Lecture 4

TODAY

SMALL TIP ANGLE SOLUTION AS A K-SPACE INTEGRAL

2D SPIRAL PULSE DESIGN

GRADIENTS

RF

NEXT

PRACTICAL ISSUES FOR 2D SPIRAL PULSES

SMALL-TIP-ANGLE SOLUTION AS A K-SPACE INTEGRAL

SMALL-TIP-ANGLE SOLUTION

$$M_{xy}(\underline{r},t) = i M_0 \int_{-\infty}^{t} \gamma B_1(\tau) \underbrace{e^{i 2\pi \underline{k}(\tau,t)\cdot \underline{r}}}_{\int_{\underline{k}} {}^3\delta(\underline{k}(\tau,t)-\underline{k})\, e^{i 2\pi \underline{k}\cdot\underline{r}}\, d\underline{k}} d\tau$$

$$= i M_0 \int_{-\infty}^{t} \gamma B_1(\tau) \int_{\underline{k}} {}^3\delta(\underline{k}(\tau,t)-\underline{k})\, e^{i 2\pi \underline{k}\cdot\underline{r}}\, d\underline{k}\, d\tau$$

$$= i M_0 \int_{\underline{k}} \underbrace{\left[\int_{-\infty}^{t} \gamma B_1(\tau)\, {}^3\delta(\underline{k}(\tau,t)-\underline{k})\, d\underline{k} \right]}_{p(\underline{k})} e^{i 2\pi \underline{k}\cdot\underline{r}}\, d\underline{k}$$

THE MAGNETIZATION IS THE INVERSE TRANSFORM OF $p(\underline{k})$

WHAT IS $p(\underline{k})$?

$$p(\underline{k}) = \int_{-\infty}^{t} \gamma B_1(\tau)\, {}^3\delta(\underline{k}(\tau,t)-\underline{k})\, d\tau$$

WE WANT THIS TO BE A UNIT DELTA, MULTIPLIED BY A WEIGHTING FUNCTION.

MULTIPLY AND DIVIDE BY $|\underline{k}'(\tau, t)|$ (t CONSTANT)

$$p(\underline{k}) = \int_{-\infty}^{t} \underbrace{\frac{\gamma B_1(\tau)}{|\underline{k}'(\tau,t)|}}_{W(\underline{k}(\tau,t))} \underbrace{{}^3\delta(\underline{k}(\tau,t) - \underline{k}) |\underline{k}'(\tau,t)|}_{\text{UNIT DELTA}} \, d\tau$$

IF WE ASSUME $W(\underline{k})$ IS SINGLE VALUED

$$p(\underline{k}) = W(\underline{k}) \underbrace{\int_{-\infty}^{t} {}^3\delta(\underline{k}(\tau,t) - \underline{k}) |\underline{k}'(\tau,t)| \, d\tau}_{S(\underline{k})}$$

$$= W(\underline{k}) S(\underline{k})$$

THEN

$$M_{xy}(\underline{r}, t) = i M_0 \int_{\underline{k}} W(\underline{k}) S(\underline{k}) e^{i \underline{k} \cdot \underline{r}} \, d\underline{k}$$

WHERE

$$W(\underline{k}(\tau,t)) = \frac{\gamma B_1(\tau)}{|\underline{k}'(\tau,t)|}$$ k-SPACE WEIGHTING (APPROXIMATE, VORONOI BETTER)

$$S(\underline{k}) = \int_{-\infty}^{t} {}^3\delta(\underline{k}(\tau,t) - \underline{k}) |\underline{k}'(\tau,t)| \, d\tau$$ k-SPACE SAMPLING (UNIFORM)

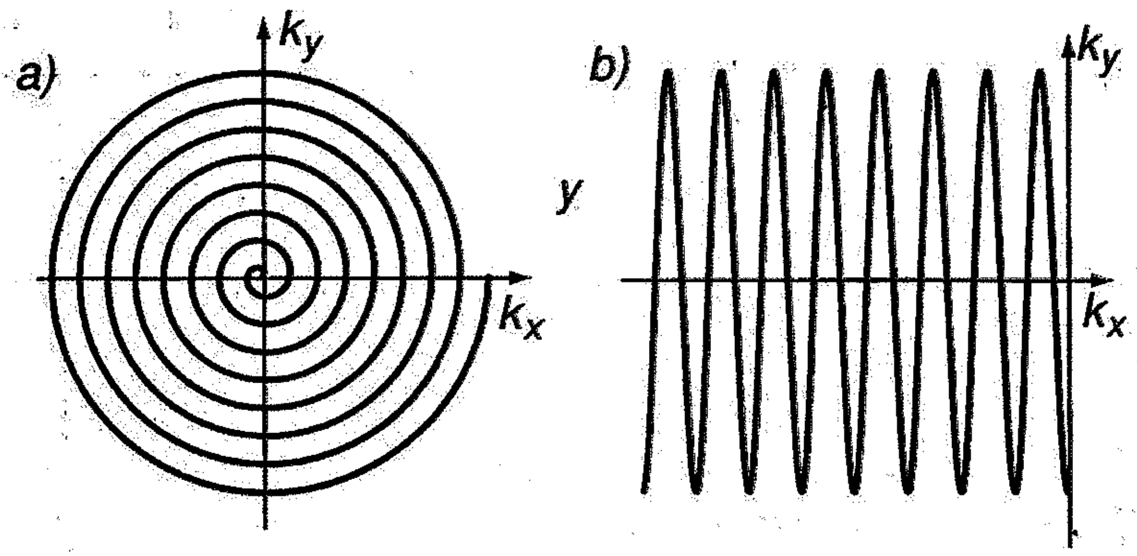
2D PULSE DESIGN

1) CHOOSE A K-SPACE TRAJECTORY THAT (APPROXIMATELY) UNIFORMLY COVERS K-SPACE

K-SPACE EXTENT $\Rightarrow$ SPATIAL RESOLUTION

SAMPLING DENSITY $\Rightarrow$ SPATIAL FOV

COMMON CHOICES



SPIRAL   EPI

SPIRAL COMMON FOR PENCIL BEAMS

EPI COMMON FOR SPECTRAL-SPATIAL PULSES

2) CHOOSE A WEIGHTING FUNCTION

THE INVERSE TRANSFORM OF WEIGHTING IS EXCITATION PROFILE

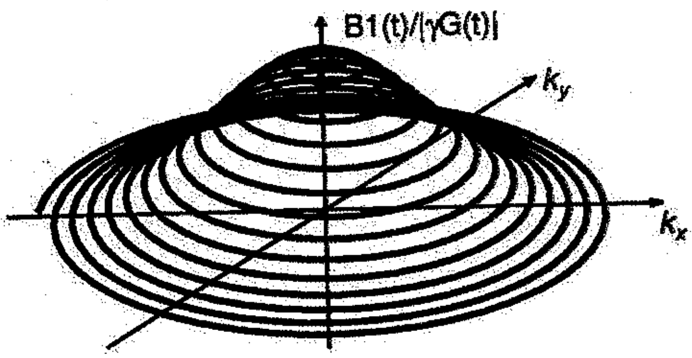
LOCALIZED EXCITATION

$\Rightarrow$ LOWPASS K-SPACE WEIGHTING

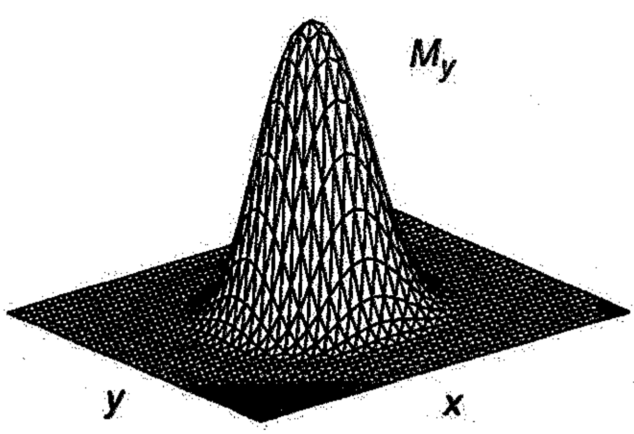
3) DESIGN THE RF

$$W(k(\tau,t)) = \frac{\gamma B_1(\tau)}{|k'(\tau,t)|} = \frac{\gamma B_1(\tau)}{|\gamma \underline{G}(\tau)|} = \frac{B_1(\tau)}{|G(\tau)|}$$

$$B_1(\tau) = |G(\tau)|\, W(k(\tau,t))$$



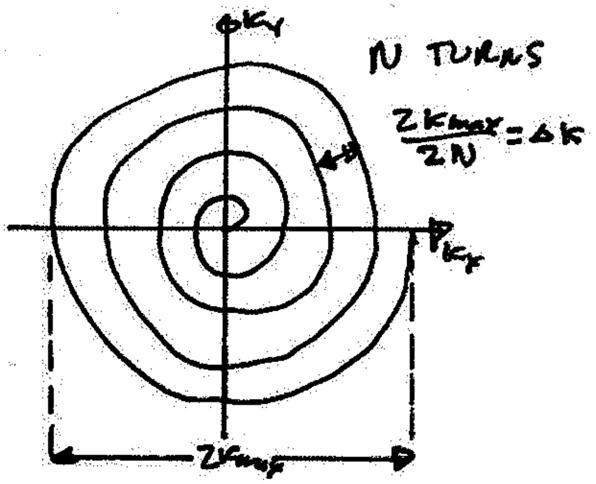
K-SPACE WEIGHTING



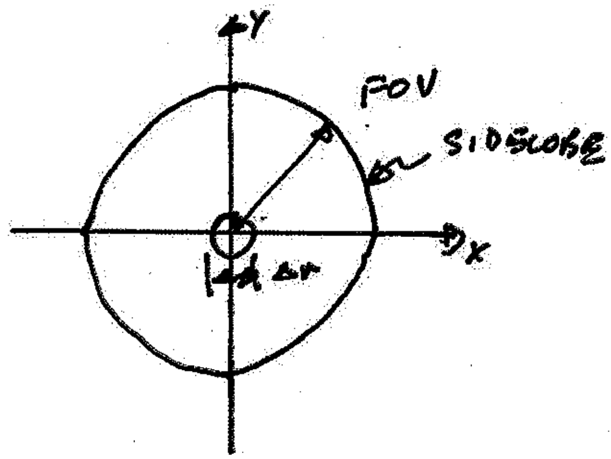
EXCITATION PROFILE

2D SPIRAL PULSE DESIGN

K-SPACE TRAJECTORY DESIGN



K-SPACE



IMPULSE RESPONSE

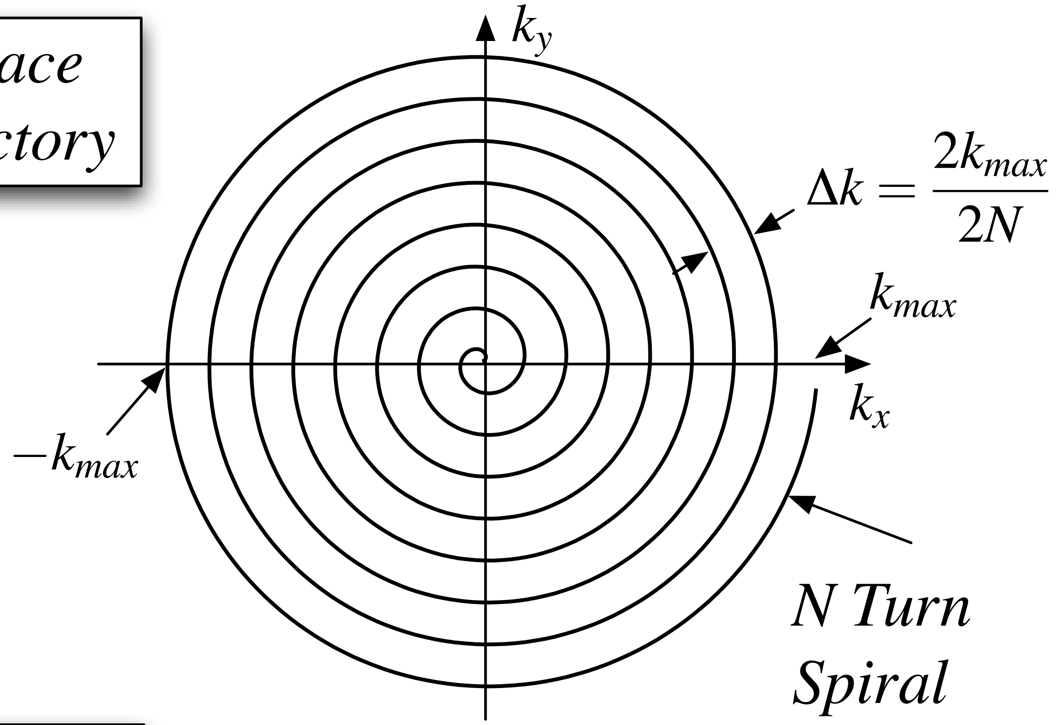
TWO MAJOR CHOICES:

- RESOLUTION Δr
  - SMALLEST VOLUME
  - MINIMUM TRANSITION WIDTH
  - $$\Delta r = \frac{1}{2k_{max}}$$
- FOV
  - DISTANCE TO CENTER OF FIRST SIDELOBE
  - $$FOV = \frac{1}{\Delta k} = \frac{2N}{2k_{max}} = 2N\Delta r$$

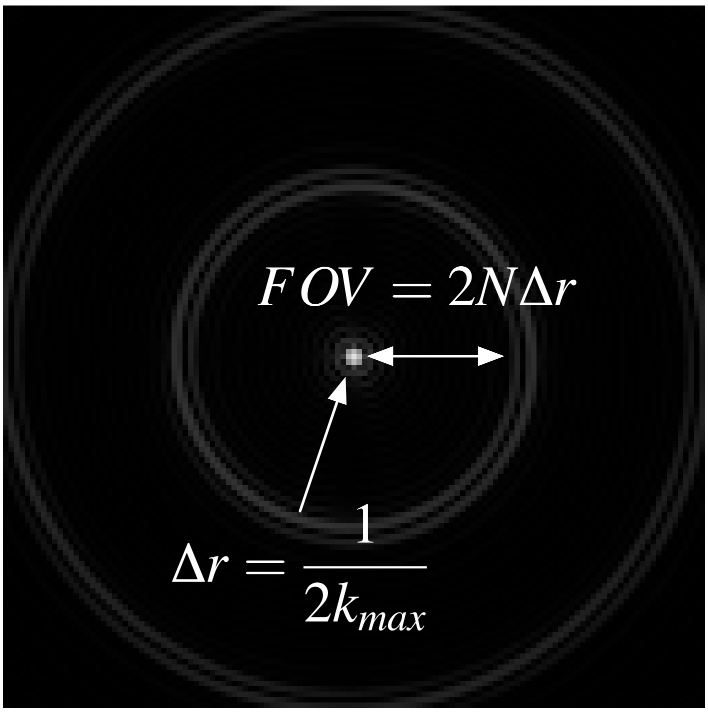
CHOOSING Δr DETERMINES k_{max}

CHOOSING FOV DETERMINES N

k-Space Trajectory



Impulse Response



2D SPIRAL GRADIENT DESIGN

LET

$$k(t) = k_x(t) + i k_y(t)$$

(CONVENTIONAL $k(t) = \frac{\gamma}{2\pi}\int_0^t G(s)ds$)

CONSTANT ANGULAR RATE INWARD SPIRAL

$$k(t) = k_{max}\left(1 - \frac{t}{T}\right) e^{i 2\pi N (1 - t/T)}$$

WHERE THE PULSE LENGTH IS T.

THE GRADIENT (COMPLEX, $G(t) = G_x(t) + i G_y(t)$) IS

$$k(t) = \frac{\gamma}{2\pi}\int_0^t G(s)ds \qquad \text{(CYCLES/cm)}$$

$$k'(t) = \frac{\gamma}{2\pi} G(t)$$

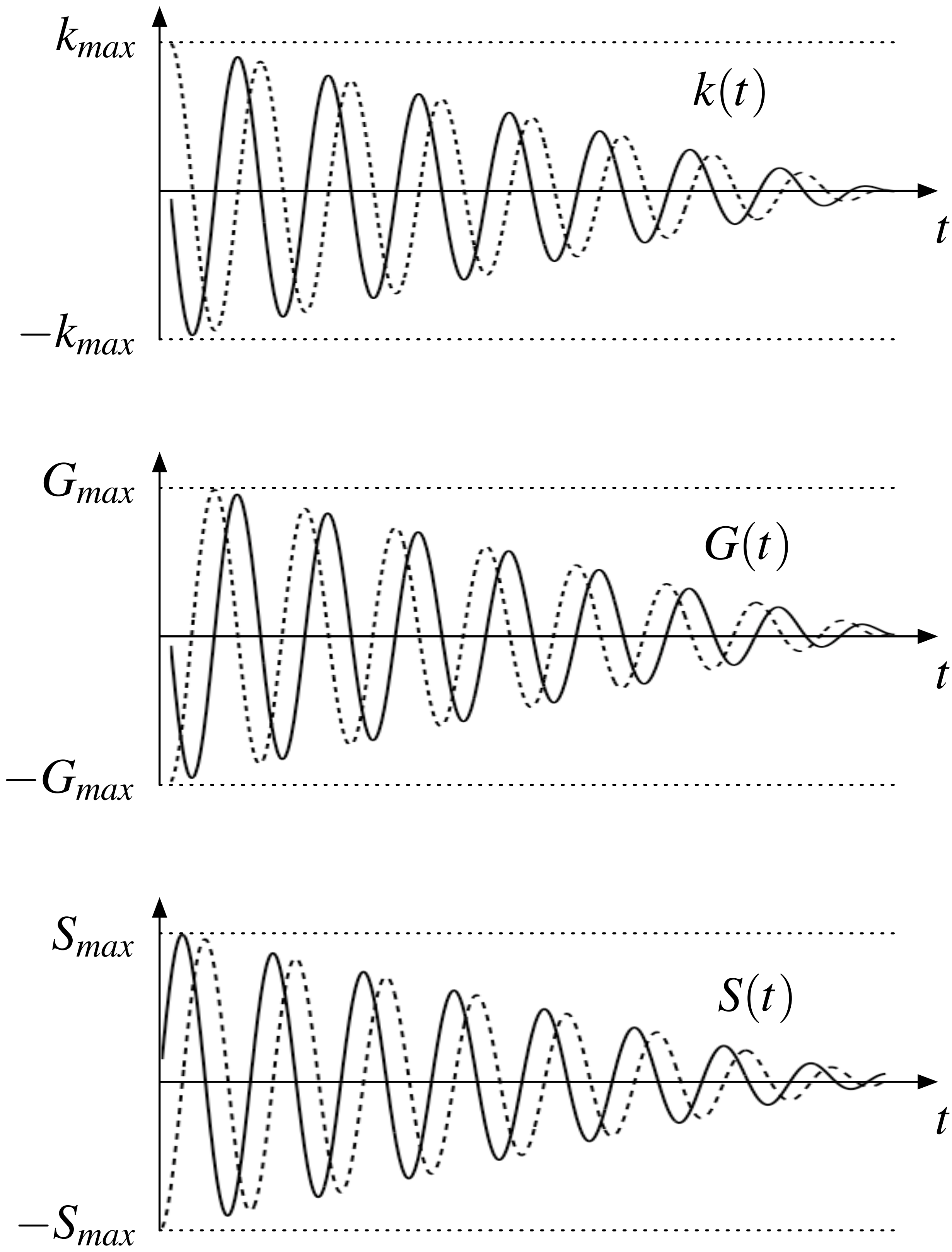
$$G(t) = \frac{2\pi}{\gamma} k'(t) \qquad \text{(G/cm)}$$

