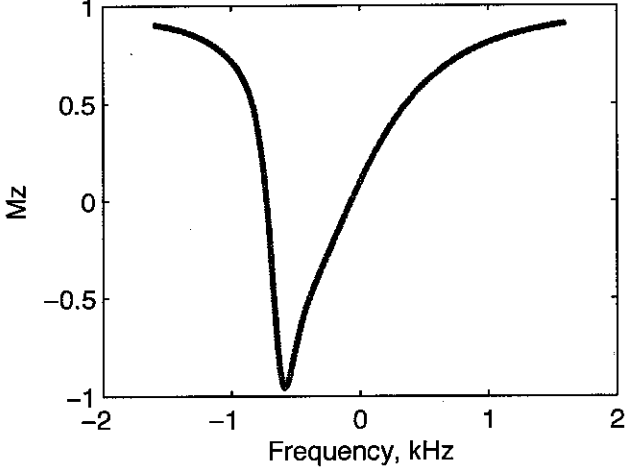
BAND EDGE CLOSEST TO BEGINNING OF  
FREQUENCY SWEEP DEFINED FIRST, OTHER  
BAND EDGE LAST

# Inversion Sequence

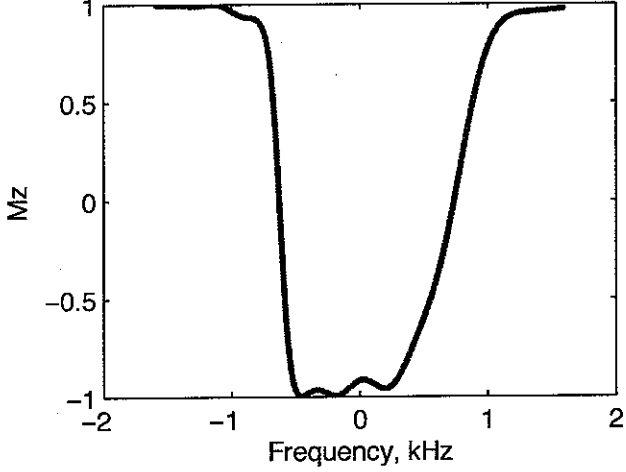
*after 7.5 ms*



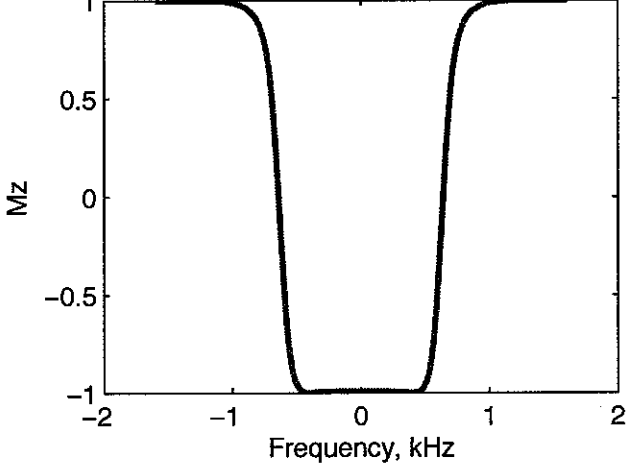
*after 10 ms*



*after 12.5 ms*



*after 20 ms*



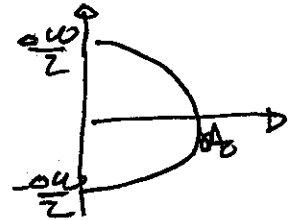
## COMMENTS

1) MANY ENVELOPE/MODULATION FUNCTIONS WORK

EX:

$$A(t) = A_0 \sin(\omega_c t)$$

$$\omega_c(t) = \frac{\Delta\omega}{2} \cos(\omega_c t)$$



WHERE  $\Delta\omega$  IS THE BANDWIDTH.

2) IF A RANGE OF ADAPTABILITY IS REQUIRED, OPTIMIZATION CAN HELP REDUCE PULSE LENGTH.

3) HYPERBOLIC SIZES NEED TO BE TRUNCATED, CAN SIGNIFICANTLY AFFECT PERFORMANCE.