Unconventional RF Pulses and Pulse Pairs

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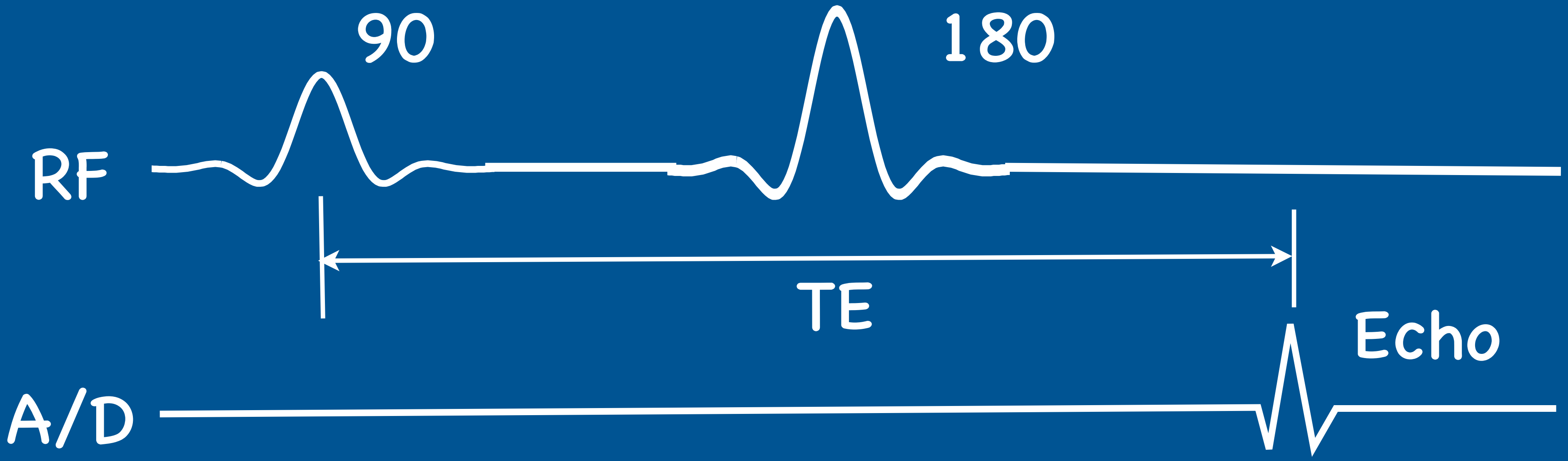
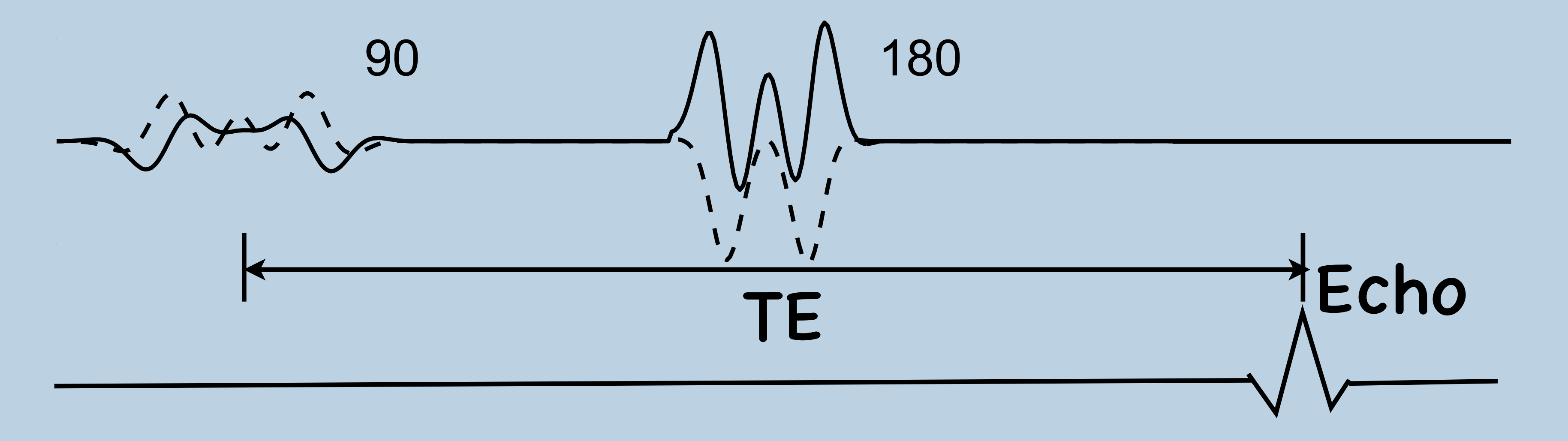
Typical Spin Echo Pulse Sequence



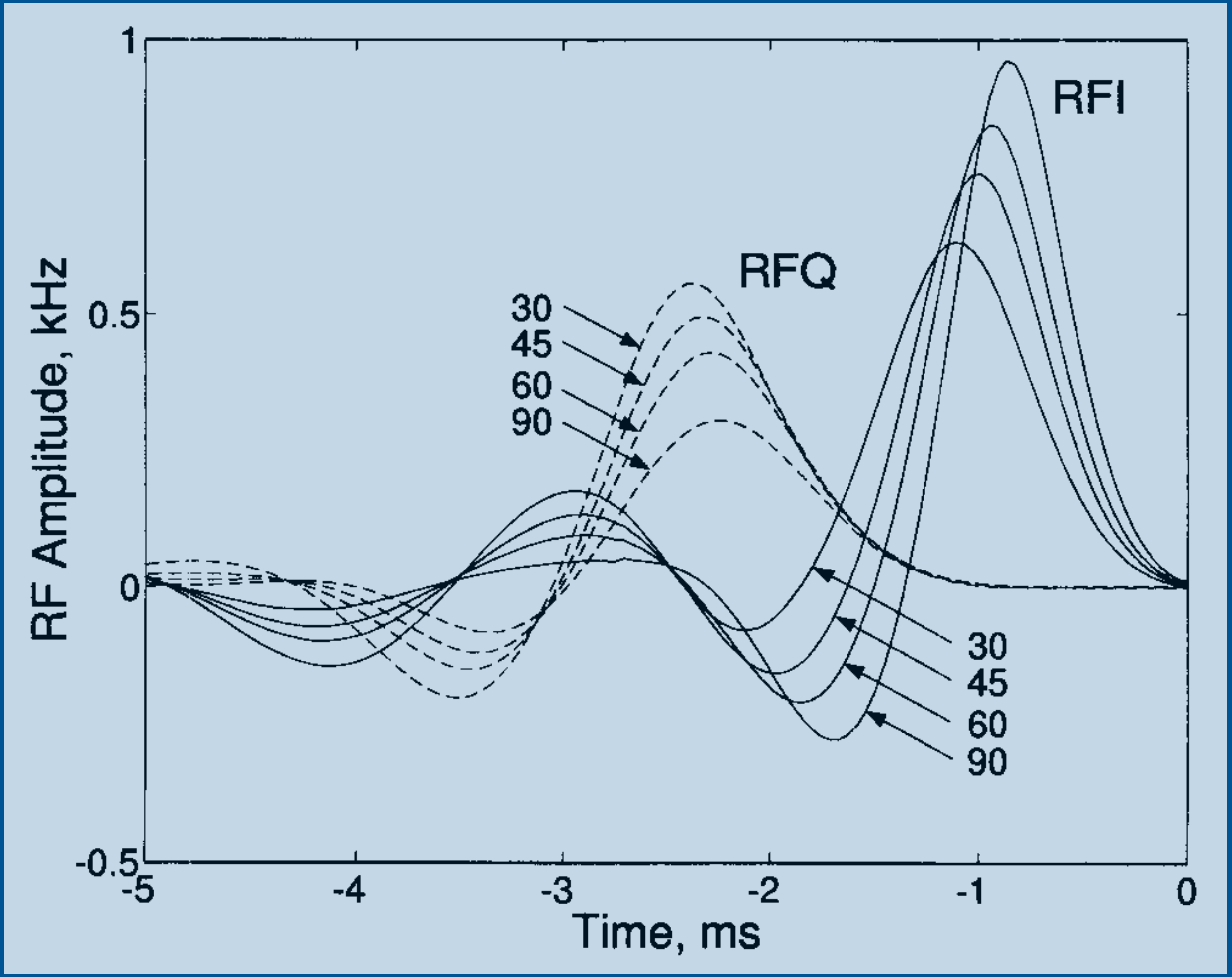
- Pulses designed independently
- Echo time limited
- Large dynamic range, demanding for RF amp

Designing Spin Echoes



- Design the spin echo instead of the pulses!
- Matched pulse pairs produce a linear phase echo
- Non-linear phase pulses have lower peak power
- Much more selective, less demanding for the RF amp

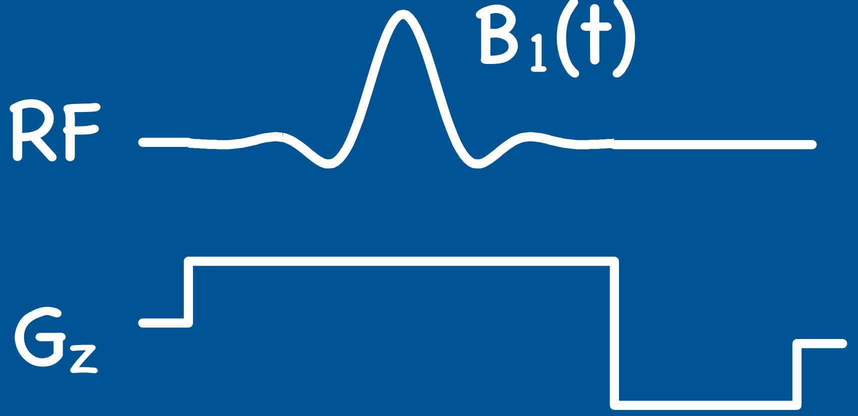
Short Echo Times

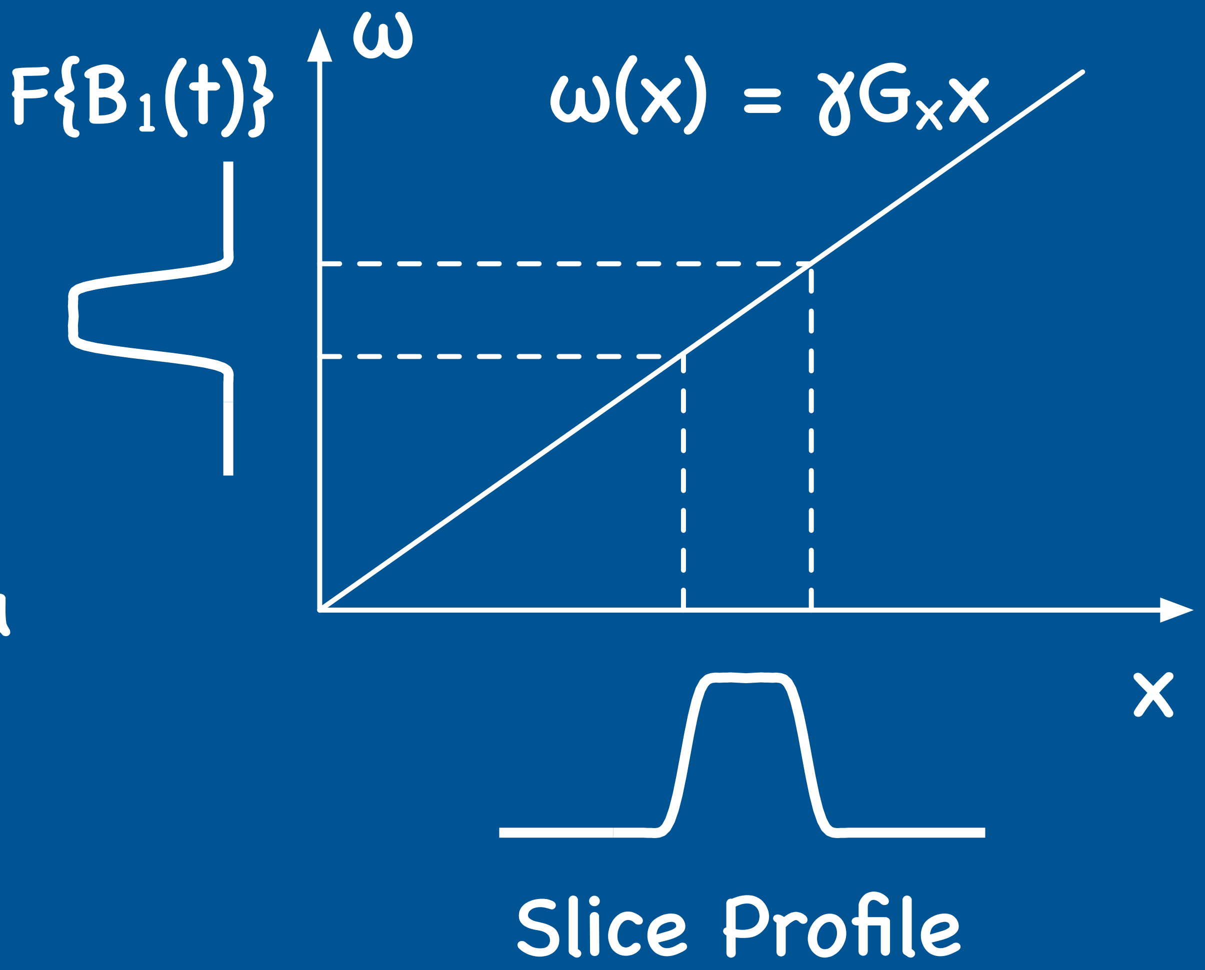


- The pulses can overlap, allowing short spin echo times
- Arbitrary flip angles
- Phase cycles

Lim, et al. Magnetic Resonance in Medicine 32:98ı103 (1994)

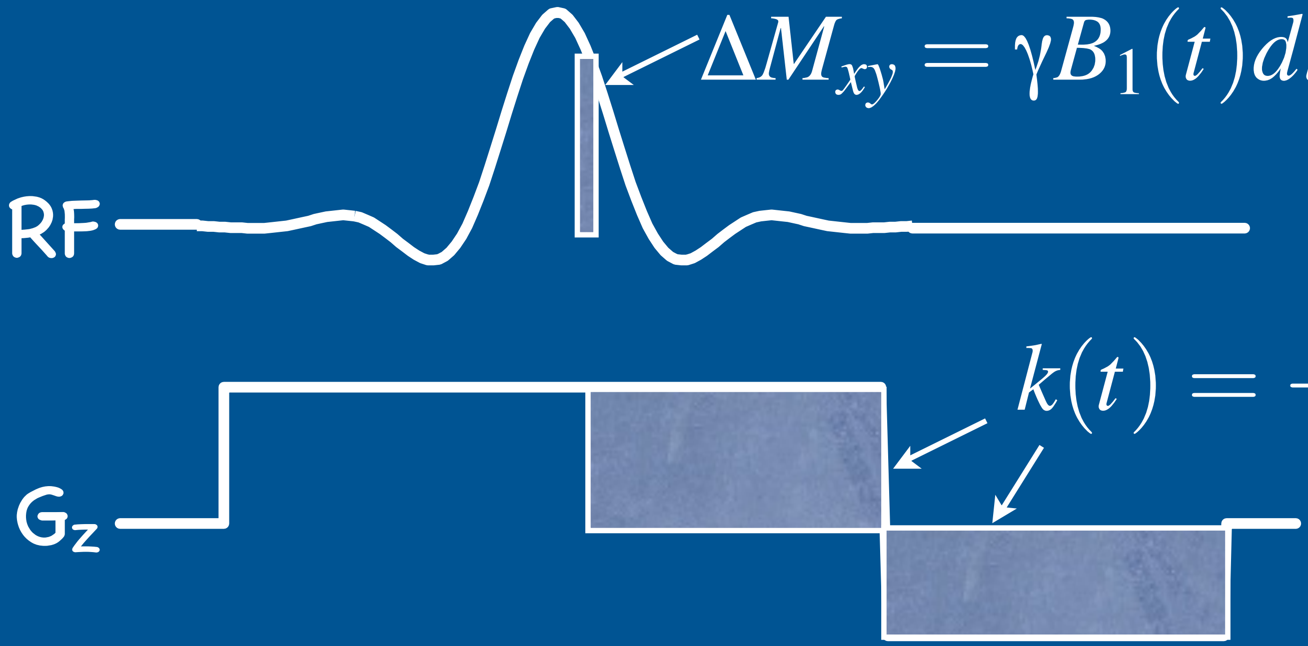
Slice Selective Excitation





Choose $B_1(t)$ with a limited range of frequencies

Small-Tip-Angle Approximation

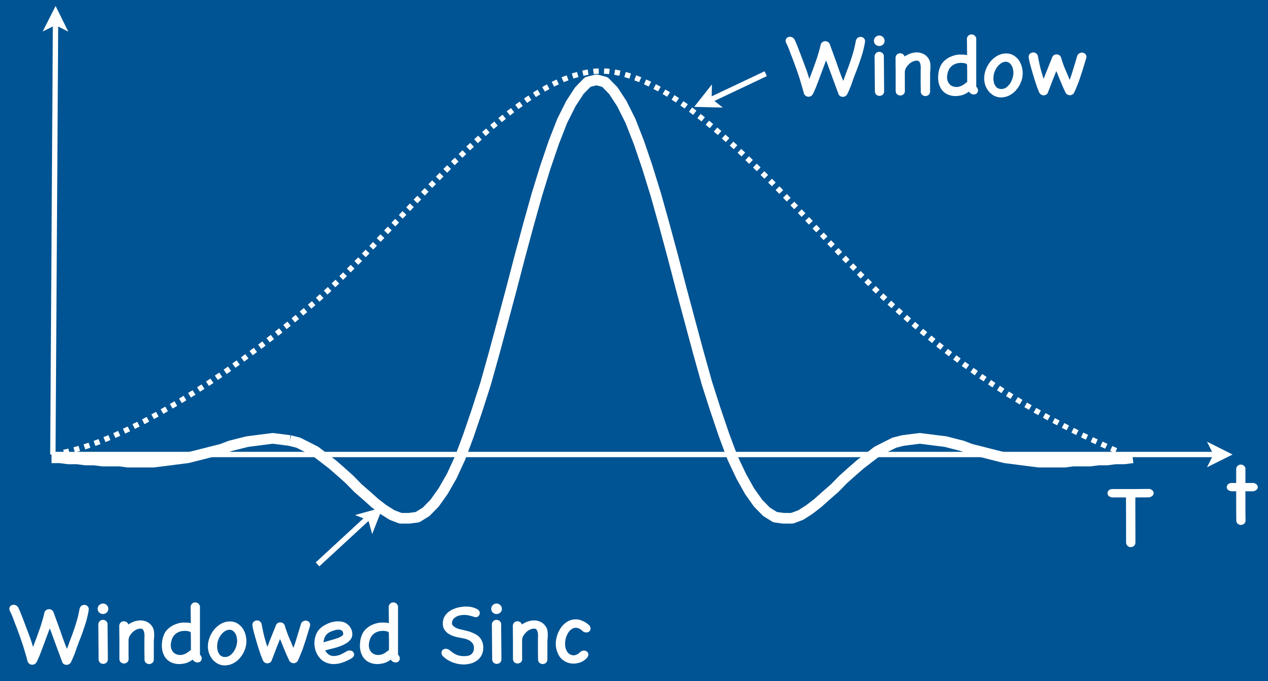


$$\Delta M_{xy} = \gamma B_1(t)\,dt$$

$$k(t) = -\gamma \int_t^T G(s)\,ds$$

$$M_{xy}(x) = \int_0^T \gamma B_1(t) e^{ik(t)x}\,dt$$

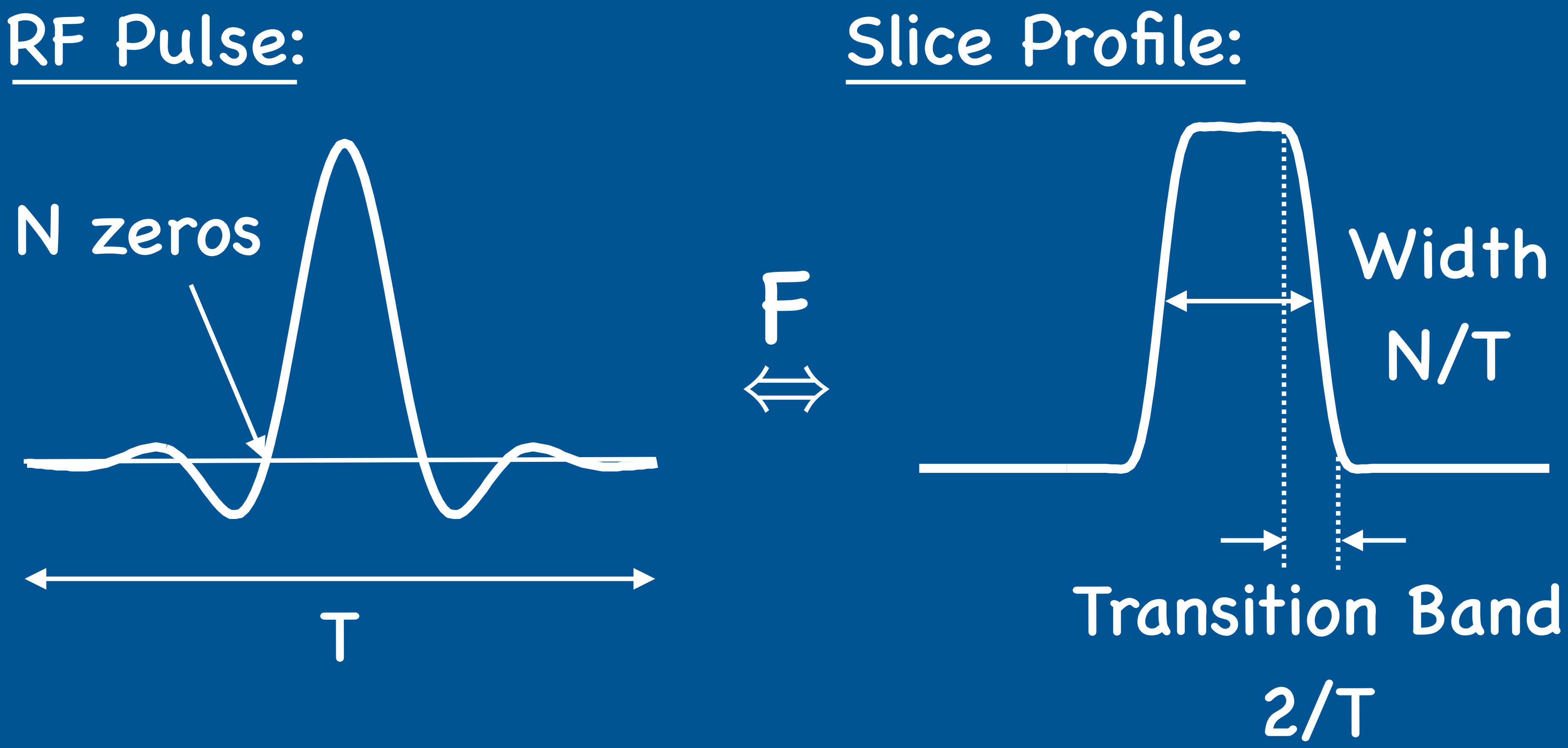
Designing Small-Tip-Angle Pulses



Key Parameters:

- Duration, T
- Number of zeros, N (here 8)

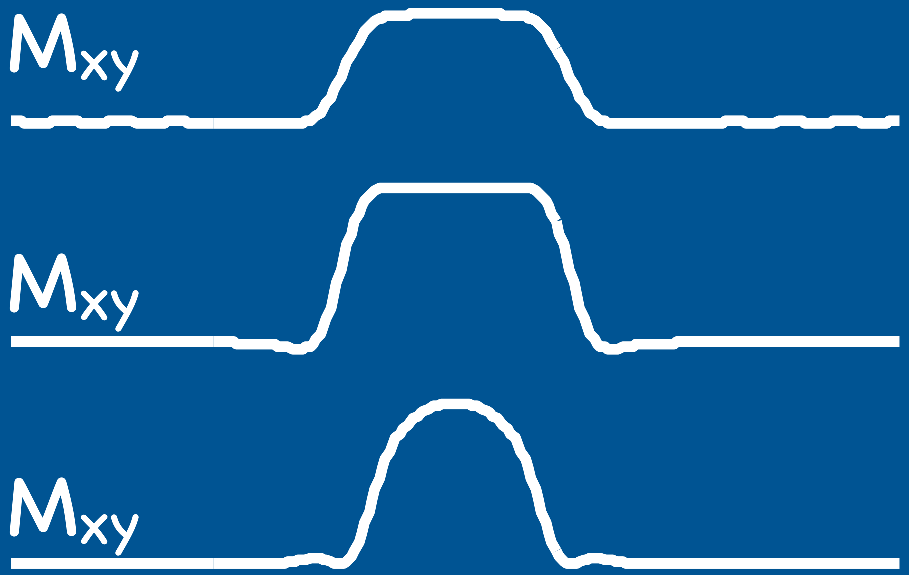
Slice Profile



N is the Time-Bandwidth Product of the pulse

Large-Flip-Angle Pulses

Fourier-based designs work well to 90°



30° Excitation

90° Excitation

180° Refocusing

Non-linear problem beyond 90°

Spin Domain

- Solve for rotations instead of magnetization
- Rotation represented by 2x2 unitary matrix

$$Q = \begin{pmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{pmatrix}$$

- α and β are

  $\alpha = \cos(\varphi/2) - i n_z \sin(\varphi/2)$

  $\beta = -i(n_x + i n_y)\sin(\varphi/2)$

- Rotation is by angle φ about axis $(n_x, n_y, n_z)^T$
- Magnitude constraint: $\alpha\alpha^* + \beta\beta^* = 1$

Magnetization from Spin Domain

Simple to compute slice profiles from spin domain:

$M_{xy} = 2\alpha^{*}\beta\, M_0$	Excitation
$M_z = (1-2\beta\beta^{*})\, M_0$	Inversion/Saturation
$M_{xy}^{+} = -\beta^{2}(M_{xy}^{-})^{*}$	Refocusing

Hard Pulse Approximation

- Represent RF as impulses separated by free precession intervals (discrete time approximation)
- Spinors are two polynomials $A_N(z)$ and $B_N(z)$ in $z=e^{i\gamma Gx\Delta t}$ where Δt is sampling time
- This is invertible, given $A_N(z)$ and $B_N(z)$ we can find $B_1(t)$

Shinnar-Le Roux Algorithm

- Design $B_N(z)$ to approximate the desired flip angle profile

  $$\beta(x) = -i(n_x+in_y)\sin(\varphi(x)/2)$$

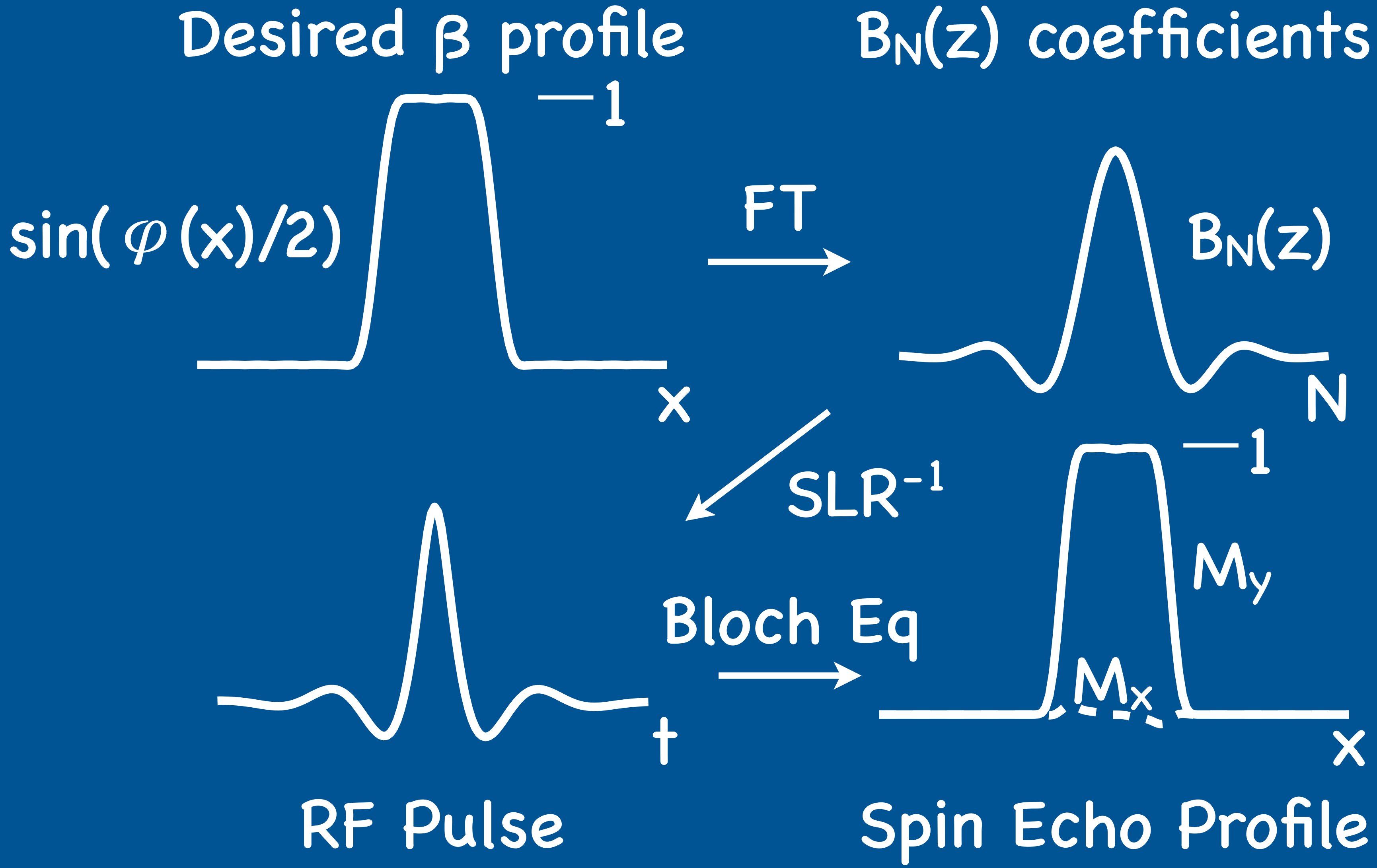
  using a Fourier design (filter design).

- Find a consistent $A_N(z)$ using magnitude constraint, minimum power (min phase)

- Solve for $B_1(t)$

  $$B_1(t) = SLR^{-1}(A_N(z),B_N(z))$$

Example: Spin Echo Pulse



Desired β profile

$B_N(z)$ coefficients

FT

SLR^{-1}

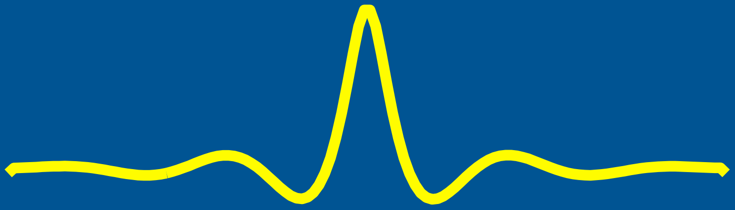
Bloch Eq

RF Pulse

Spin Echo Profile

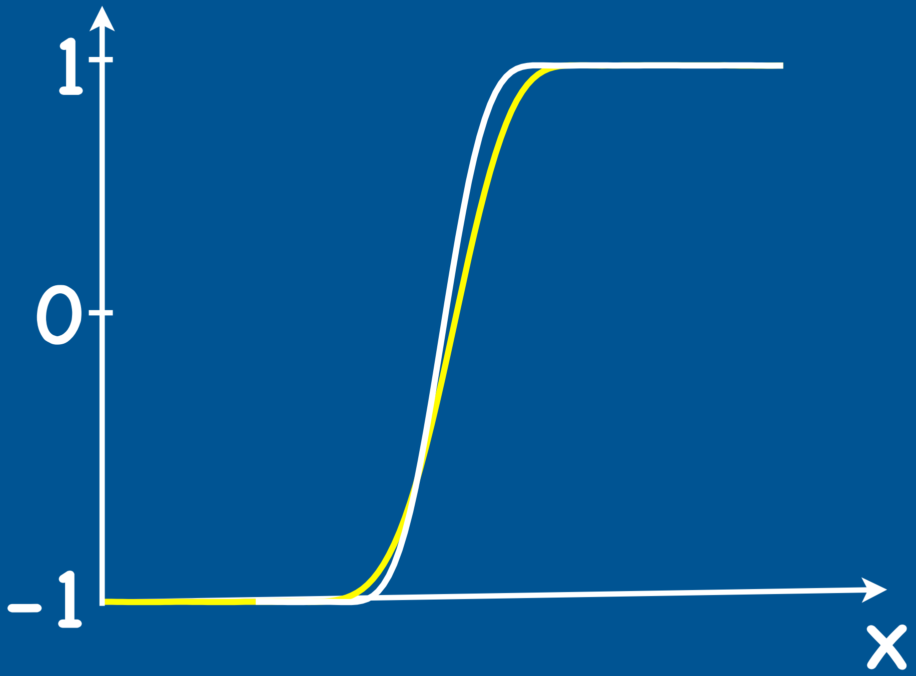
Other Phase Profiles

Linear Phase



Minimum Phase

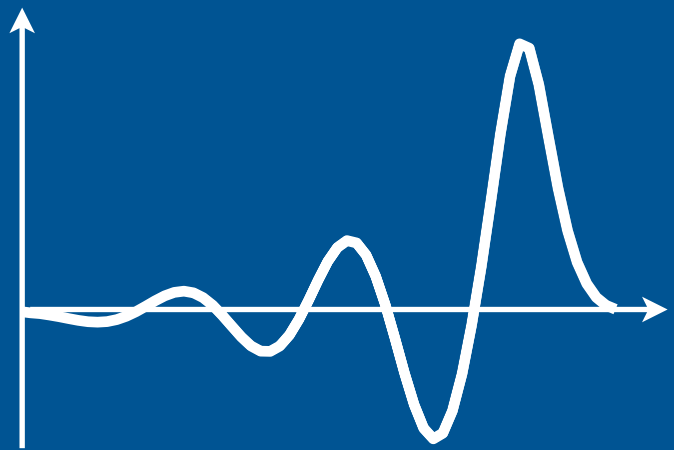
Inversion Profile



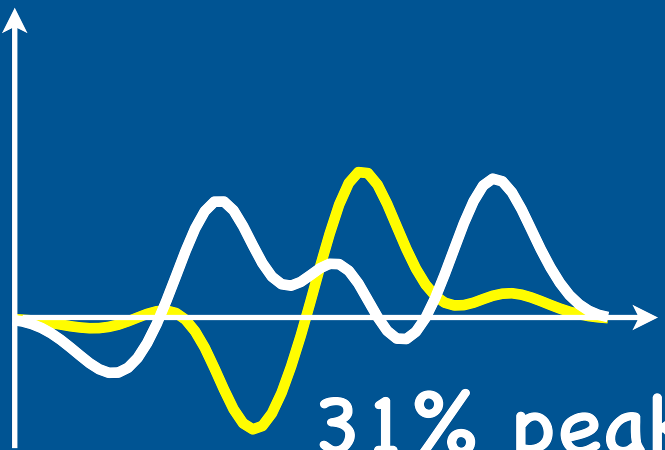
Minimum phase pulse can have half the transition width

Other Phase Profiles

Minimum Phase



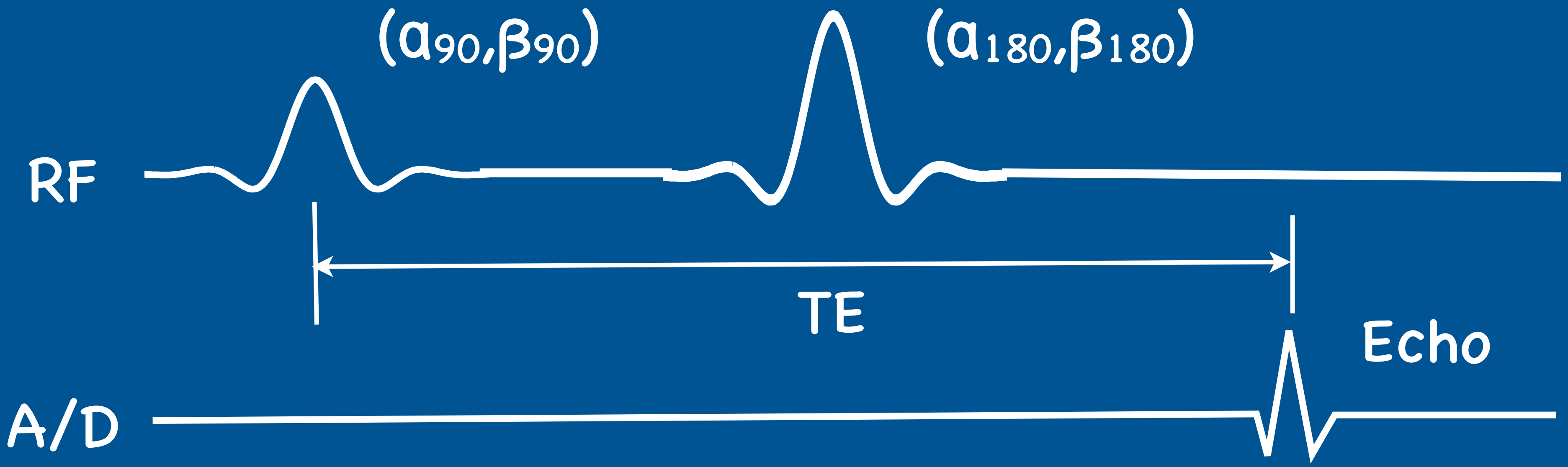
Non-linear Phase



31% peak power

- Linear, minimum phase: high peak power
- Optimized non-linear phase reduces peak power
- Same total power (SAR)
- Identical magnitude profile

Designing Spin Echoes



- M_{xy} at the echo is

$$M_{xy} = (2\alpha_{90}\beta^*_{90})(-\beta^2_{180})\, M_0$$

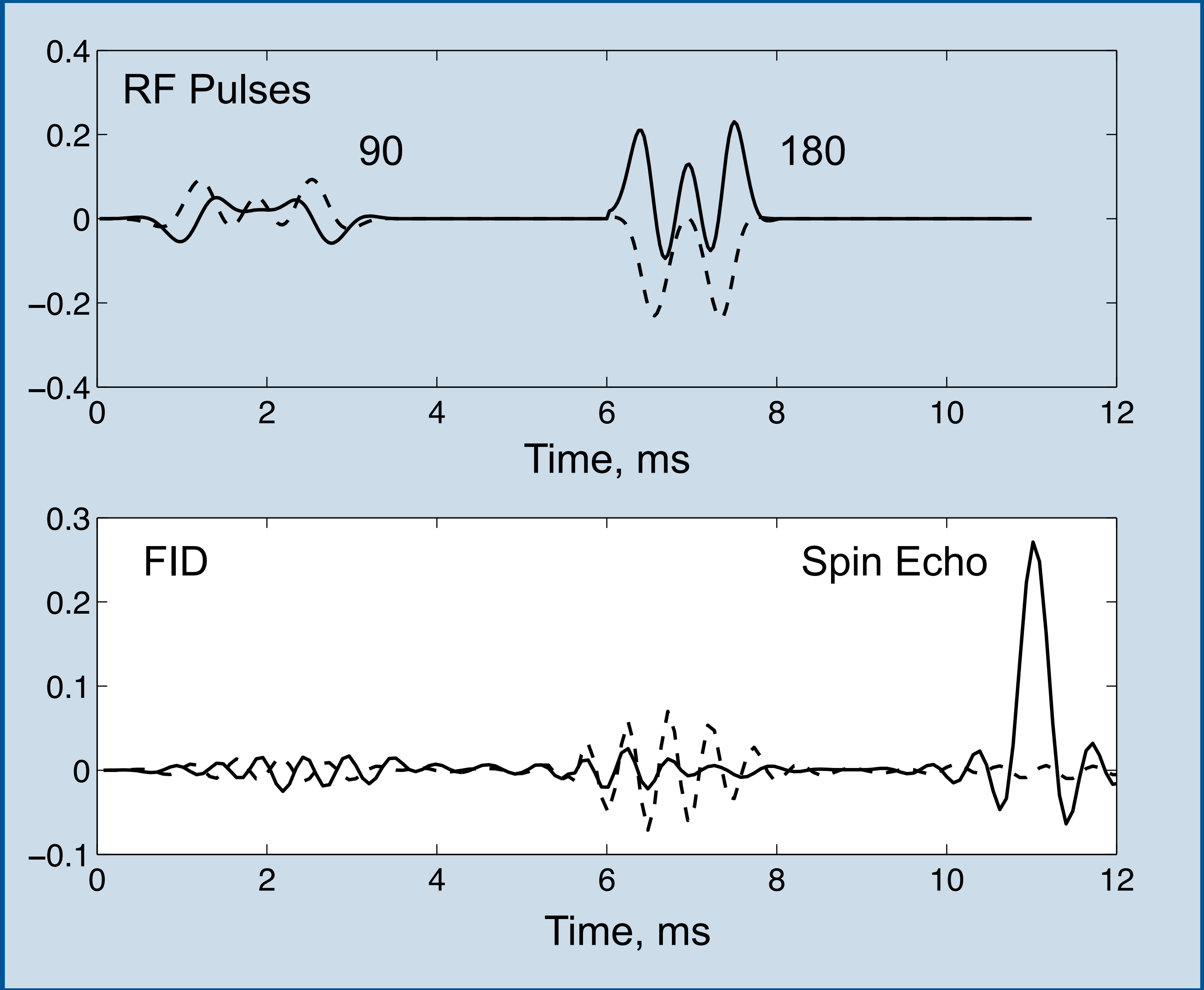
- Choose

$$\beta_{90} = (1/\sqrt{2})(i\beta^2_{180})$$

Matches profile and phase

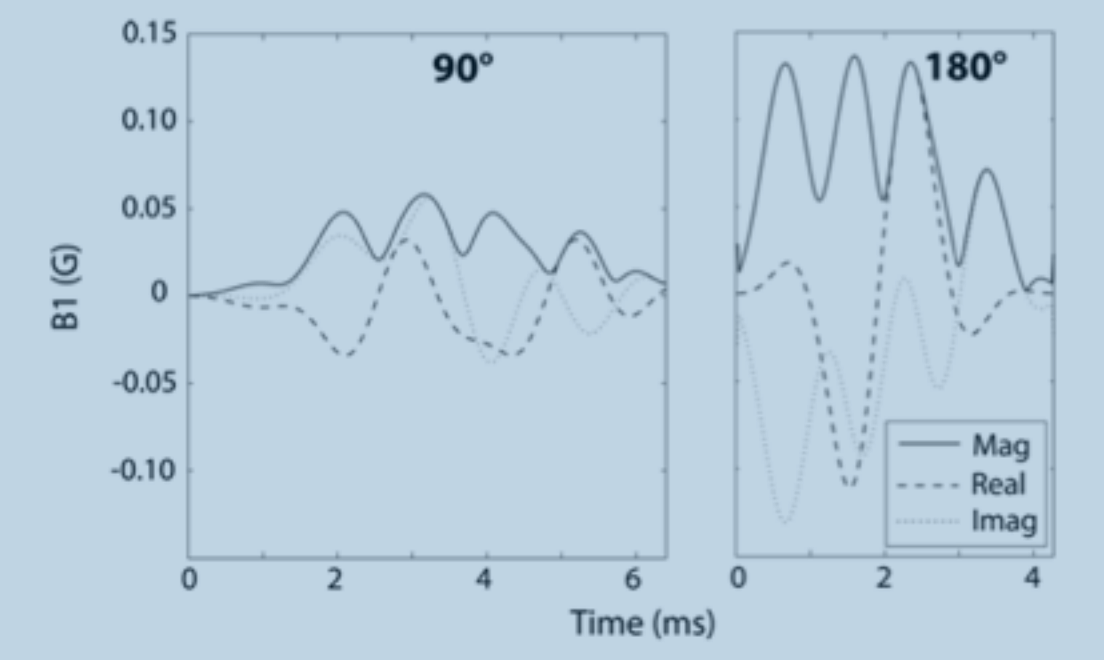
$$M_{xy} \simeq |\beta_{180}|^4\, M_0$$

Non-Linear Phase Example



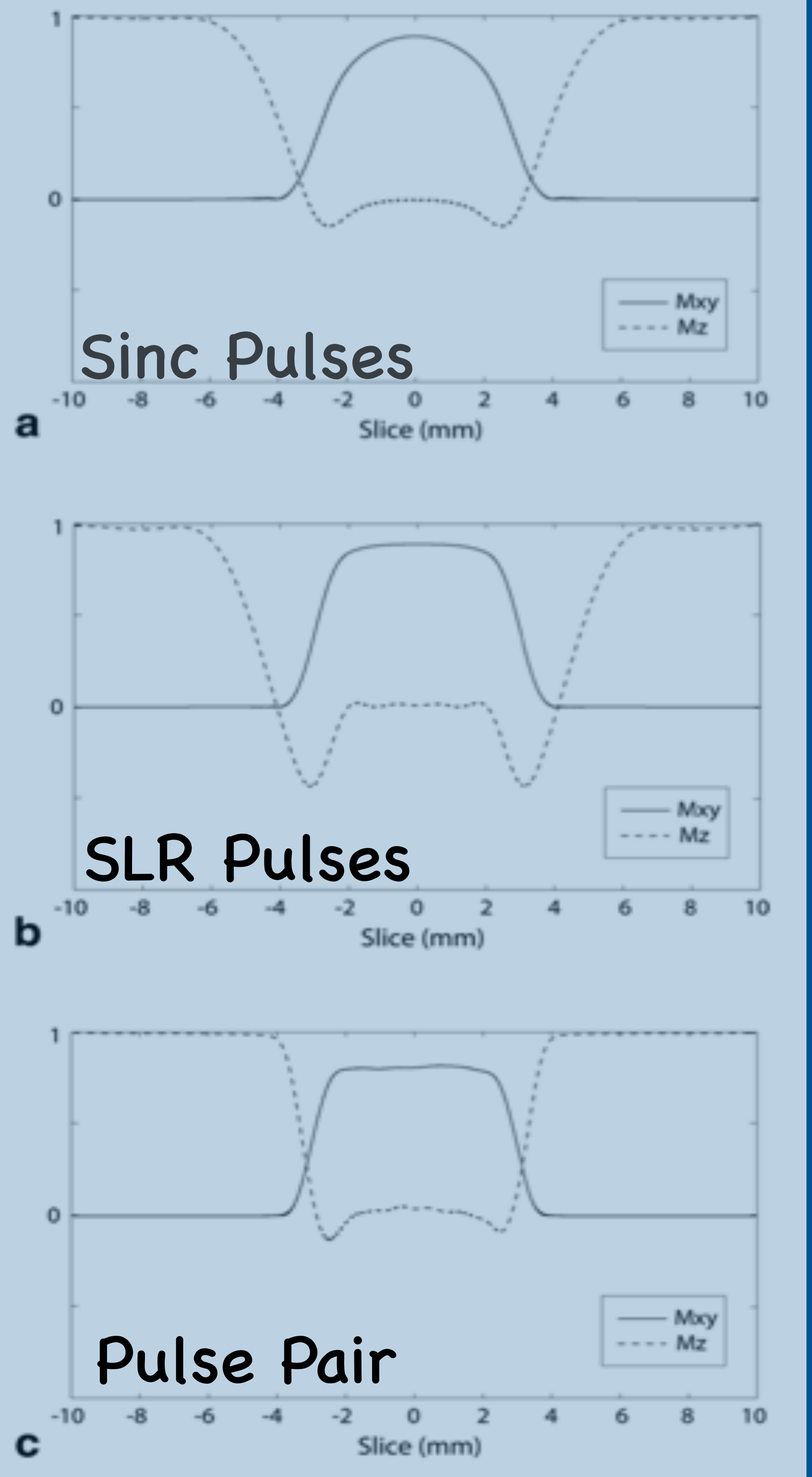
- Phases exactly cancel
- Perfect linear phase spin echo
- 90 is twice as long as 180

Near-Contiguous Spin Echoes

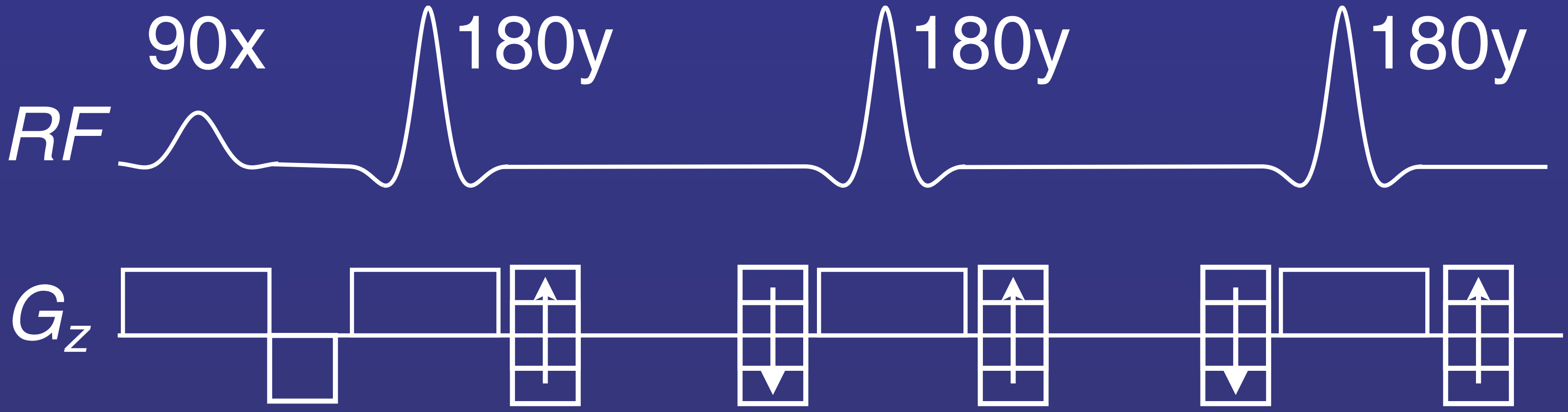


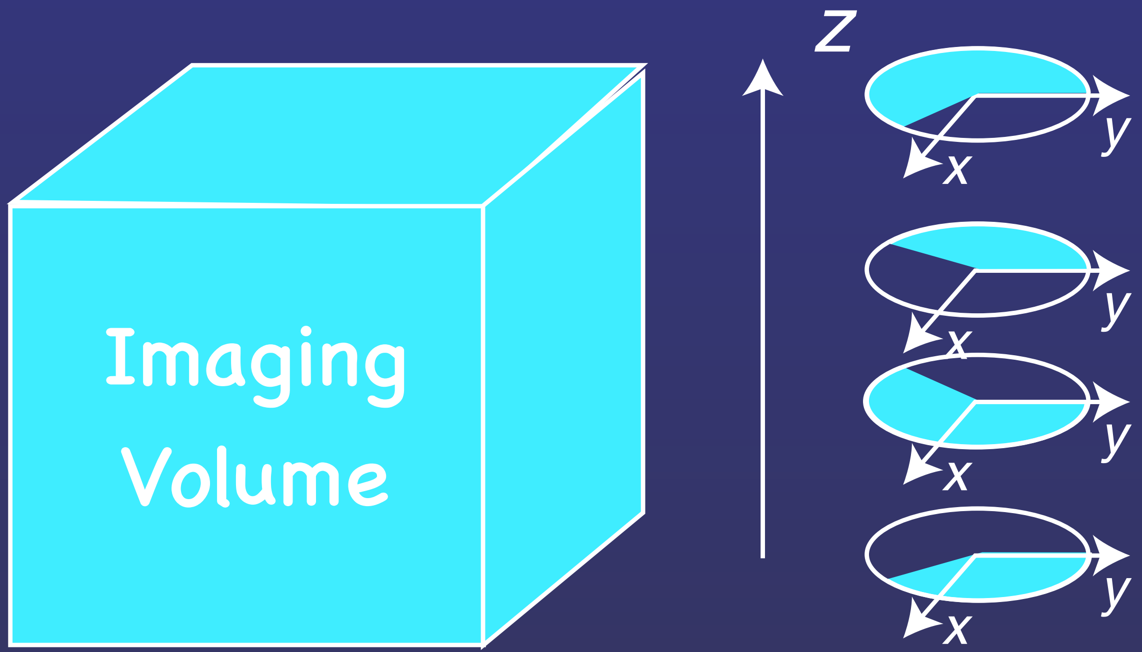
- Used for arterial spin labeling, where the sharp transition is important

Zun, et al. Magn. Reson. in Med, online (2013)



Non-Linear Phase CPMG

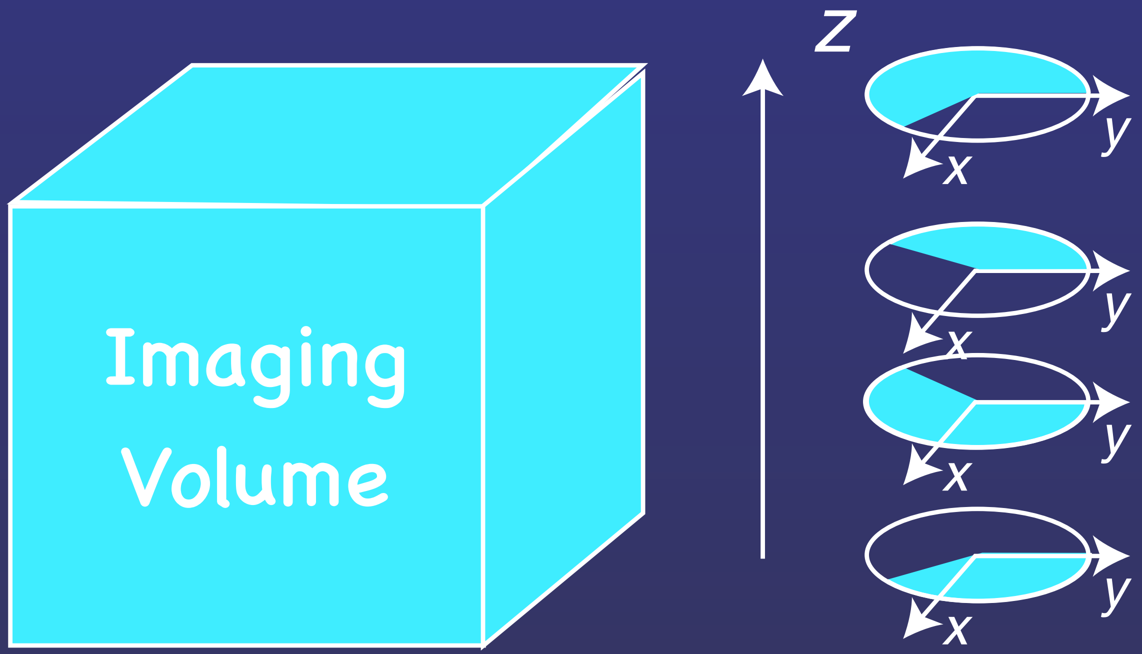




Goal:

Different absolute phase at each z position

Non-Linear Phase CPMG



Goal:
Different absolute phase at each z position

CPMG Echoes

$(2\alpha^*_{90}\beta_{90})M_0$ $(2\alpha_{90}\beta^*_{90})(-\beta^2_{180})\ M_0$

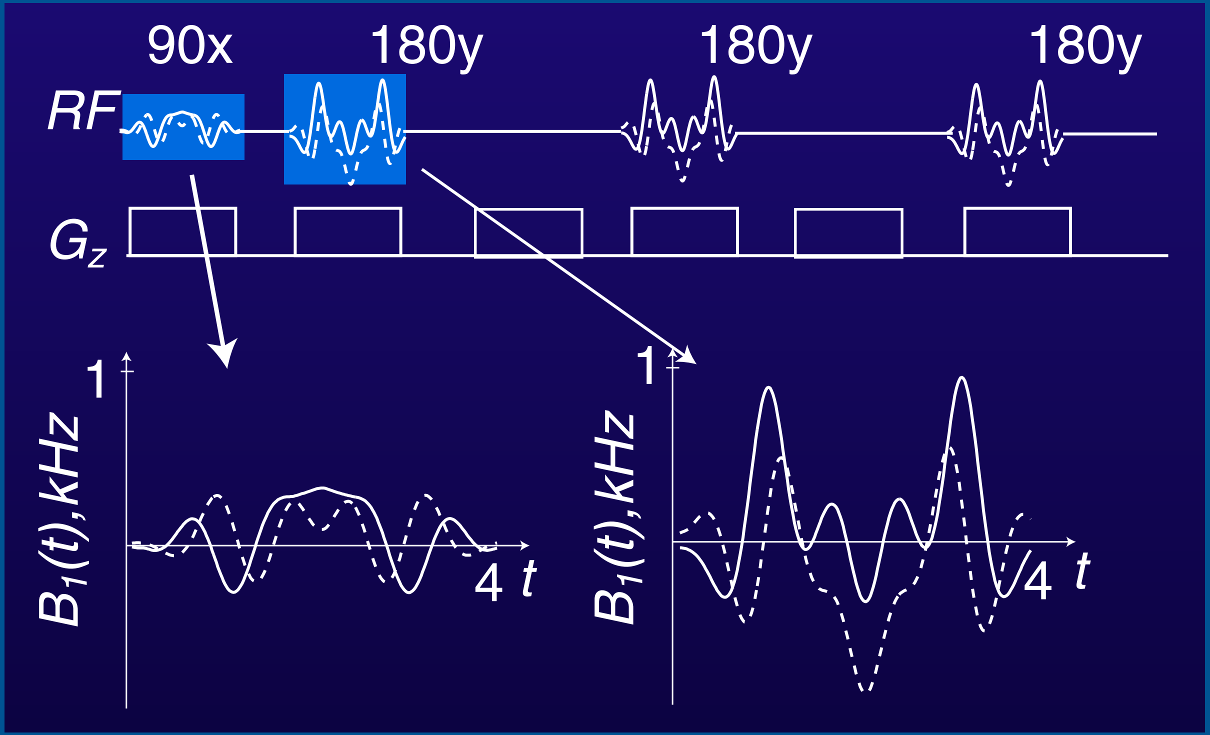
- Choose

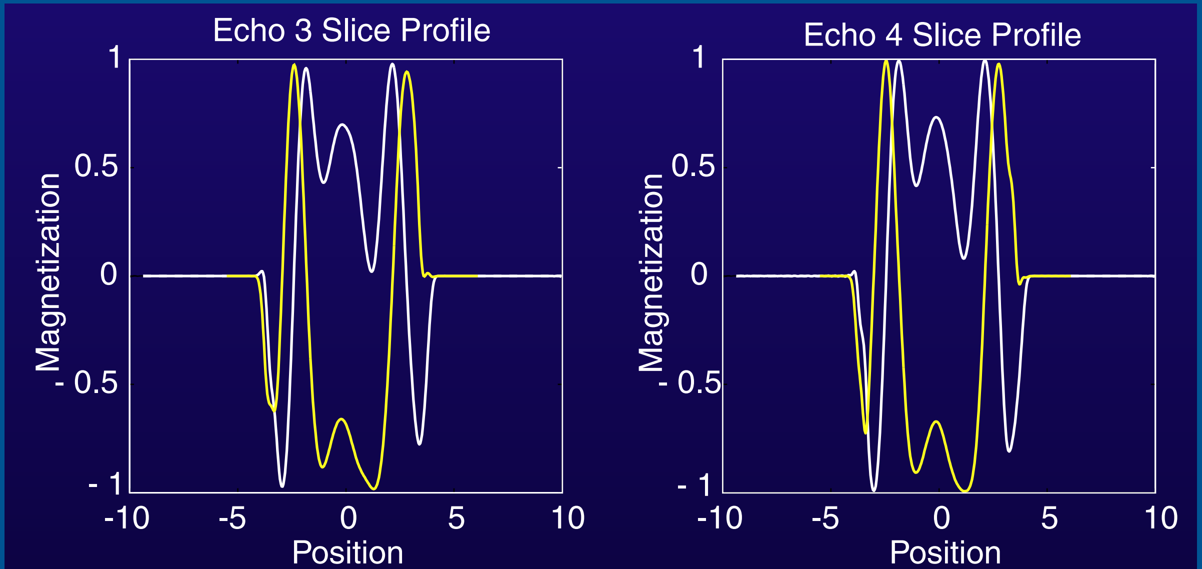
$$\beta_{90} = (1/\sqrt{2})(i\beta_{180}) \quad => \quad M_{xy,90} \simeq i\beta_{180}\ M_0$$

$$M_{xy} \simeq i\beta_{180}\ |\beta_{180}|^2\ M_0$$

Echo has same phase profile as initial M_{xy}

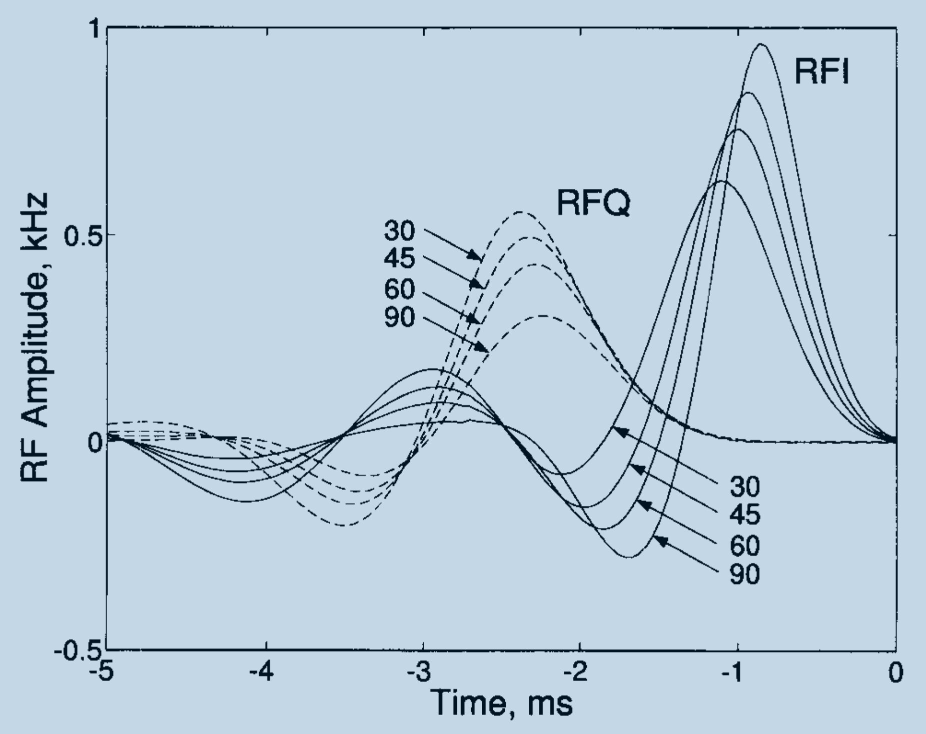
Non-Linear Phase CPMG





- Echoes all have same profile and polarity
- Works for any phase profile, including frequency sweeps (SPEN)

Overlapping Pulses

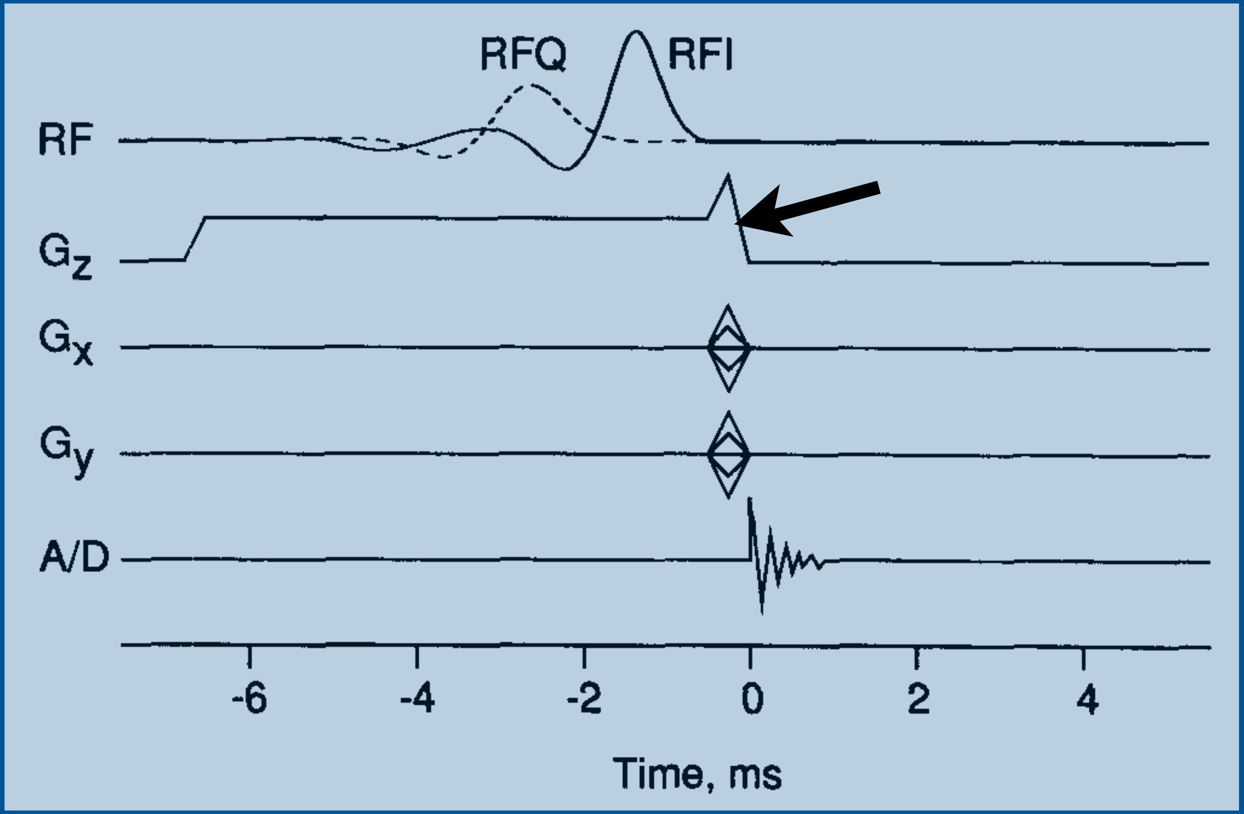


2.5 ms Echo Time
(immediately after end of pulse)

- The two pulses don't need to be distinct
- Give $(\alpha_{90}, \beta_{90})$ and $(\alpha_{180}, \beta_{180})$
  $\beta_{pp} = z^{D} \beta_{90} \alpha^{*}_{180} + \alpha_{90} \beta_{180}$
  $\alpha_{pp} = z^{-D} \alpha_{90} \alpha_{180} - \beta_{90} \beta^{*}_{180}$
- D is the echo delay (in samples)
- Find $B_1(t)$ via SLR back recursion

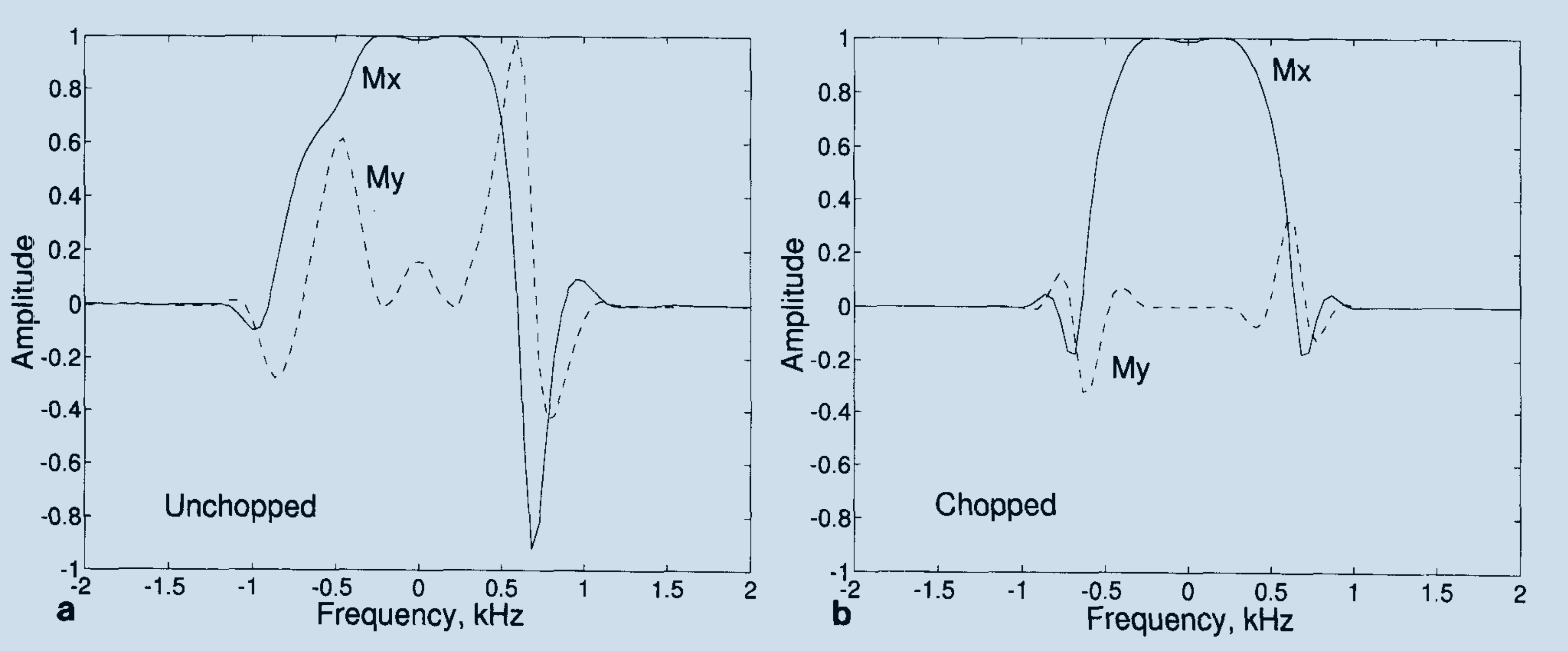
Lim, et al. Magnetic Resonance in Medicine 32:98–103 (1994)
Balchandani, et al. Magnetic Resonance in Medicine 67:1077–1085 (2012)

Pulse Sequence



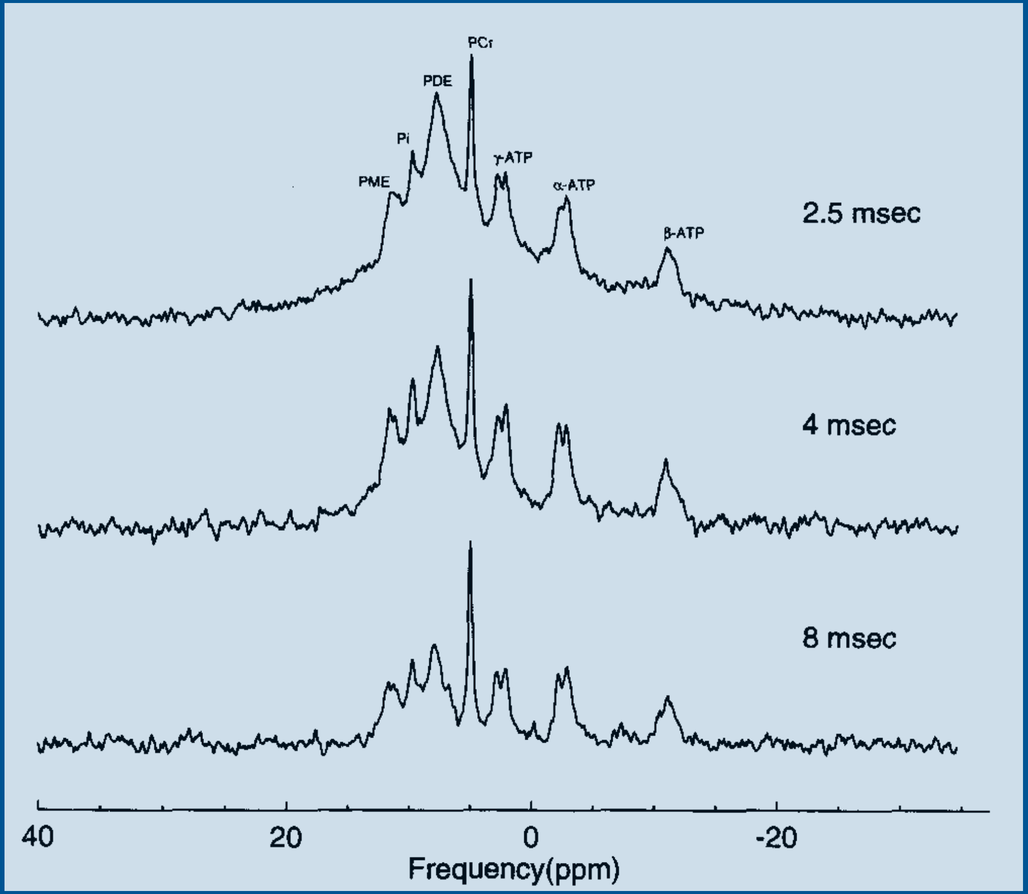
- z-Gradient needs added area
- Same as if it was on constantly until echo time
- Conjugate phase suppresses crushed component from 180

Slice Profiles



- Single pulse has spin echo and crushed signal
- Crushed component subtracts out

Slice Selective ^{31}P Spectra



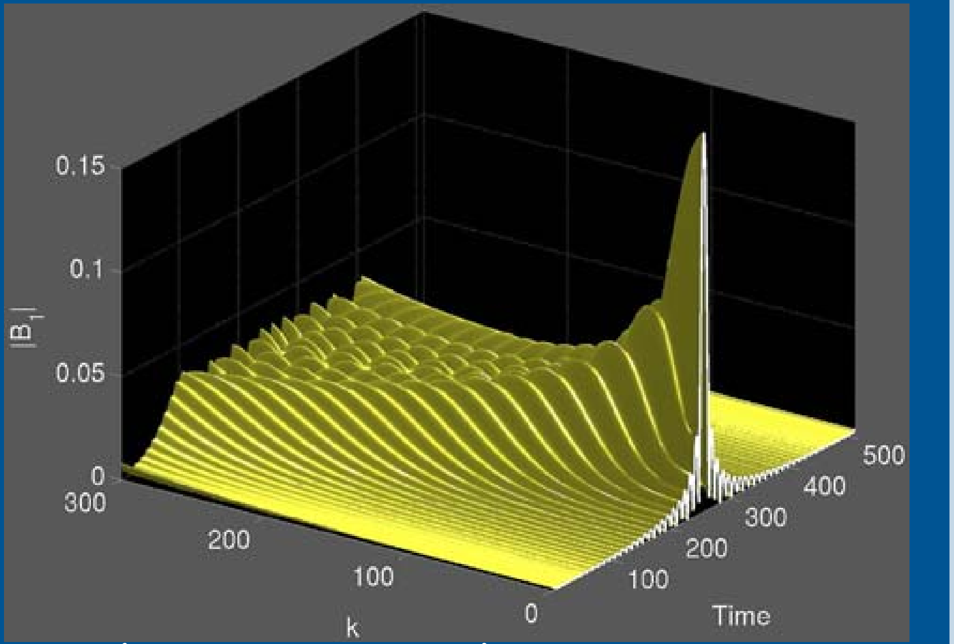
- Pulse pairs with different echo delays
- Axial 3.3 cm sllice of brain
- 1.5T, 250 ms A/D, 2 s TR

Lim, et al. Magnetic Resonance in Medicine 32:98–103 (1994)

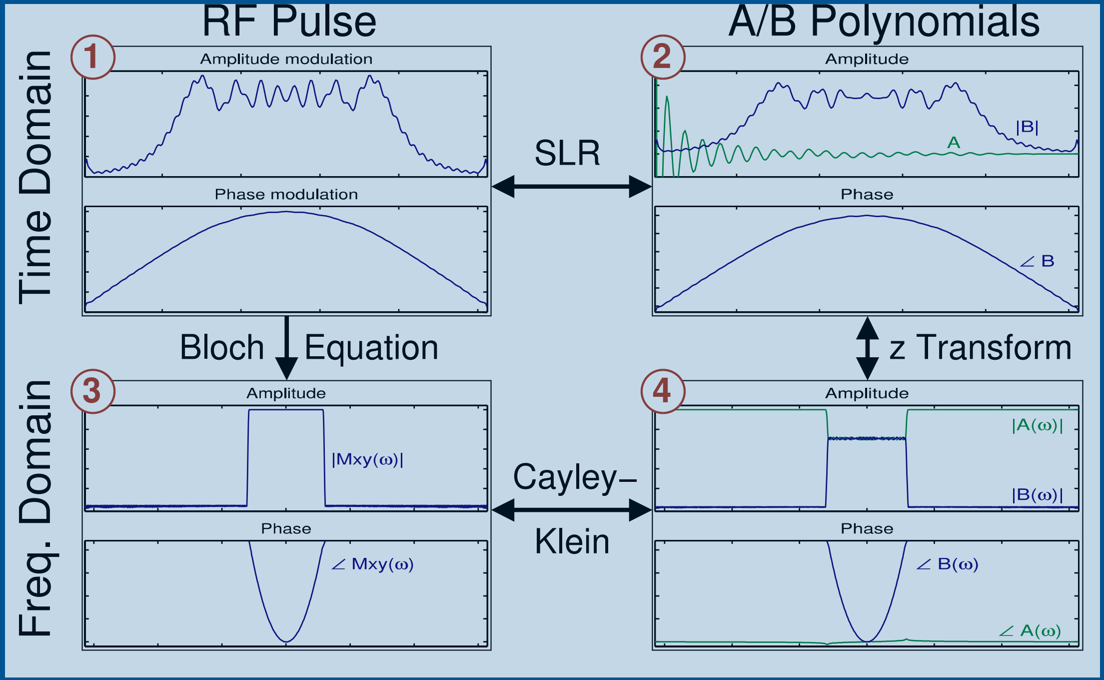
Non-Linear Phase β's

- Many ways to design non-linear phase β
  Widely studied for saturation pulses
- Many options:
  - Add quadratic phase to a linear phase β
  - Use complex Remez algorithm
  - Design a minimum phase pulse, factor, and zero flip
  - Use an adiabatic pulse as a prototype

Quadratic Phase



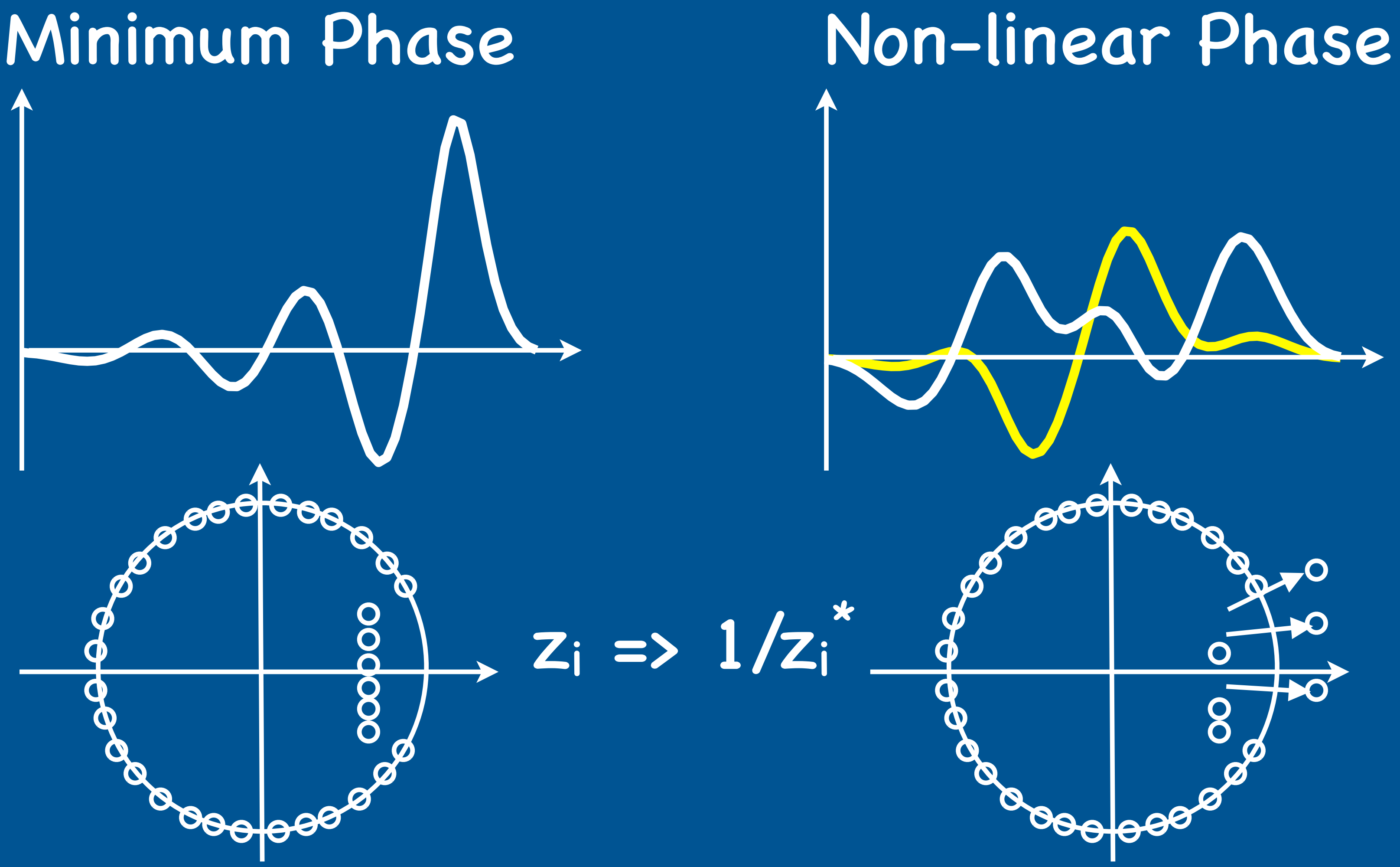
(Quadratic Phase)



- Complex Remez algorithm will produce an optimal design for a specified phase and amplitude profile
- Paper by Schulte et al. tells you what to ask for!

R.F. Schulte et al. Journal of Magnetic Resonance 166 (2004) 111–122

Zero Flipping



$z_i \Rightarrow 1/z_i^*$

TBW= N has 2^N possible phase profiles!

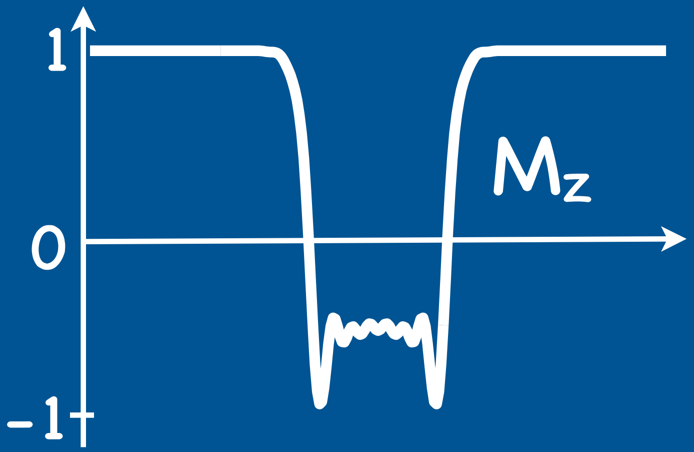
Max peak amplitude reduced ~$1/\sqrt{N}$

Adiabatic Pulses

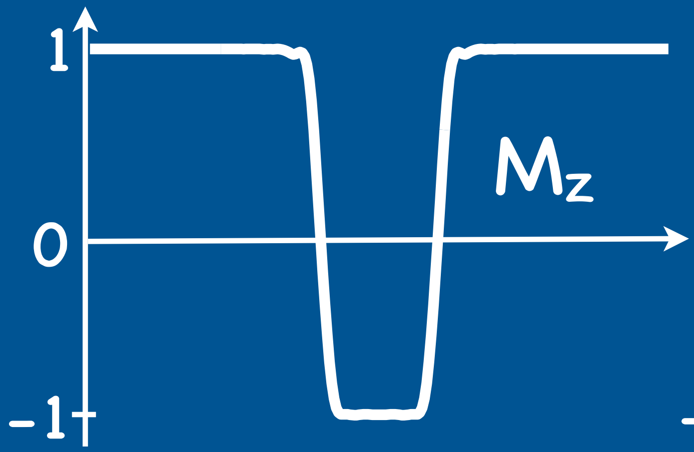
Slice selective inversion

Insensitive to amplitude

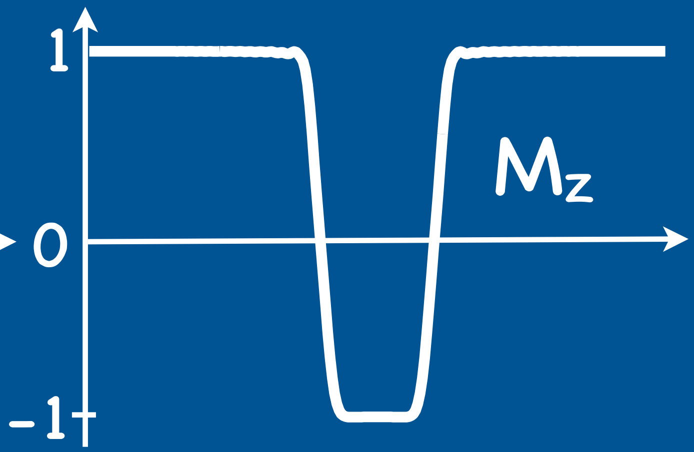
$$B_1(t) = A_0 \mathrm{sech}(\beta t) e^{-i\mu\beta \tanh(\beta t) t}$$



50%

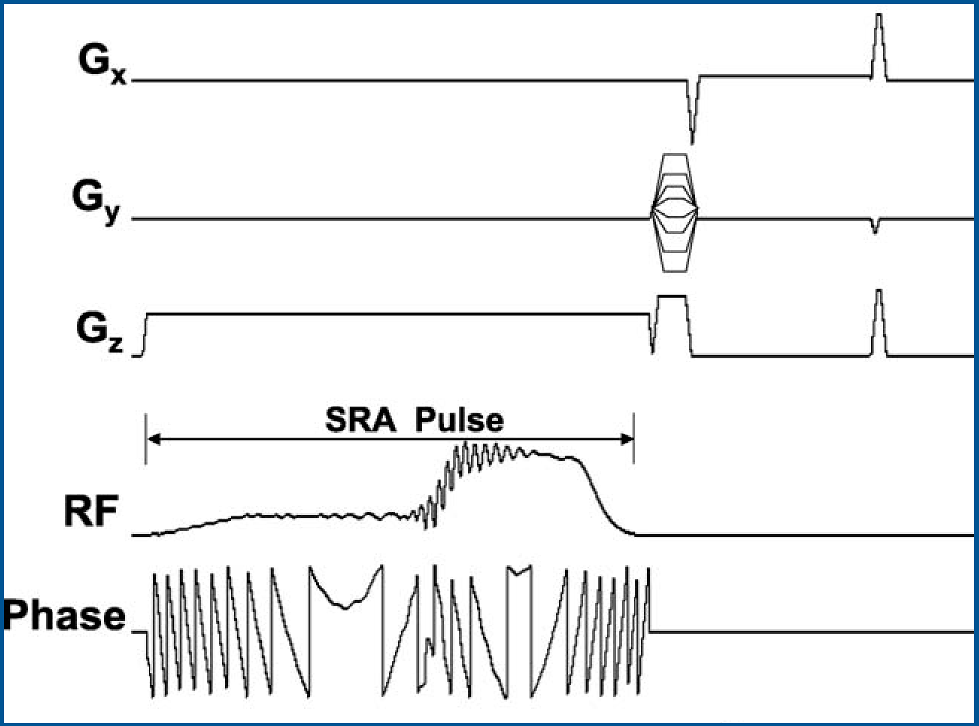


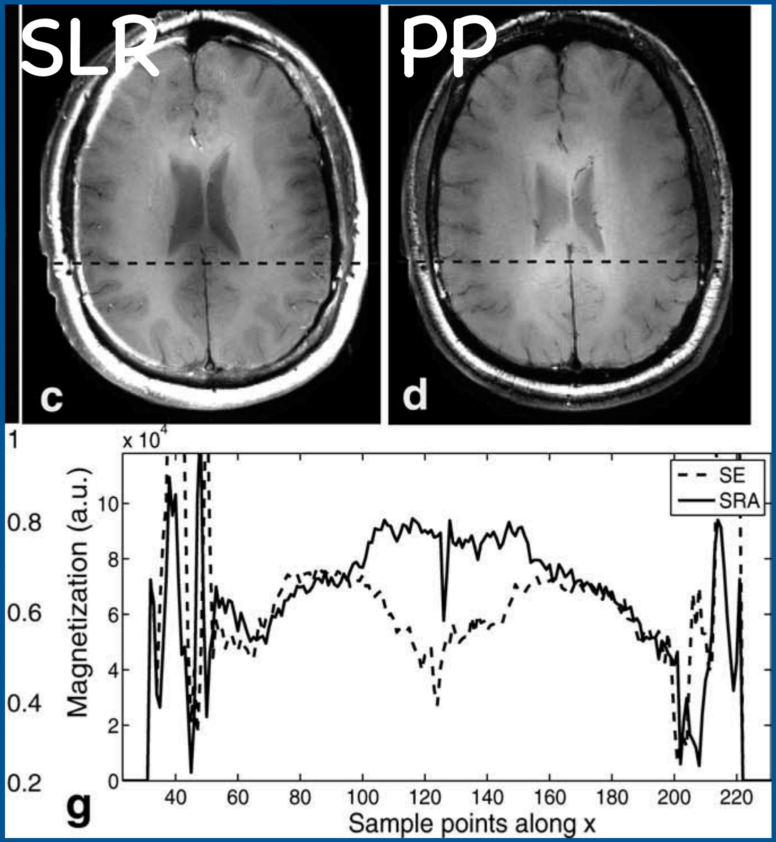
100%



200%

Adiabatic Pulse Pairs





- Pulse pair with adiabatic 180, and overlapping phase compensated 90
- Compensates for B1 at 7T

Balchandani, et al. Magnetic Resonance in Medicine 67:1077–1085 (2012)

True Self Refocusing Pulses

- Another option: start with a minimum phase excitation, and add phase to α
- Original Mxy is

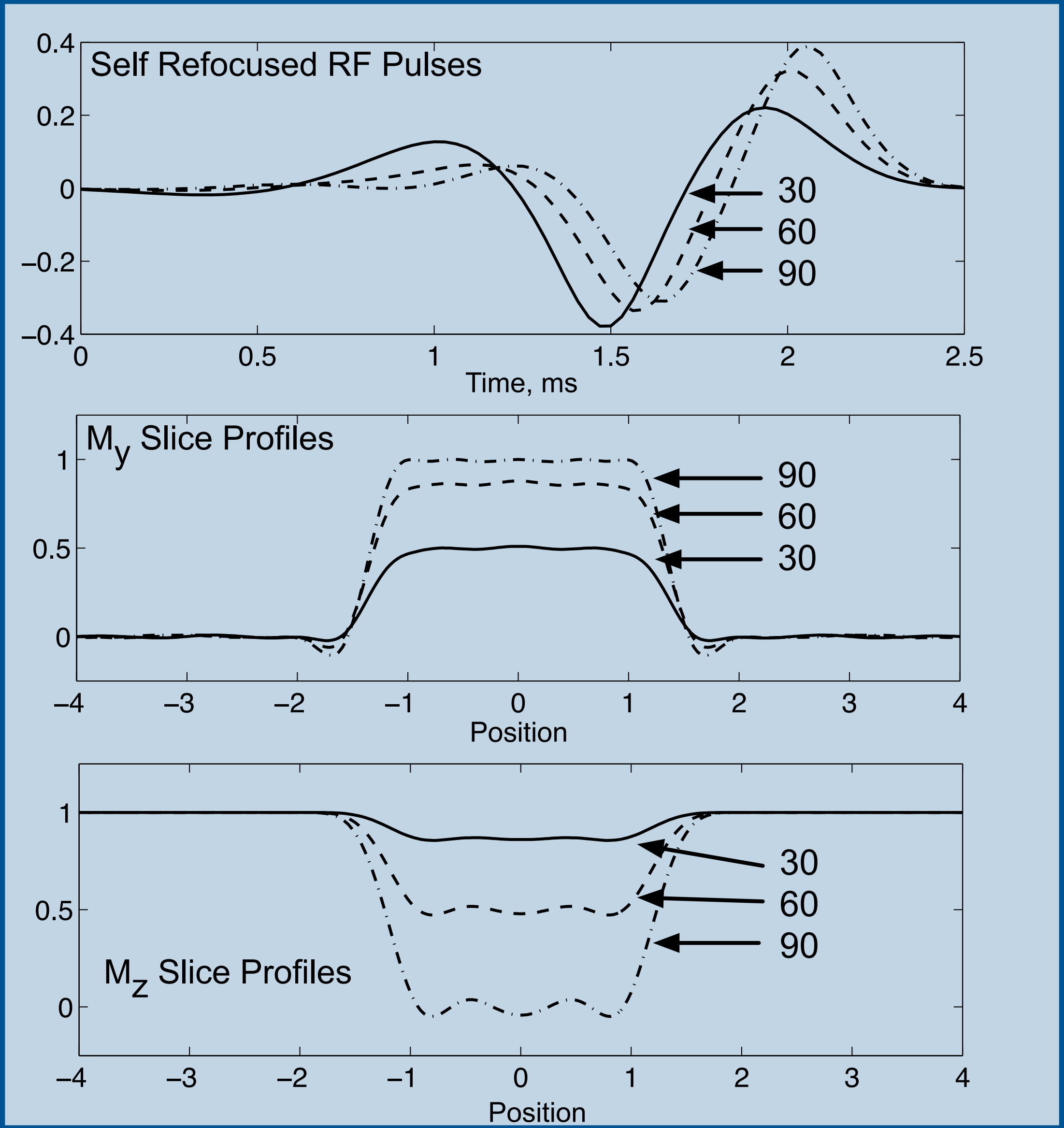
$$M_{xy} = 2\alpha^{*}\beta \, M_0$$

- Add phase compensation p to α to delay echo

$$M_{xy} = 2(\alpha p)^{*}\beta \, M_0$$

- One cycle of linear phase across slice delays echo by one main lobe width
- Adds one 180 in power (expensive!)

True Self Refocusing Pulses

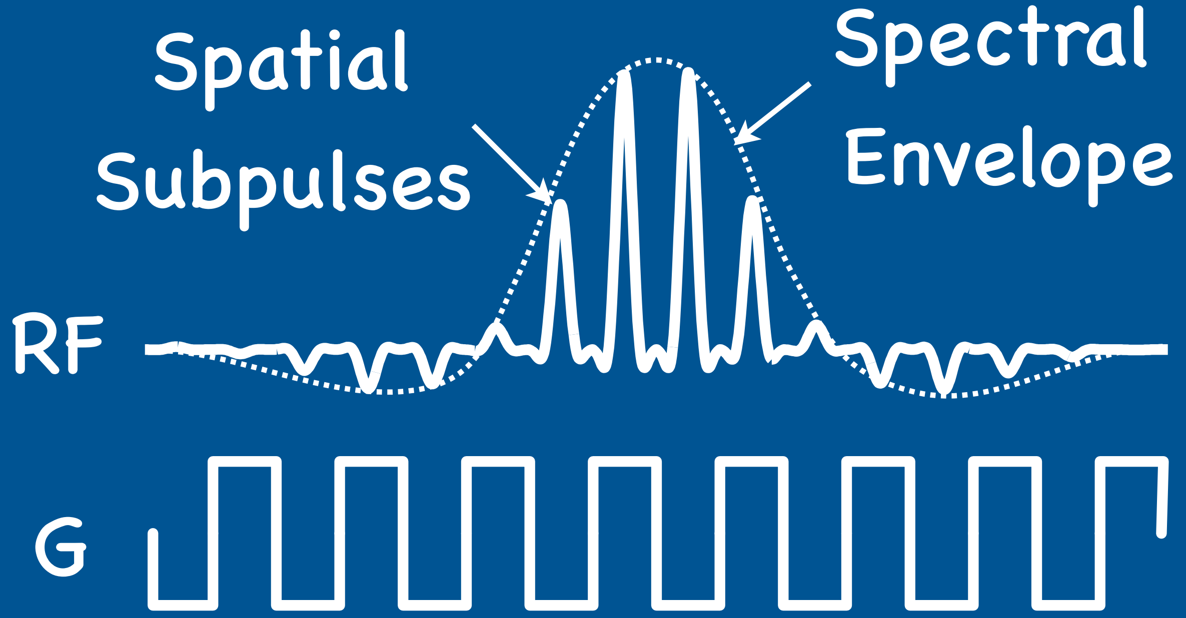


- Zero echo time
- No crushed or unrefocused signal
- Echo delayed by adding phase, and RF power

Multi-Dimensional Pulses

- Excitation and acquisition are duals
- Any acquisition technique can be used as an excitation pulse (spiral, EPI)
- EPI RF pulses are most useful for MRS, Spectral-Spatial pulses
- Do two things with one pulse!

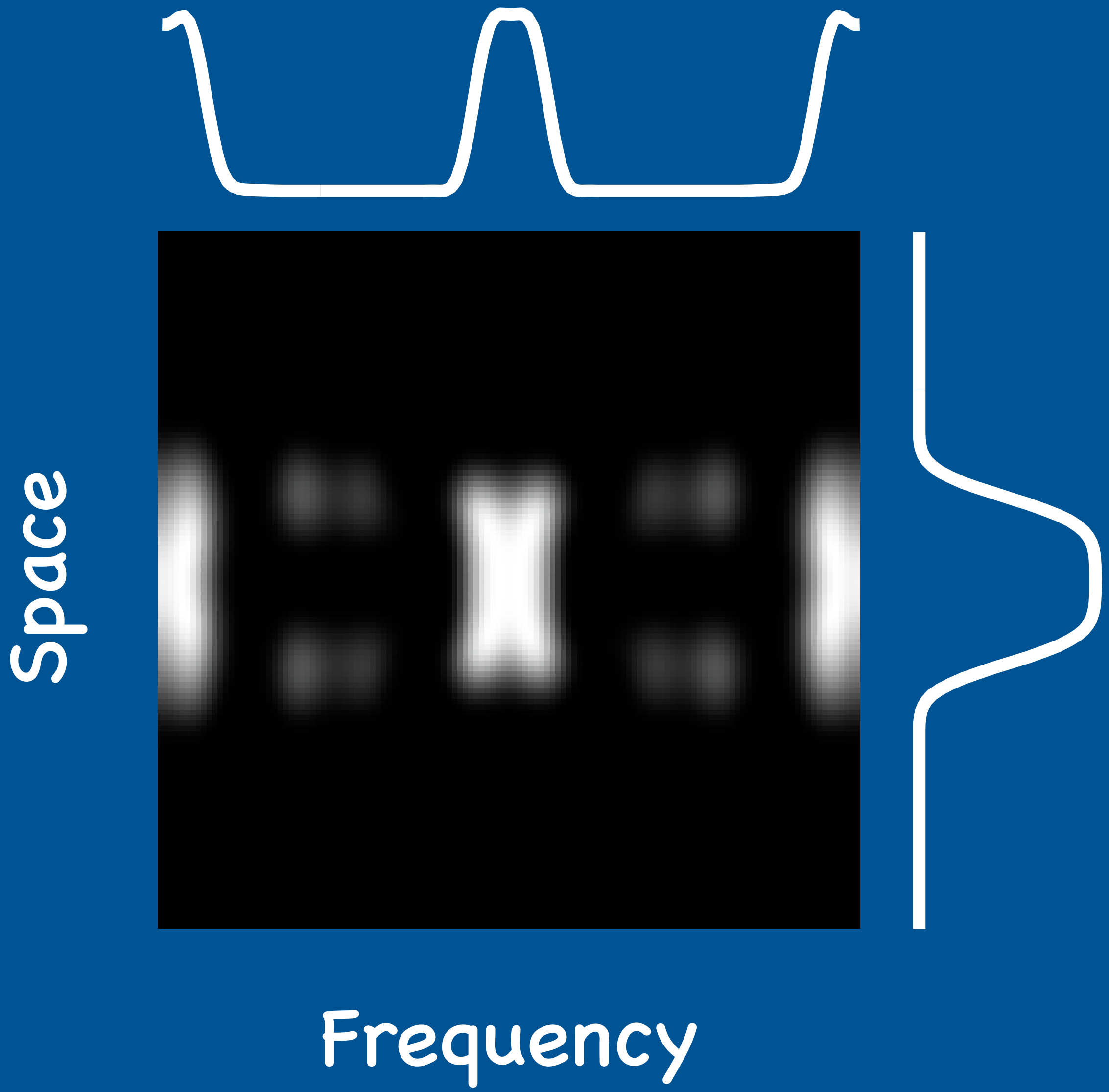
Spectral-Spatial Pulses



Product Pulse Design:

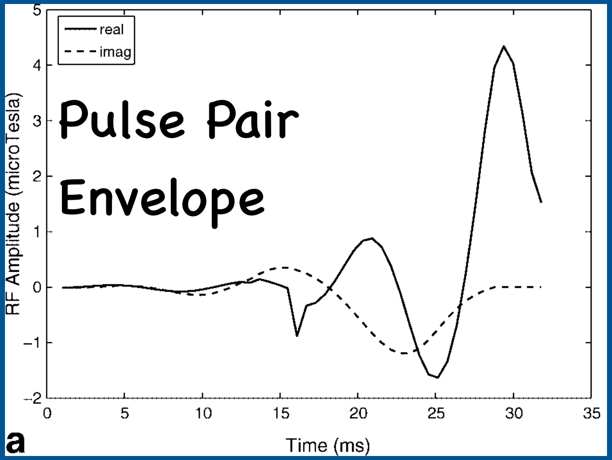
- Sampled spectral pulse (large tip angle)
- Spatial subpulses (small tip angle)
- Also, full 2D design

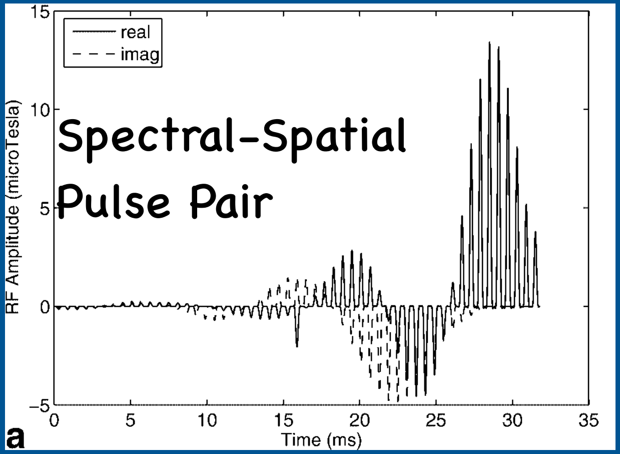
Spectral-Spatial Profile

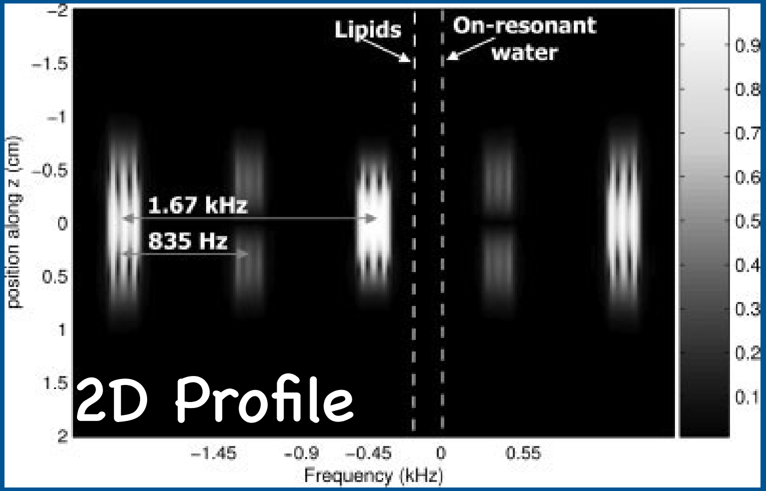


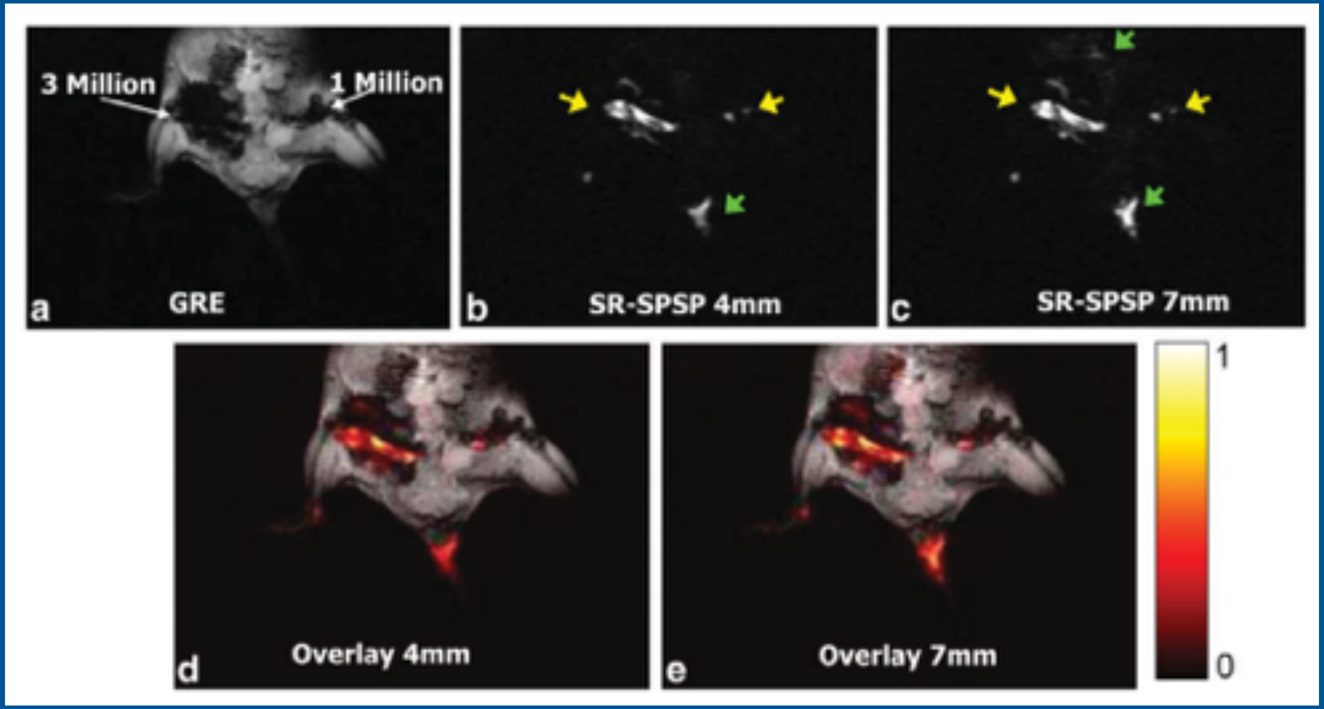
- Periodic in frequency
- N/2 “Ghost” sidelobes
- Continuous in space

Positive Contrast SPIO Imaging


Pulse Pair Envelope


Spectral-Spatial Pulse Pair


2D Profile



- Spin echo image of frequency shifted signals
- Positive contrast

Balchandani, et al. Magnetic Resonance in Medicine 62:183–192 (2009)

Conclusions

- Much to be gained by focusing on the MR signal instead of the RF pulses
- Benefits include
  - Sharper profiles
  - Shorter echo time
  - Better control of the signal