

EE469B: Assignment 2 Solutions

Due Wednesday, Oct. 7

Introduction This assignment concerns the design of small-tip-angle 2D excitation pulses based on spiral k-space trajectories. You will design a 2D RF pulse using two different density compensation methods and simulate the excitation profiles. You will also look at the characteristics of the first aliasing sidelobe.

We are going to pick a particular set of design parameters. The goals of this design are

- Gradient waveform that would allow 0.5 cm diameter cylinder to be excited when no k-space apodization is used,
- First sidelobe at 8 cm,
- Gradient system with 4 G/cm amplitude, 15 G/cm/ms slew rate (SR 150),
- RF waveform that excites 4 cm cylinder with minimal ripple.

This is an example of a 2D pulse you might use to restrict the FOV for high-resolution imaging of an interior volume.

1. Design of 2D Spiral Gradients We will design the gradient waveform in three steps. The gradient will be in inward going spiral.

- a) How far out in k-space (k_{max}) should the trajectory go for 0.5 cm resolution?

Solution

$$\Delta r = \frac{1}{2k_{max}}$$

so

$$k_{max} = \frac{1}{2\Delta r} = \frac{1}{(2)(0.5 \text{ cm})} = 1 \text{ cycle/cm.}$$

- b) How many turns N should the spiral have to put the sidelobe at 8 cm?

Solution

$$FOV = 2N\Delta r$$

so

$$N = \frac{FOV}{2\Delta r} = \frac{8 \text{ cm}}{(2)(0.5 \text{ cm})} = 8 \text{ turns.}$$

- c) Design a constant-angular rate k-space trajectory with this k_{max} and number of turns N with 2048 samples.

```
>> t = [1:2048]/2048;  
>> ka = kmax * (1-t) .* exp(i*2*pi*N*(1-t));
```

where $\text{real}(ka)$ is k_x and $\text{imag}(ka)$ is k_y .

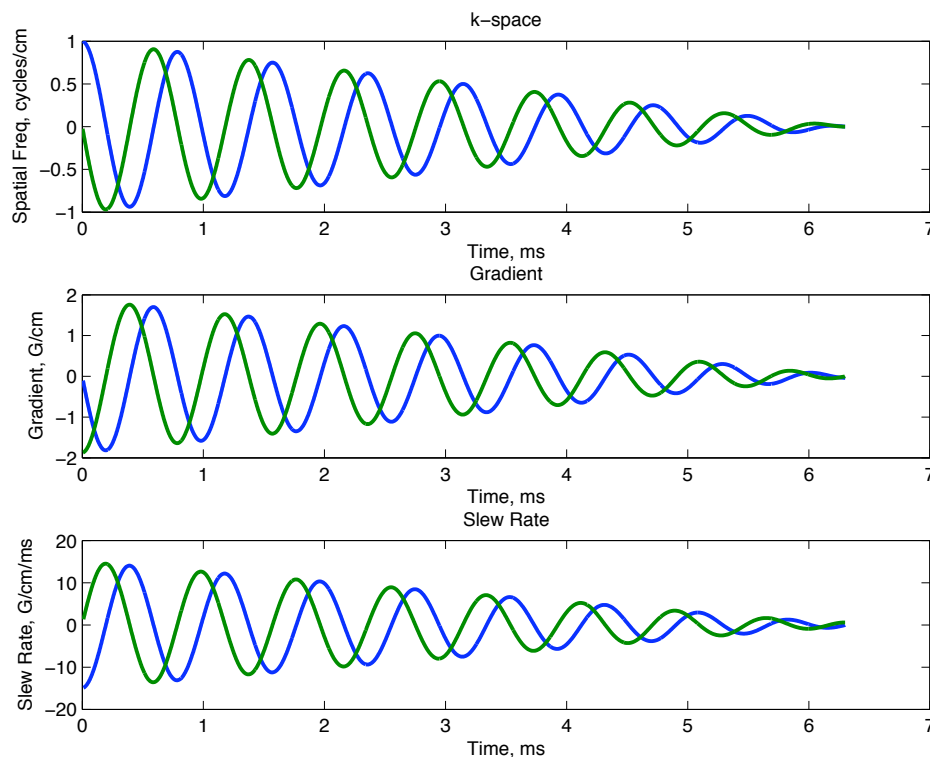
Solution Nothing to plot, yet!

