Lecture 3

TODAY

ONE DIMENSIONAL PULSE DESIGN EXAMPLES

MULTIPLE EXCITATIONS

EXCITATION AND RECEPTION

NEXT

TWO DIMENSIONAL PULSES

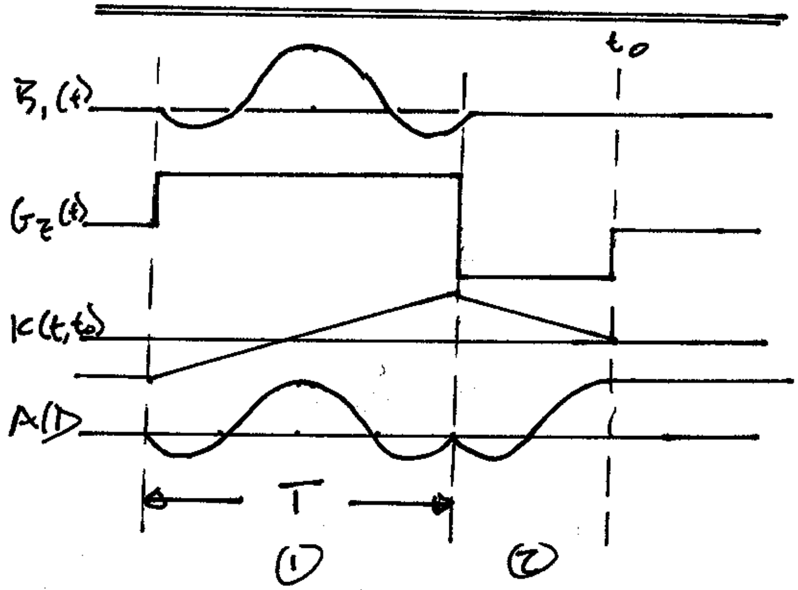
SMALL-TIP-ANGLE SOLUTION

$$m_{xy}(\underline{r},t) = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{-i\int_{\tau}^{t}\gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

$$\underline{k}(\tau,t) = -\frac{\gamma}{2\pi}\int_{\tau}^{t}\underline{G}(s)\, ds$$

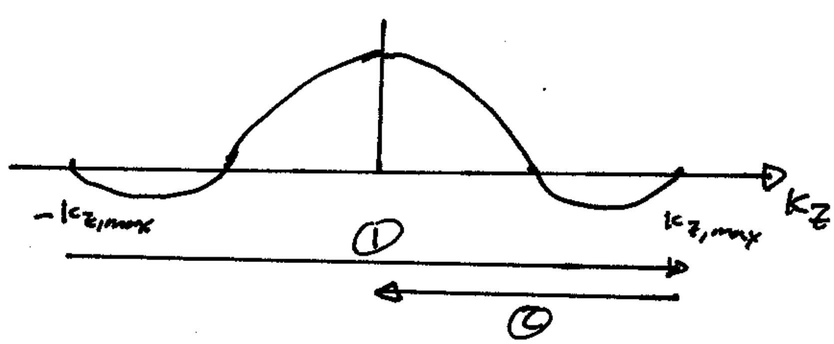
$$m_{xy}(\underline{r},t) = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{i2\pi \underline{k}(\tau,t)\cdot \underline{r}}\, d\tau$$

ONE-DIMENSIONAL EXAMPLE



$$\underline{k}(t,t_0) = -\frac{\gamma}{2\pi}\int_{t}^{t_0}\underline{G}(s)\, ds$$

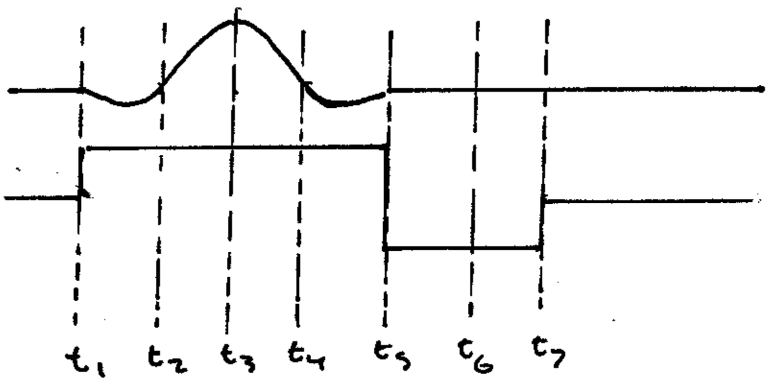
$$k_{z,max} = \frac{T}{2}\frac{\gamma}{2\pi}G_z$$



THIS GIVES MAGNETIZATION AT $t = t_0$, THE END OF THE PULSE.