SLEW RATE

$$S(t) = G'(t)$$

$$= \frac{2\pi}{\gamma} k''(t)$$

Constant Angular Rate Spiral



GRADIENT SYSTEM LIMITS ARE

$$|G(t)| \leq G_{max}$$

$$|S(t)| \leq S_{max}$$

CONSTANT ANGULAR RATE SPIRAL HITS THE LIMIT ONLY AT THE END

NOT EFFICIENT USE OF GRADIENT SYSTEM!

SOLUTION IS TO TRACE OUT TRAJECTORY AT A DIFFERENT RATE

$$k(\tau(t))$$

SPEED UP THE TRAJECTORY WHEN IT IS BELOW THE CONSTRAINT, SLOW IT DOWN WHEN IT IS OVER CONSTRAINT

$$G(t) = \frac{d}{dt}\left(\frac{2\pi}{\gamma} k(\tau(t))\right) \leq G_{max}$$

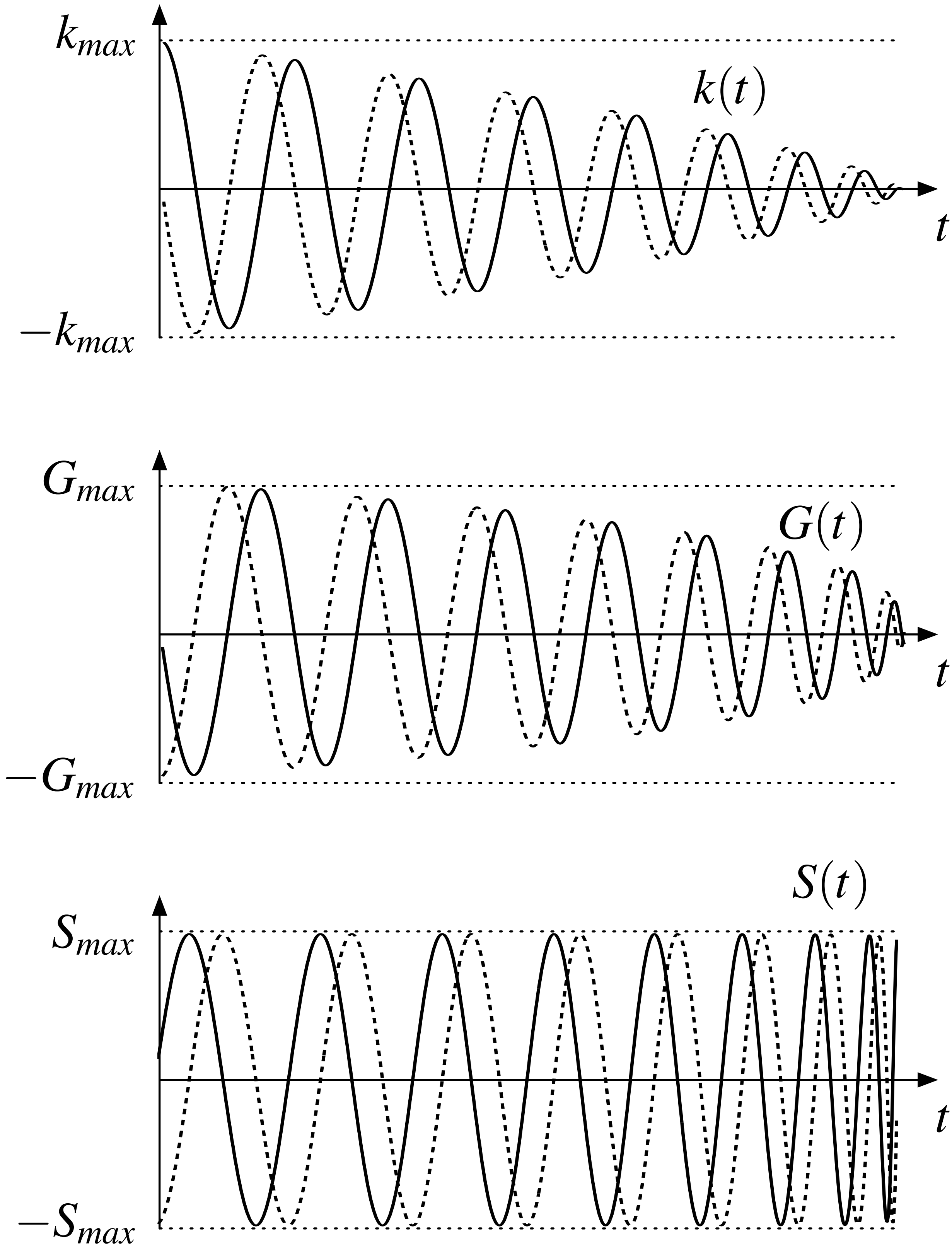
$$S(t) = \frac{d^2}{dt}\left(\frac{2\pi}{\gamma} k(\tau(t))\right) \leq S_{max}$$

THE MATLAB MFILE csg.m DOES THIS

$$k = csg(ka, G_{max}, S_{max})$$

TYPICALLY $1/\sqrt{2}$ SHORTER (CONSTANT SLEW RATE)

Constant Slew Rate Spiral



DESIGN $W(\underline{k})$

UNIFORM k-SPACE WEIGHTING

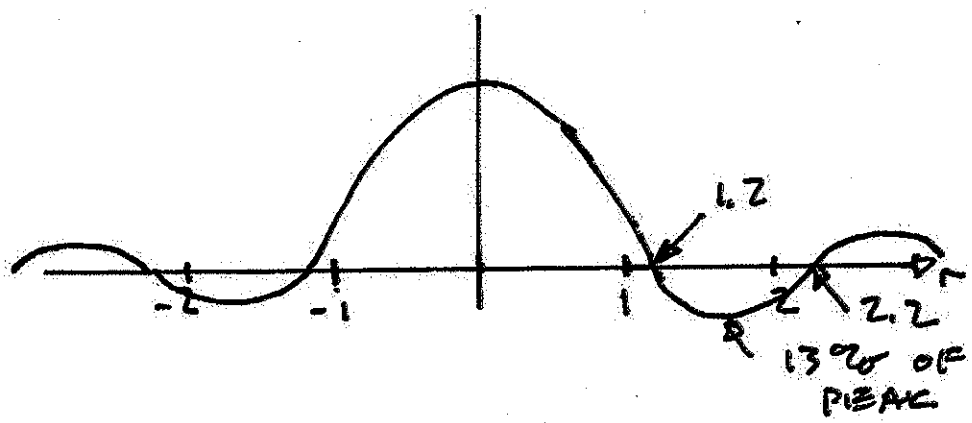
$$W(\underline{k}) = \mathrm{rect}\left(\frac{k_r}{2k_{max}}\right)$$

RESOLUTION LIMIT

$$m_{xy}(\underline{r}) = (2k_{max})^2 \,\mathrm{jinc}((2k_{max})r)$$
$$= \left(\frac{1}{\Delta r}\right)^2 \mathrm{jinc}\left(\frac{r}{\Delta r}\right)$$

WHERE

$$\mathrm{jinc}(r) = \frac{J_1(\pi r)}{2r}$$



SLIGHTLY BROADER THAN SINC

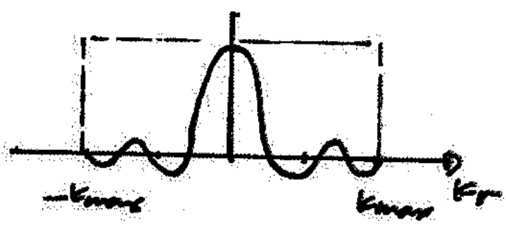
DECAYS FASTER $\sim r^{-3/2}$

TRUNCATED JINC WEIGHTING

$$W(\underline{k}) = \mathrm{jinc}\left(N \frac{k_r}{k_{max}}\right) \cdot \mathrm{rect}\left(\frac{k_r}{2k_{max}}\right)$$

$$m_{xy}(\underline{r}) \approx \mathrm{rect}\left(\frac{2k_{max}}{2N} r\right) ** \mathrm{jinc}(2k_{max} r)$$

$$= \mathrm{rect}\left(\frac{r}{2N\Delta r}\right) ** \mathrm{jinc}\left(\frac{r}{\Delta r}\right)$$



MINIMUM TRANSITION WIDTH

"SPACE - BANDWIDTH" IS (WIDTH IN SPACE) TIMES (WIDTH IN SPATIAL FREQUENCY)

FOR THIS CASE

$$SBW = (2N\Delta r)(2k_{max}) = 2N\left(\frac{1}{2k_{max}}\right)2k_{max}$$

$$= 2N$$

SAME AS TIME BANDWIDTH, BUT DOESN'T FALL ON ZEROS OF $\mathrm{jinc}(\cdot)$

WINDOWED JINC WEIGHTING

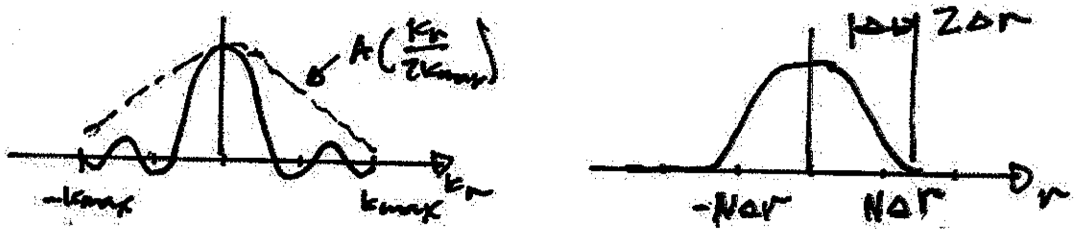
$$W(\underline{k}) = \mathrm{jinc}\left(N\frac{k_r}{K_{max}}\right) \cdot A\left(\frac{k_r}{2k_{max}}\right)$$

WHERE $A(\cdot)$ IS A WINDOW FUNCTION

CONVENTIONAL 1D WINDOWS WORK (HAMMING)
BUT ARE NOT OPTIMAL

THEN

$$m_{xy}(\underline{r}) \sim \mathrm{rect}\left(\frac{r}{2N\Delta r}\right) ** a\left(\frac{r}{\Delta r}\right)$$



DOUBLE TRANSITION WIDTH
SMOOTHER RESPONSE

CALCULATION OF THE RF WAVEFORM

GIVEN $W(\underline{k})$ AND $k(t)$

$$W(\underline{k}(t)) = \frac{\gamma B_1(t)}{|\underline{k}'(t)|} = \frac{\gamma B_1(t)}{|\gamma G(t)|}$$

$$= \frac{B_1(t)}{|G(t)|}$$

SO

$$\underline{B_1(t) = |G(t)|\, W(\underline{k}(t))}$$

THIS NEEDS TO BE SCALED FOR FLIP ANGLE.

NOTE THAT $|\underline{k}'(t)|^{-1}$ IS AN <u>ESTIMATE</u> OF THE DENSITY COMPENSATION FUNCTION $d(t)$

GOOD IF TRAJECTORY IS GEOMETRICALLY UNIFORM

ADAQUATE FOR SINGLE-SHOT SPIRALS

<u>NOT</u> ADAQUATE FOR INTERLEAVED SPIRALS