- d) This k-space trajectory can be traced out at different rates, to produce different gradient waveforms. If the rate is constant and the duration of the trajectory is T ms, the gradient and slew rates required can be computed using $ktog(k, dt)$ and $ktos(k, dt)$. These are available on the web site. For example if the pulse were 5 ms long,

```
>> dt = 5 / 2048;
>> g = ktog(ka, dt);
>> s = ktos(ka, dt);
```

where g is in G/cm, and s is G/cm/ms. How long should the trajectory be to meet the gradient system limits? Which constraints are in effect? Plot k_a , g , and s versus time, with the axes properly labeled.

Solution By checking the constraints iteratively, the pulse duration T is about 6.3 ms. The k-space, gradient, and slew rate waveforms are plotted below.



- e) The constant angular rate trajectory doesn't use the gradient system efficiently. Ideally we want to always be operating at either the slew or amplitude limit. The m-file `csg(k, gmax, smax)` computes a new k-space trajectory that meets the amplitude and slew limits, and reports the new (shorter) duration of the pulse. Redesign your gradients

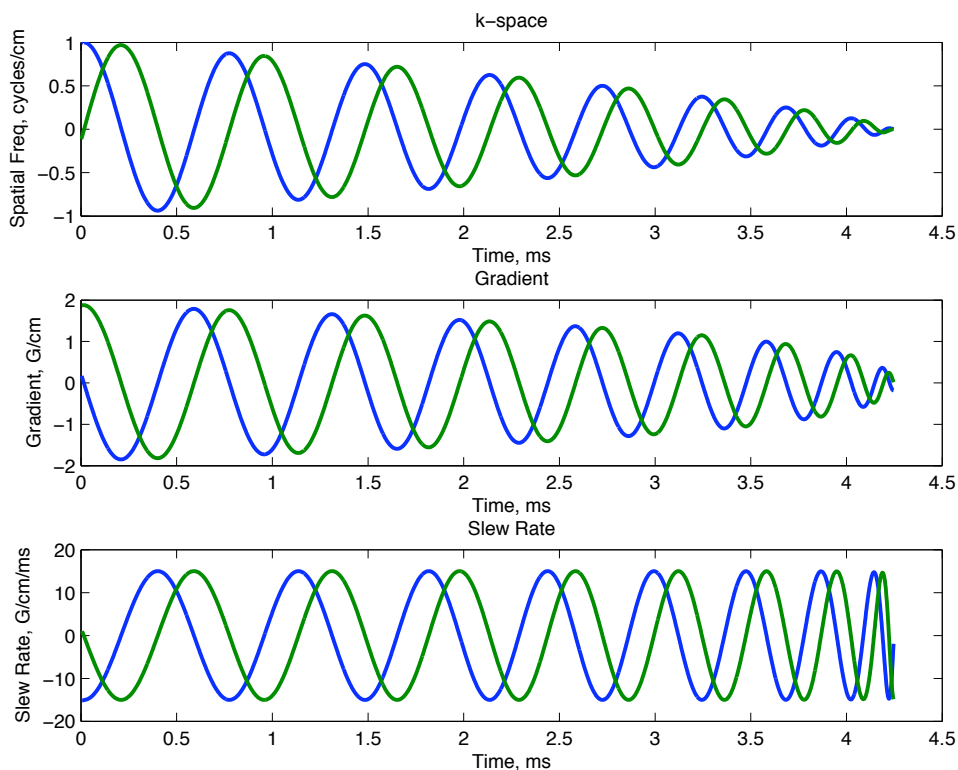
```
>> k = csg(ka, 4, 15)
```

How long is the pulse now? Plot the k-space trajectory, gradients, and slew rate as a function of time, with properly labeled axes. Make sure the `rf_tools` directory is not in your path. The class version of `csg` is different than the `rf_tools` version.

We now have a gradient waveform for the RF pulse! That was easy enough. Note that the gradients you've designed are not quite ready for the scanner, the waveforms don't start and end at zero.

Solution

The pulse is now about 4.25 ms long. The plots are shown below. The gradient amplitude constraint is never reached. The trajectory is entirely slew-rate limited.