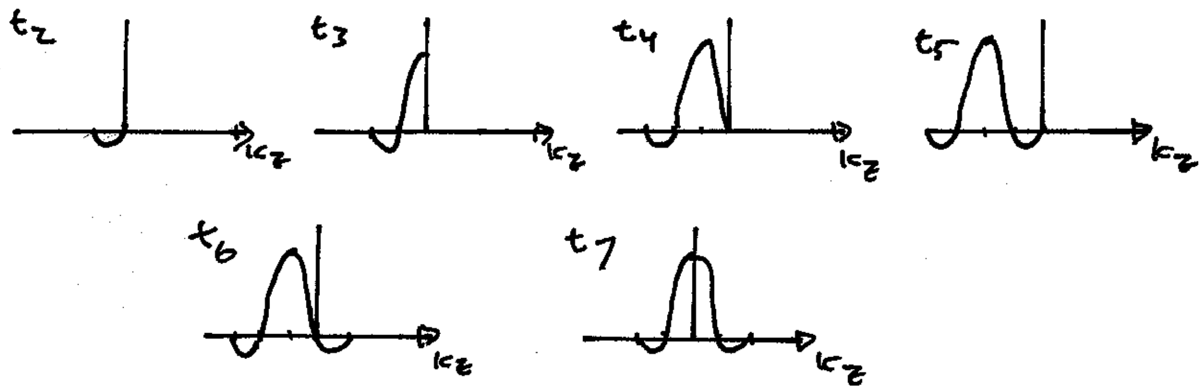
LOOKS LIKE YOU SCAN ACROSS k-SPACE, THEN RETURN TO ORIGIN...

Evolution of Magnetization During Pulse



Consider $t_2, t_3 \ldots t_7$ the <u>end</u> of a pulse

What k-space weighting is produced?



RF goes in at DC ($k_z=0$)!

Gradients move previously applied weighting around.

Think of the RF as "writing" an analog waveform in k-space.

Same idea applies to reception

Readout gradients shift object's Fourier transform

(mult by complex exponential in image space

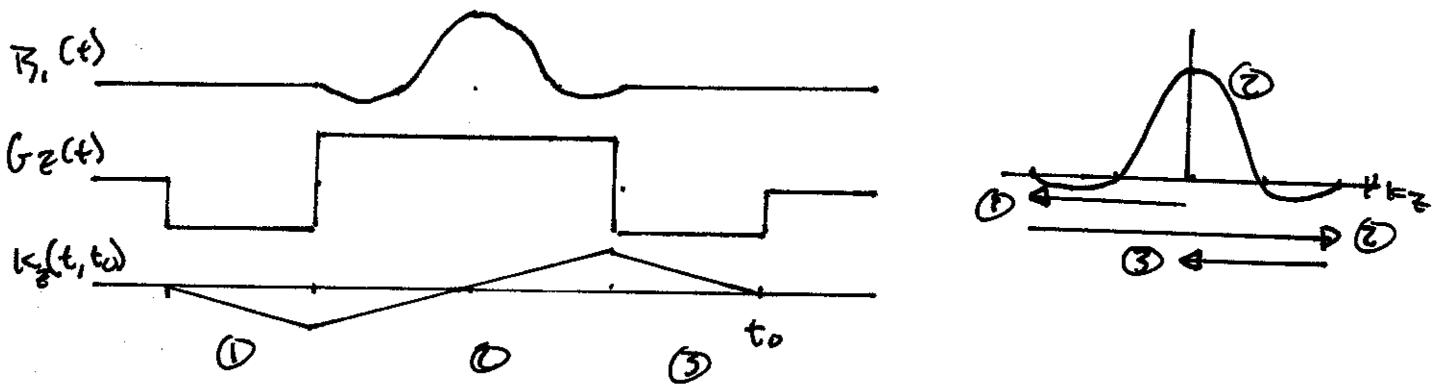
$\Rightarrow$ shift in k-space)

RF coil is non-selective

Receive whatever signal is at $k_z=0$

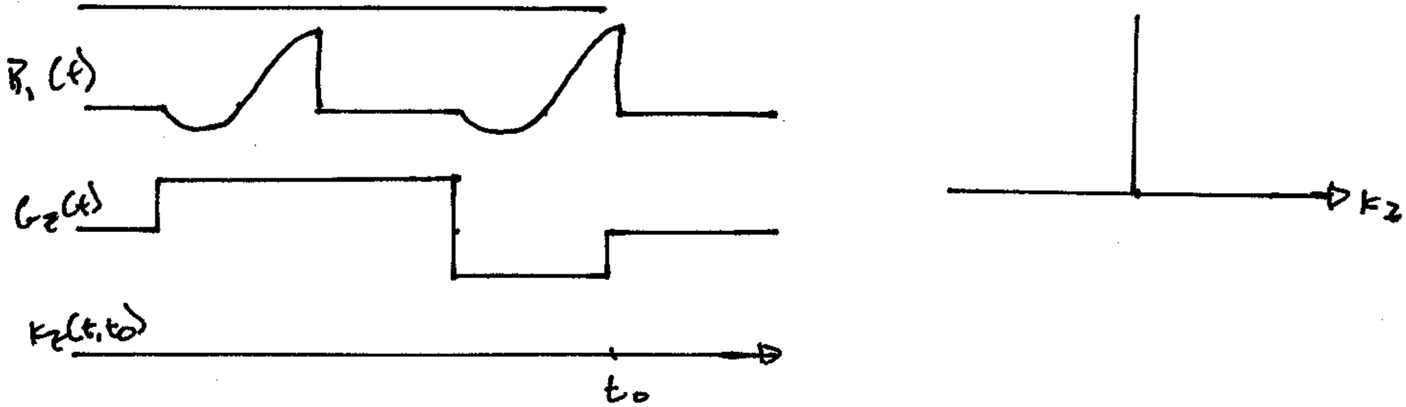
More on this later...

OTHER 1D EXAMPLES

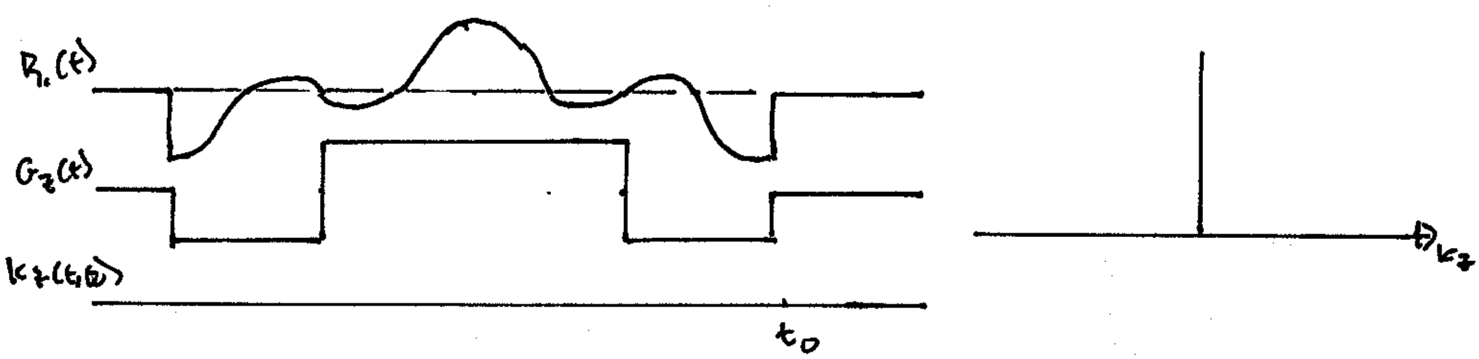


STARTS AND ENDS AT $k_z = 0$.
IMPORTANT SPECIAL CASE LATER ON.

WHAT DOES THIS DO?



HOW ABOUT THIS ONE?



Multiple Excitations

Most acquisition methods require several repetitions to make an image

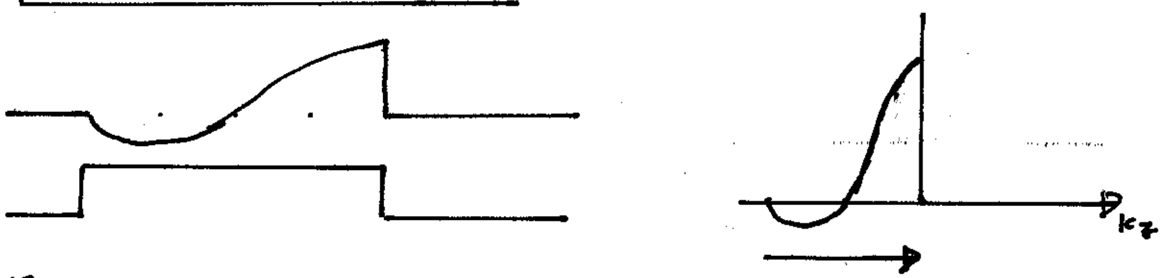
128 phase encodes

16 spiral interleaves

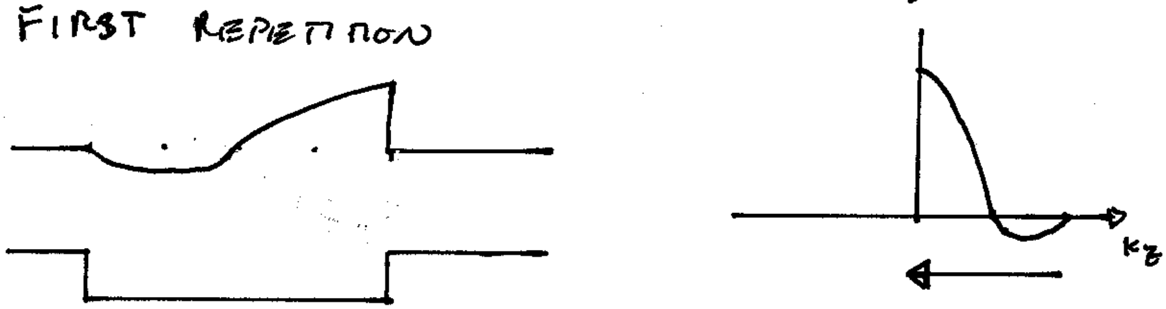
Data is combined to reconstruct an image

Same idea works for excitation

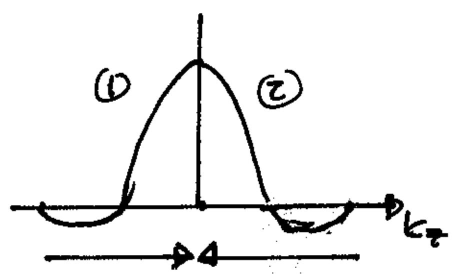
Simple 1D example



First repetition



Second repetition

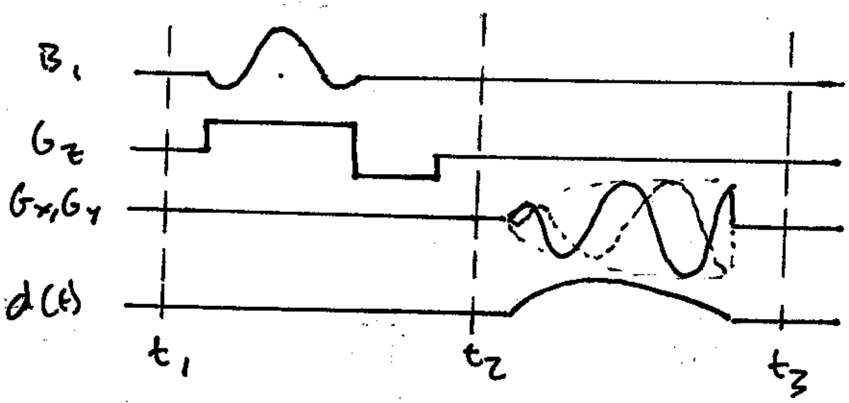


Sum the data from two acquisitions.

Same profile as slice select pulse.

Zero echo time.

EXCITATION AND RECEPTION



DENSITY, APODIZATION (K-SPACE)

MAGNETIZATION AT TIME t_2

$$m_{xy}(\underline{r}, t_2) = i\, m_0(\underline{r}) \int_{t_1}^{t_2} \gamma B_1(\tau)\, e^{-i\int_\tau^{t_2} \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

SIGNAL FOR $t > t_2$

$$s(t) = \int_{\underline{R}} m_{xy}(\underline{r}, t_2)\, e^{-i\int_{t_2}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\underline{r}$$

RECONSTRUCTED SIGNAL AT $\underline{r} = \underline{r_0}$

$$\hat{m}_{xy}(r_0) = \int_{t_2}^{t_3} s(t) \underbrace{d(t)}_{\text{DENSITY}} \underbrace{e^{i\int_{t_2}^{t} \gamma \underline{G}(s)\cdot r_0\, ds}}_{\text{CONJUGATE PHASE KERNEL}} dt$$

SUBSTITUTE FOR $s(t)$

$$\hat{m}_{xy}(r_0) = \int_{t_2}^{t_3} \left[\int_{\underline{R}} m_{xy}(\underline{r}, t_2)\, e^{-i\int_{t_2}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\underline{r} \right] \left[d(t)\, e^{i\int_{t_2}^{t} \gamma \underline{G}(s)\cdot r_0\, ds} \right] dt$$

$$= \int_{\underline{R}} m_{xy}(\underline{r}, t_2) \underbrace{\left[\int_{t_2}^{t_3} d(t)\, e^{-i\int_{t_2}^{t} \gamma \underline{G}(s)\cdot (\underline{r}-r_0)\, ds}\, dt \right]}_{\text{ACQUISITION IMPULSE RESPONSE}} d\underline{r}$$

SUBSTITUTE FOR $m_{xy}(\underline{r}, t_2)$

$$\hat{m}_{xy}(r_0) = \int_{\underline{R}} i\, m_0(\underline{r}) \left[\int_{t_1}^{t_2} \gamma B_1(\tau)\, e^{-i\int_{\tau}^{t_2} \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau \right] \left[\int_{t_2}^{t_3} d(t)\, e^{-i\int_{t_2}^{t} \gamma \underline{G}(s)\cdot (\underline{r}-\underline{r}_0)\, ds}\, dt \right] d\underline{r}$$

$\underline{k}$-SPACE DEFINITION

$$\underline{k}(t_1, t_2) = -\frac{\gamma}{2\pi}\int_{t_1}^{t_2} \underline{G}(s)\, ds = \frac{\gamma}{2\pi}\int_{t_2}^{t_1} \underline{G}(s)\, ds \qquad \left(\begin{array}{c} \text{IF } t_2 = 0, \\ \text{THIS IS CONVENTIONAL} \\ \underline{k}(t) \end{array} \right)$$

SUBSTITUTE FOR $\underline{k}(\cdot,\cdot)$

$$\hat{m}_{xy}(r_0) = \int_{\underline{R}} i\, m_0(\underline{r}) \left[\int_{t_1}^{t_2} \gamma B_1(\tau)\, e^{i 2\pi \underline{k}(\tau, t_2)\cdot \underline{r}}\, d\tau \right] \left[\int_{t_2}^{t_3} d(t)\, e^{-i 2\pi \underline{k}(t, t_2)\cdot (\underline{r}-\underline{r}_0)}\, dt \right] d\underline{r}$$

EXCITATION AND RECEPTION ARE SYMMETRIC

EXCITATION AND RECEPTION + RECONSTRUCTION ARE ALMOST COMPLETELY SYMMETRIC!

THE RF WEIGHTING $\gamma B_1(t)$ CORRESPONDS EXACTLY TO $d(t)$, THE DENSITY WEIGHTING AND K-SPACE APODIZATION USED IN RECONSTRUCTION

SAME K-SPACE TRAJECTORIES CAN BE USED FOR EITHER ONE.

DIFFERENCES

ON EXCITATION, THE VOXEL POSITION IS SET

ON RECEPTION, THE VOXEL POSITION CAN BE MOVED (THIS IS IMAGE RECONSTRUCTION)

ONE IS FORWARD, ONE IS INVERSE TRANSFORM
<u>NOT</u> FUNDAMENTAL
ARTIFACT OF CHOOSING t_2 AS REFERENCE TIME

GENERALLY, RF PULSE <u>REVERSED</u> ACQUISITION
PRACTICAL CONSIDERATIONS THE SAME (DEPHASING, T_2)

SHIFTING THE SLICE (VOLUME)

SO FAR, EXCITATION IS AT ISOCENTER

$$m_{xy}(\underline{r}, t) = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau) e^{i 2\pi \underline{k}(\tau,t)\cdot \underline{r}} d\tau$$

WE CAN SHIFT THE EXCITATION BY APPLYING

$$B_1(\tau) e^{-i 2\pi \underline{k}(\tau,t)\cdot \underline{r}_0}$$

THEN

$$i m_0 \int_{-\infty}^{t} \left(\gamma B_1(\tau) e^{-i 2\pi \underline{k}(\tau,t)\cdot \underline{r}_0}\right) e^{i 2\pi \underline{k}(\tau,t)\cdot \underline{r}} d\tau$$

$$= i m_0 \int_{-\infty}^{t} \gamma B_1(\tau) e^{i 2\pi \underline{k}(\tau,t)\cdot(\underline{r}-\underline{r}_0)} d\tau$$

$$= m_{xy}(\underline{r}-\underline{r}_0, t)$$

FOR A CONSTANT GRADIENT G_z

$$k_z(\tau, t) = -\frac{\gamma}{2\pi}\int_{\tau}^{t} G_z \, ds = -\frac{\gamma}{2\pi} G_z (t-\tau)$$

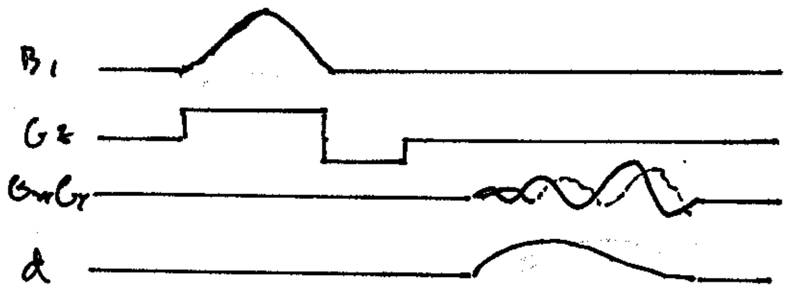
AND A SLICE POSITION z_0

$$e^{-i 2\pi k(\tau,t) z_0} = e^{-i 2\pi\left(-\frac{\gamma}{2\pi} G_z\right)(t-\tau)} = \underbrace{e^{i(\gamma G_z z_0)t}}_{\text{PHASE}} \underbrace{e^{-i(\gamma G_z z_0)\tau}}_{\substack{\text{FREQUENCY} \\ \gamma G_z z_0}}$$

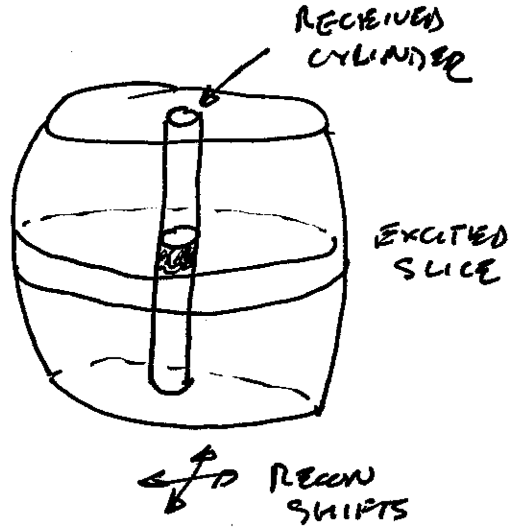
SAME IDEA AS CONJUGATE PHASE RECONSTRUCTION.

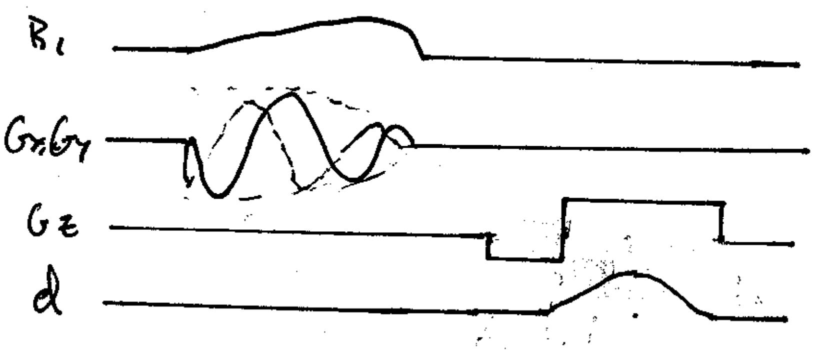
ANY IMAGING METHOD CAN BE FLIPPED AROUND TO MAKE AN RF PULSE!

EXAMPLE: 2D SPIRAL



SLICE SELECT EXCITE
SPIRAL RECEIVE





SPIRAL EXCITE
SLICE SELECT RECEIVE