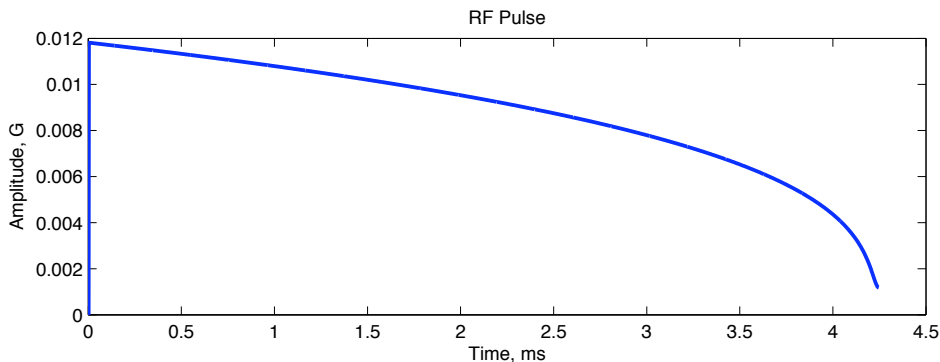


2. Design of RF Pulse Next we turn to designing the RF waveform. If the desired weighting is flat, the RF waveform should be the same as the density compensation used for reconstructing spiral data. As we discussed in class, a good approximation for a single-shot spiral is to use the inverse of the magnitude of the gradient as the density, and the magnitude of the gradient as the density compensation

```
>> rf = abs(g)
>> rf = rf/sum(rf)
```

This normalizes the rf to a flip angle of 1 radian. Plot this RF pulse. Note that since this pulse corresponds to uniform k-space weighting, it will have the smallest selective volume, but will ring excessively. Plot your RF pulse with the axes properly labeled.

Solution



3. Simulation of 2D Pulses Next we simulate our RF pulse. The simulator you used last time will also handle 2D pulses and 2D simulations. As before, the RF is scaled so that $\text{sum}(rf)$ is the flip angle in radians. The gradient waveform is supplied as a complex waveform. If $g = g_x + i \cdot g_y$ in G/cm, then the input to the simulator is

```
>> gs = gamma*g*dt;
```

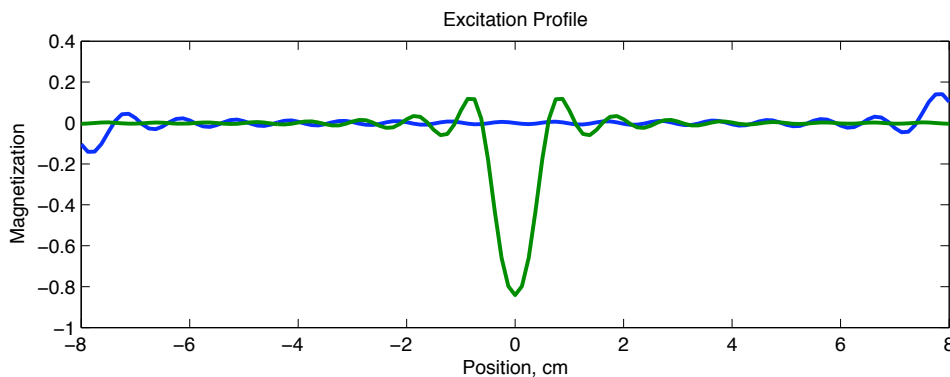
where $\text{gamma} = 2 \cdot \pi \cdot 4.257$ krad/G, and dt the same as above. Finally, the vector of positions to simulate is x , in cm. To simulate a 1D profile through the 2D volume,

```
>> x = [-8:0.25:8];
>> mxy1 = ab2ex(abrm(rf,gs,x));
```

This simulates the response from -8 cm to 8 cm in 0.25 cm steps. Do the simulation, and plot magnitude of the response as a function of x . Are the sidelobes in the right places compared to where you would calculate them to be? Is the mainlobe the right width?

Solution

The cross section through the excitation profile is plotted below.



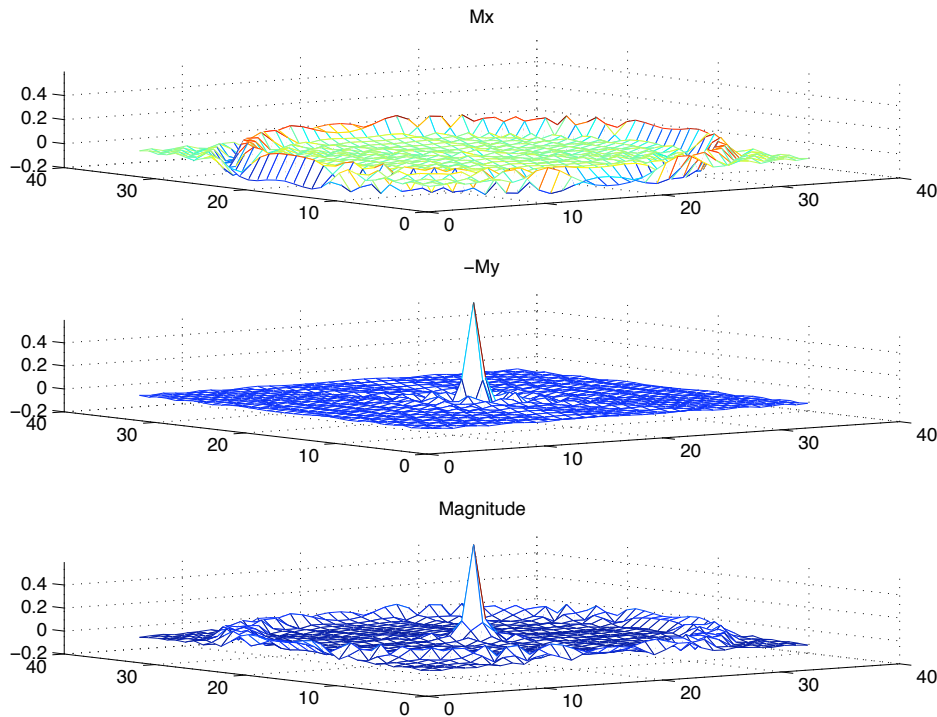
The sidelobes appear to be in the right place, ± 8 cm, and the mainlobe width is about 1 cm. Note that M_y is negative because the simulator assume right handed rotations, and protons precess in the left-handed direction. If we wanted to make this behave exactly like protons, we would make gamma negative, both for the RF and for the gradient.

We can also simulate the 2D profile. In this case we provide an additional y vector for the second dimension. Each x, y pair is simulated, and the result stored in a 2D matrix,

```
>> x = [-8:0.5:8];
>> y = [-8:0.5:8];
>> mxy2 = ab2ex(abrm(rf,gs,x,y));
>> mesh(abs(mxy2))
```

Plot the absolute value, the real part, and the imaginary part of the excitation profile (you may need to multiply by -1 to make it more visible).

Solution The surface plots look like:



4. Single shot sidelobe is in quadrature with the main lobe From the simulations in the previous section, you should have noticed that the main lobe was in the imaginary component, and the sidelobe is in the real component (i.e. they are in quadrature). Provide an argument that this is always true for a single shot spiral gradient waveform.

hint: Would this also be true of a two interleave spiral? This problem is closely related to the partial k-space reconstruction problem for spirals.

Solution

The single interleave spiral There are a couple of ways of looking at this. One is to create a two interleave spiral by adding the original spiral to another copy of the spiral that has been rotated by 180 degrees. If $S(k_x, k_y)$ is the original spiral, and we look at the sum and differences of the two interleaves, we get

$$\begin{aligned} S_2^+(k_x, k_y) &= S(k_x, k_y) + S(-k_x, -k_y) \\ S_2^-(k_x, k_y) &= S(k_x, k_y) - S(-k_x, -k_y) \end{aligned}$$

$S_2^+(k_x, k_y)$ is an even function

$$S_2^+(k_x, k_y) = S_2^+(-k_x, -k_y)$$

so it has a real transform. The main lobe and all the sidelobes are real. In addition, it has twice the radial sampling density, so the first sidelobe is twice as far out as the original spiral (i.e. $2FOV$, where $FOV = 1/\Delta k_r$ of the original spiral). $S_2^+(k_x, k_y)$ on the other hand is an odd function

$$S_2^-(k_x, k_y) = -S_2^+(-k_x, -k_y)$$

so its transform is imaginary. In addition it is radially modulated by a sequence with alternating sign $1, -1, 1, -1, \dots$. This corresponds to shifting the main lobe to a radius that is half of that for the $S_2^+(k_x, k_y)$ first sidelobe ($2FOV$), so it is the same radius as the sidelobe for the original spiral (FOV). Also note that this results in no contribution at low frequency.

The original spiral can be written as a linear combination of $S_2^+(k_x, k_y)$ and $S_2^-(k_x, k_y)$,

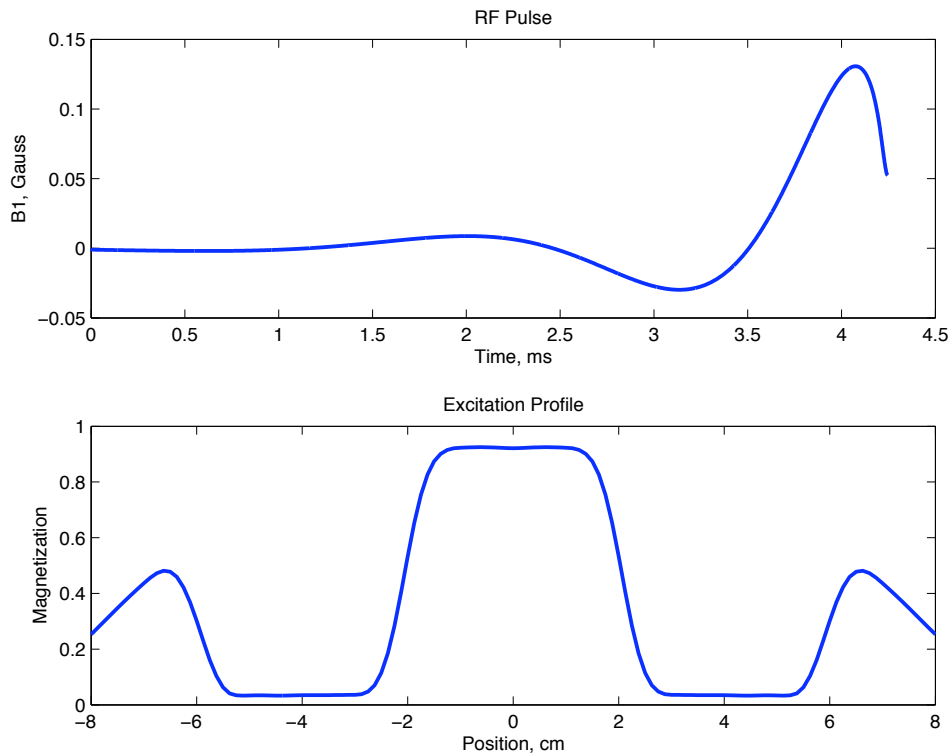
$$S(k_x, k_y) = \frac{1}{2} \left(S_2^+(k_x, k_y) + S_2^-(k_x, k_y) \right)$$

This has the mainlobe at the origin (from $S_2^+(k_x, k_y)$), the quadrature sidelobe at a radius FOV (from $S_2^-(k_x, k_y)$), and a real sidelobe at a radius $2FOV$, and subsequent sidelobes at multiples of FOV , alternating between real and imaginary.

An alternate approach is to consider the samples along a diameter. The transform of these samples is the projection of the excitation profile, by the central section theorem. A similar argument to the preceding one can then be used to solve for the sidelobe locations and phases.

5. Improved profile Design an RF pulse that is approximately 4 cm in diameter that produces a windowed jinc k-space weighting. Plot the RF pulse as a function of time, the magnitude of the 1D profile from -8 to 8 cm, and mesh plots of the real and imaginary components over the same range.

Solution The RF pulse, and the cross section through the 2D excitation profile look like



Note that the profile is very nicely defined, but the baseline is not right. We'll deal with this in the next assignment. Note also that both the mainlobe and the sidelobe have gotten broader by the same factor compared to the plots in question 3 above.

The most common problem here was sampling the jinc properly in kr , but then using a window function that was sampled uniformly in time. The result is a pulse with not quite as nice a profile.

The surface plots look like:

