Lecture 11

LAST TIME

GIVEN $B_N(z)$, FINDING $B_1(t)$

TODAY

DESIGNING $B_N(z)$

EQUAL RIPPLE

LEAST SQUARES

ALTERNATE PHASE PROFILES

LINEAR PHASE

MIN/MAX PHASE

NON-LINEAR PHASE

BASIC PROBLEM

RF PULSE DESIGN WITH SLR CONSISTS OF:

1) DESIGNING $B_N(z)$

2) CHOOSING A COMPATIBLE, MINIMUM POWER $A_N(z)$.

3) PERFORMING THE BACK RECURSION

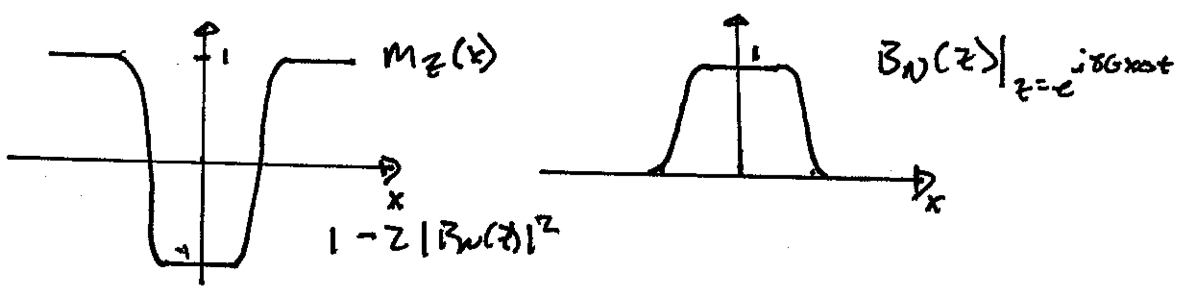
ALL DETERMINED ONCE $B_N(z)$ HAS BEEN DESIGNED.

HOW DO WE DESIGN $B_N(z)$?

SEVERAL ISSUES:

1) GOAL IS SPECIFIED IN TERMS OF M_{xy}, OR M_z

$B_N(z)$ IS $\sim \sin(\theta(x)/2)$



THE GOAL IS A NON-LINEAR FUNCTION OF THE INPUT $B_N(z)$.

WE NEED TO FIGURE OUT WHAT TO ASK FOR TO GET WHAT WE WANT.

2) THE PARAMETERS WE CARE ABOUT ARE

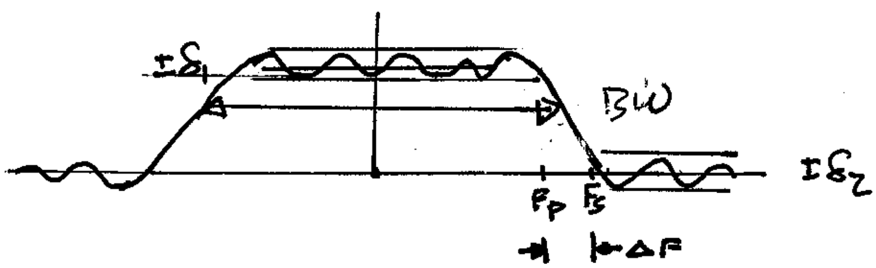
PASSBAND ERROR (δ_1)

STOPBAND ERROR (δ_2)

SLICE WIDTH IN FREQUENCY (BW)

PULSE LENGTH (T)

GIVEN THESE WE WOULD LIKE MINIMUM TRANSITION WIDTH $(F_S - F_P) = \Delta F$



WHAT FILTER DESIGN PROGRAMS WANT IS

PASSBAND EDGE (F_P)

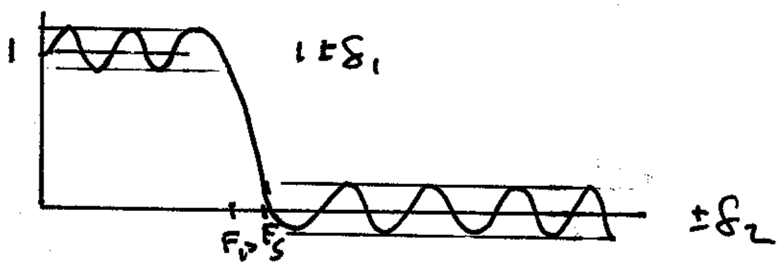
STOPBAND EDGE (F_S)

PASSBAND/STOPBAND ERROR RATIO (δ_1 / δ_2)

WE NEED TO RELATE WHAT WE WANT TO WHAT WE NEED TO SPECIFY.

BASIC IDEA

EQUAL-RIPPLE FILTERS (PARKS-McCLELLAN) ARE DETERMINED BY BAND EDGES AND RIPPLE AMPLITUDES



ALL WE HAVE TO DO IS FIGURE OUT WHAT THE EFFECTIVE RIPPLE PRODUCED IN THE SLICE PROFILE OF INTEREST IS.

δ_1^e – PASSBAND MAGNETIZATION RIPPLE

δ_2^e – STOPBAND MAGNETIZATION RIPPLE

THIS WILL DEPEND ON THE PROFILE. ONCE WE HAVE THESE RELATIONS, WE CAN INVERT THEM TO DETERMINE WHAT (δ_1, δ_2) TO SPECIFY.

EXAMPLE: INVERSION PULSES

INVERSION PROFILE

$$M_z^+(x) = \left(1 - 2|\beta(x)|^2\right) M_0$$
$$= \left(1 - 2|B_N(z)|^2\right) M_0 \Big|_{z = e^{i\gamma G x \Delta t}}$$

THIS CASE WE CAN ACTUALLY SOLVE FOR EXPLICITLY BY DESIGNING $M_z^+(x)$, AND FACTORING IT. WE WILL RETURN TO THIS.

OUT-OF-SLICE RIPPLE

AN INPUT RIPPLE OF δ_2 PRODUCES

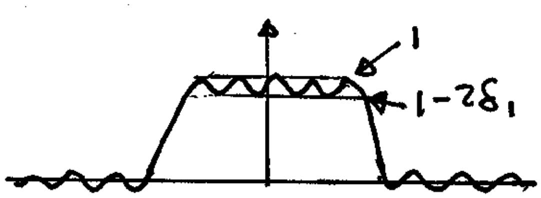
$$\delta_2^e = 2\delta_2^2$$

SO

$$\underline{\delta_2 = \sqrt{\delta_2^e / 2}}$$

IN-SLICE RIPPLE

$B_N(z)$ IS SCALED TO BE LESS THAN 1



MAXIMUM RIPPLE OCCURS IN M_z WHEN $B_N(z)$ IS MINIMUM

$$M_z = \left(1 - 2(1-2\delta_1)^2\right) M_0$$

$$= \left(1 - 2\left(1 - 4\delta_1 + 4\delta_1^2\right)\right) M_0$$

$$\simeq (-1 + 8\delta_1) M_0$$

SO

$$\delta_1^e = 8\delta_1$$

$$\delta_1 = \tfrac{1}{8}\delta_1^e$$

FOR EXAMPLE, IF WE WANT AN INVERSION PROFILE WITH $\delta_1^e = 0.01$ AND $\delta_2^e = 0.01$, WE NEED TO DESIGN $B_N(z)$ WITH

$$\delta_2 = \sqrt{\delta_2^e/2} = \sqrt{0.01/2} = 0.07$$ MUCH LARGER!

$$\delta_1 = \tfrac{1}{8}\delta_1^e = \frac{0.01}{8} = 0.0013$$ MUCH SMALLER!

MORE IMPORTANTLY, THE RATIO

$$\frac{\delta_2}{\delta_1} = 53$$

FAR FROM THE UNITY RATIO OF M_z.

SIMILAR RELATIONS CAN BE DERIVED FOR OTHER TYPES OF PULSES

CASE	δ_1	δ_2	
SMALL TIP	δ_1^e	δ_2^e	
$\pi/2$	$\sqrt{\delta_1^e/2}$	$\delta_2^e/\sqrt{2}$	(NOT USABLE)
INVERSION	$\delta_1^e/8$	$\sqrt{\delta_2^e/2}$	
SPIN ECHO	$\delta_1^e/4$	$\sqrt{\delta_2^e}$	
SATURATION	$\delta_1^e/2$	$\sqrt{\delta_2^e}$	

FROM PAULY, LE ROUX, et al, IEEE TMI 10(1), p 53-65, 1991

WE KNOW RIPPLE AMPLITUDES δ_1, δ_2

HOW DO WE FIND PASSBAND EDGES?

FROM DIGITAL FILTER DESIGN

$$\underbrace{T}_{\text{DURATION}} \underbrace{(\Delta F)}_{\substack{\text{TRANSITION} \\ \text{IN HZ}}} = \underbrace{D_\infty(\delta_1, \delta_2)}_{\substack{\text{CONSTANT, FCN OF} \\ \delta_1 \; \delta_2}}$$

OR

$$\underbrace{T(BW)}_{\substack{\text{TIME} \\ \text{BANDWIDTH}}} \underbrace{\left(\frac{\Delta F}{BW}\right)}_{\substack{\text{FRACTIONAL} \\ \text{TRANSITION} \\ \text{WIDTH}}} = D_\infty(\delta_1, \delta_2)$$

D_∞ HAS BEEN DETERMINED EMPIRICALLY FOR EQUI-RIPPLE FILTERS

INTUITIVELY WE EXPECT

$$\Delta F \sim \frac{1}{T}$$

FOR A SINC, AND

$$\Delta F \sim \frac{2}{T}$$

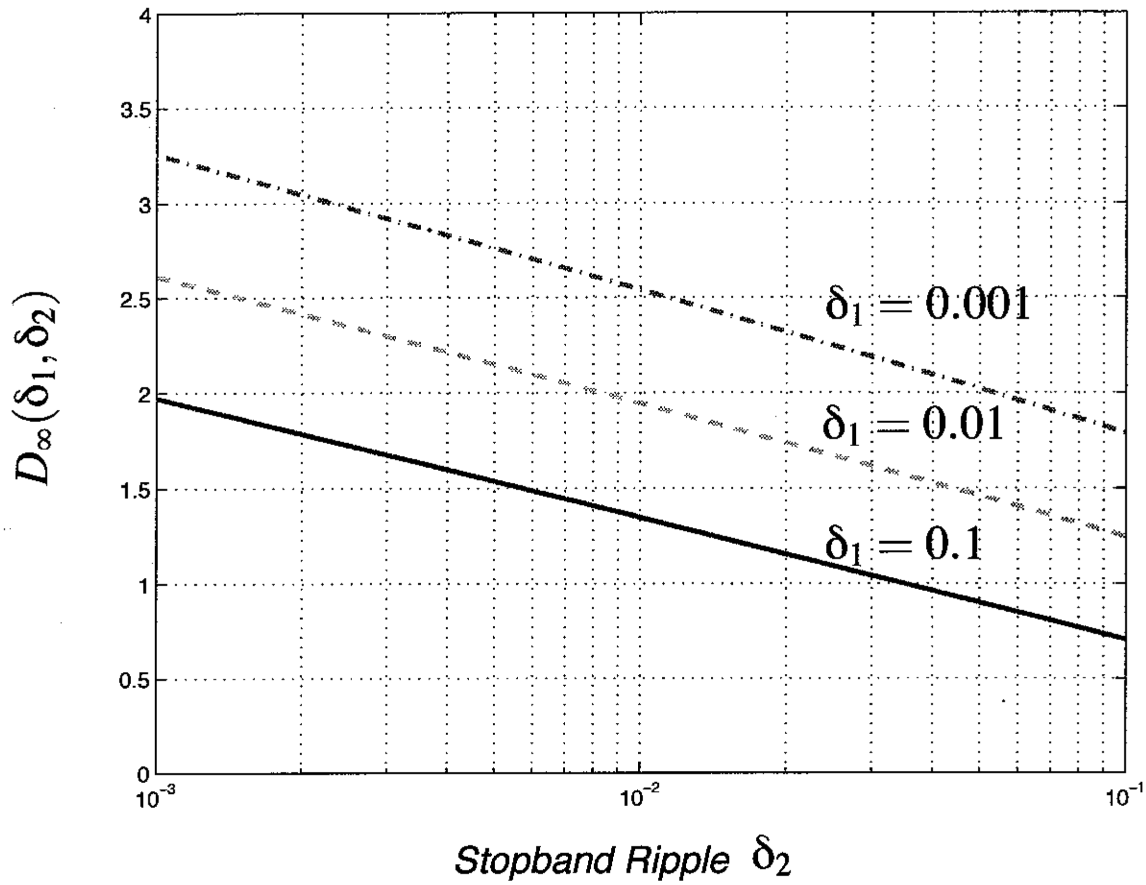
FOR A WINDOWED SINC

SO D_∞ SHOULD BE ON THE ORDER OF 1 TO 2, THIS IS A GOOD ESTIMATE!

BETTER FILTER DESIGNS HAVE LOWER D_∞

LIMIT TO HOW MUCH CAN BE GAINED.

$$D_\infty(\delta_1, \delta_2)$$



$$D_\infty(\delta_1, \delta_2) = (a_1 L_1^2 + a_2 L_1 + a_3) L_2 + (a_4 L_1^2 + a_5 L_1 + a_6)$$

Where

$$L_1 = \log_{10} \delta_1 \quad \text{and} \quad L_2 = \log_{10} \delta_2$$

and

$$\mathrm{a}_1 = 5.309 \times 10^{-3}$$
$$\mathrm{a}_2 = 7.114 \times 10^{-2}$$
$$\mathrm{a}_3 = -4.761 \times 10^{-1}$$
$$\mathrm{a}_4 = -2.66 \times 10^{-3}$$
$$\mathrm{a}_5 = -5.941 \times 10^{-1}$$
$$\mathrm{a}_6 = -4.278 \times 10^{-1}$$

DESIGN EXAMPLE

INVERSION PULSE

$T = 4\,ms$

$BW = 2\,KHZ$ $\qquad T(BW) = 8$

$\delta_1^e = 0.01$

$\delta_2^e = 0.01$

FROM PREVIOUS EXAMPLE

$$\delta_1 = \delta_1^e / 8 = 0.00125$$

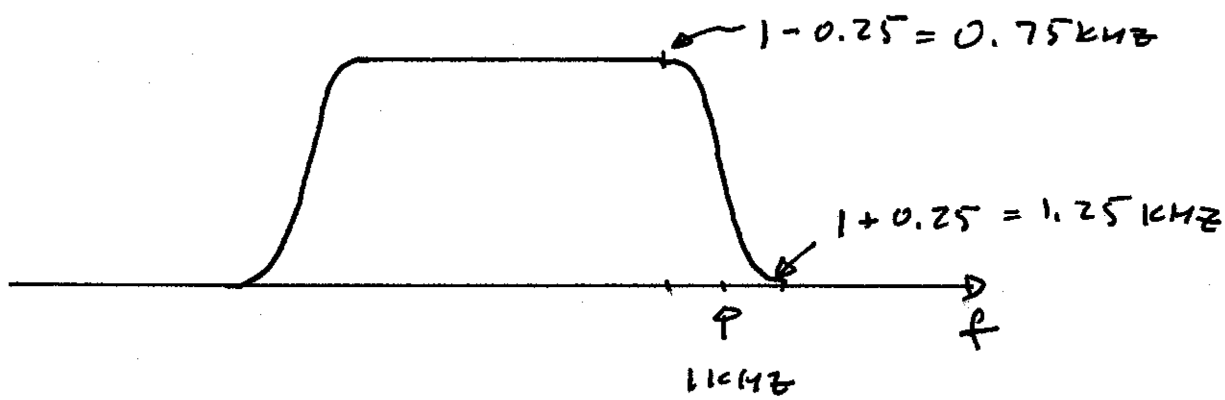
$$\delta_2 = \sqrt{\delta_1^e / 2} = 0.0707$$

THEN

$$D_\infty(0.00125, 0.0707) = 2$$

THE TRANSITION WIDTH IS THEN

$$\Delta F = \frac{D_\infty(\delta_1, \delta_2)}{T} = \frac{2}{4\,ms} = 500\,Hz$$



IN MATLAB, DESIGN THIS FILTER WITH

firpm.m

(WAS remez.m)

INPUTS

(SAMPLING FREQ)/2
256 SAMPLES IN 4ms

$f = [0 \quad 750 \quad 1250 \quad 32000]/32000;$

$m = [1 \quad 1 \quad 0 \quad 0]$

$w = [1 \quad \delta_1/\delta_2] = [1 \quad 0.0177]$

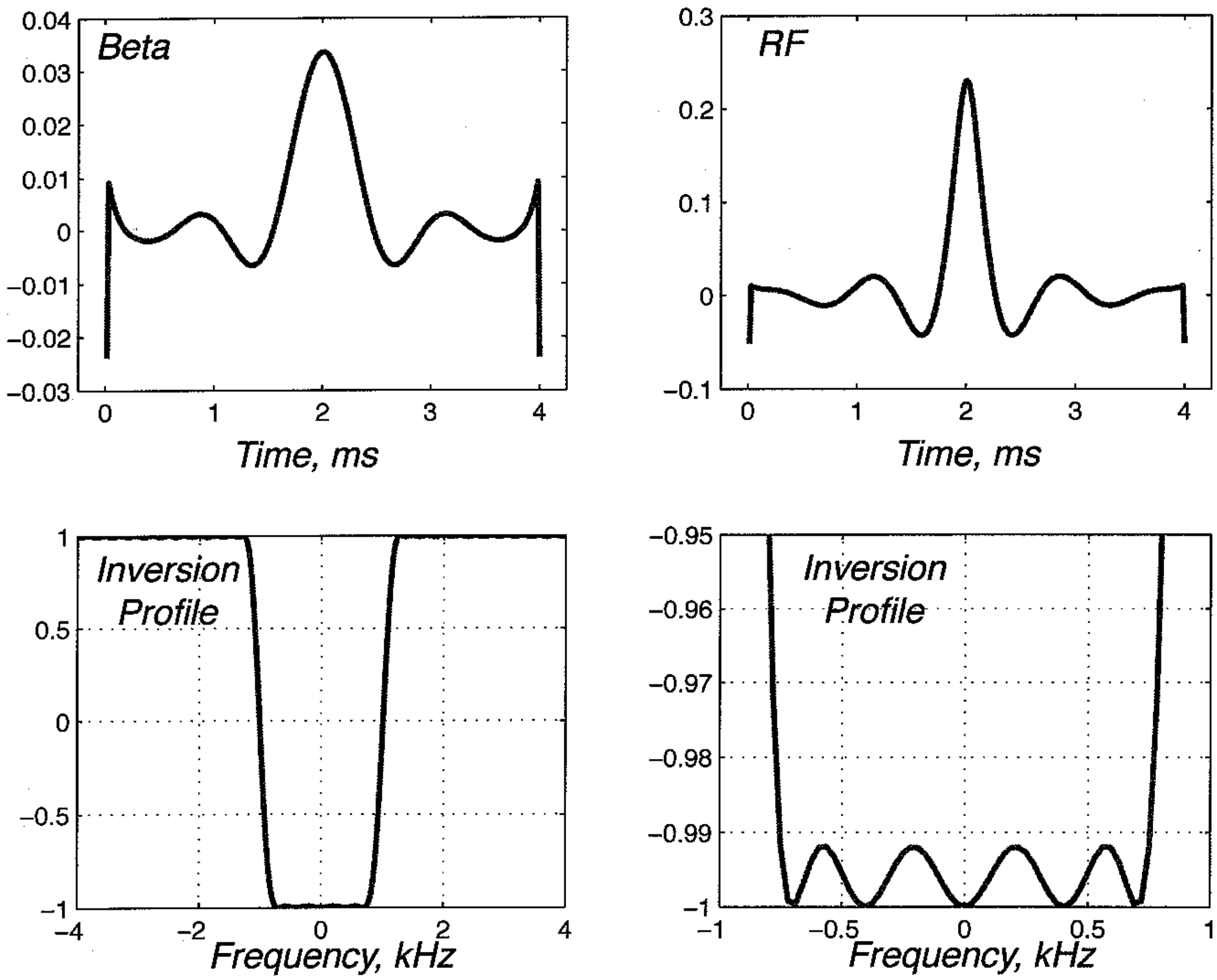
THEN

N-1

$b = \text{firpm}(255, f, m, w)$

THIS IS BETA. SCALE TO $\sin(\theta/2)$, THEN APPLY INVERSE SLR.

PM Inversion



DRAWBACKS OF PM DESIGNS

1) LARGE SPIKES COMMON AT FIRST/LAST SAMPLES (CONOLLY WINGS)

2) SPIKES GET LARGER AS N INCREASES

3) INTEGRATED ABSOLUTE VALUE OF STOPBAND ($1-|m_z|$ FOR EXAMPLE) CAN BE LARGE.

ANOTHER ALTERNATIVE IS WEIGHTED LEAST SQUARES

$$b = \mathrm{firls}(N, f, m, w)$$

SAME INPUTS AS REMEZ.

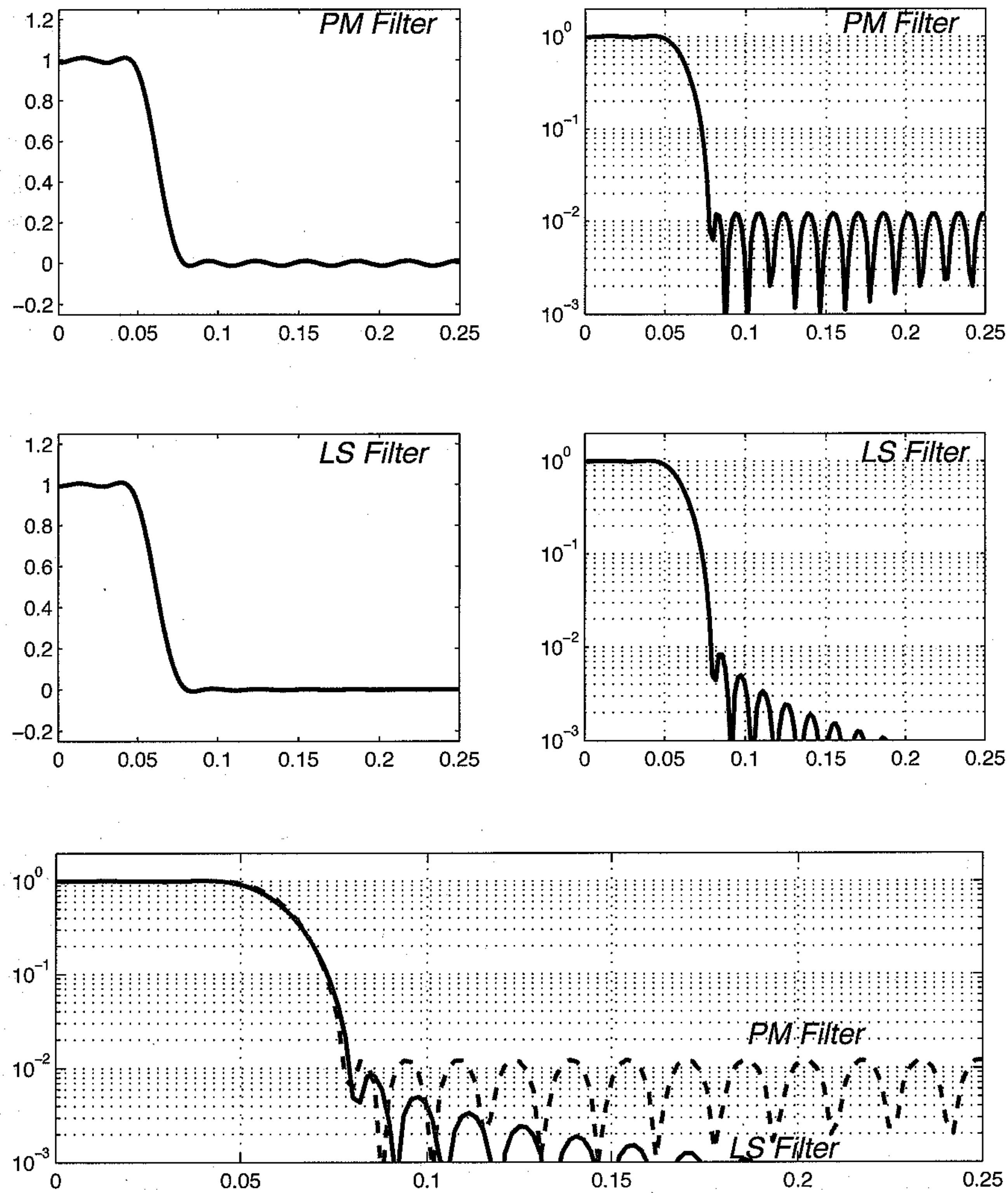
$D_\infty(\delta_1, \delta_2)$ WILL BE DIFFERENT FOR FIRLS, BUT NOT KNOWN.

FORTUNATELY THE $D_\infty(\delta_1, \delta_2)$ FOR PM FILTERS IS REASONABLY CLOSE.

<u>FIRLS DESIGNS ARE RECOMMENDED</u> UNLESS THERE ARE OTHER IMPORTANT FACTORS.

PM vs LS Filter Design

Time-Bandwidth = 8, 1% pass/stopband ripples
Same band edges



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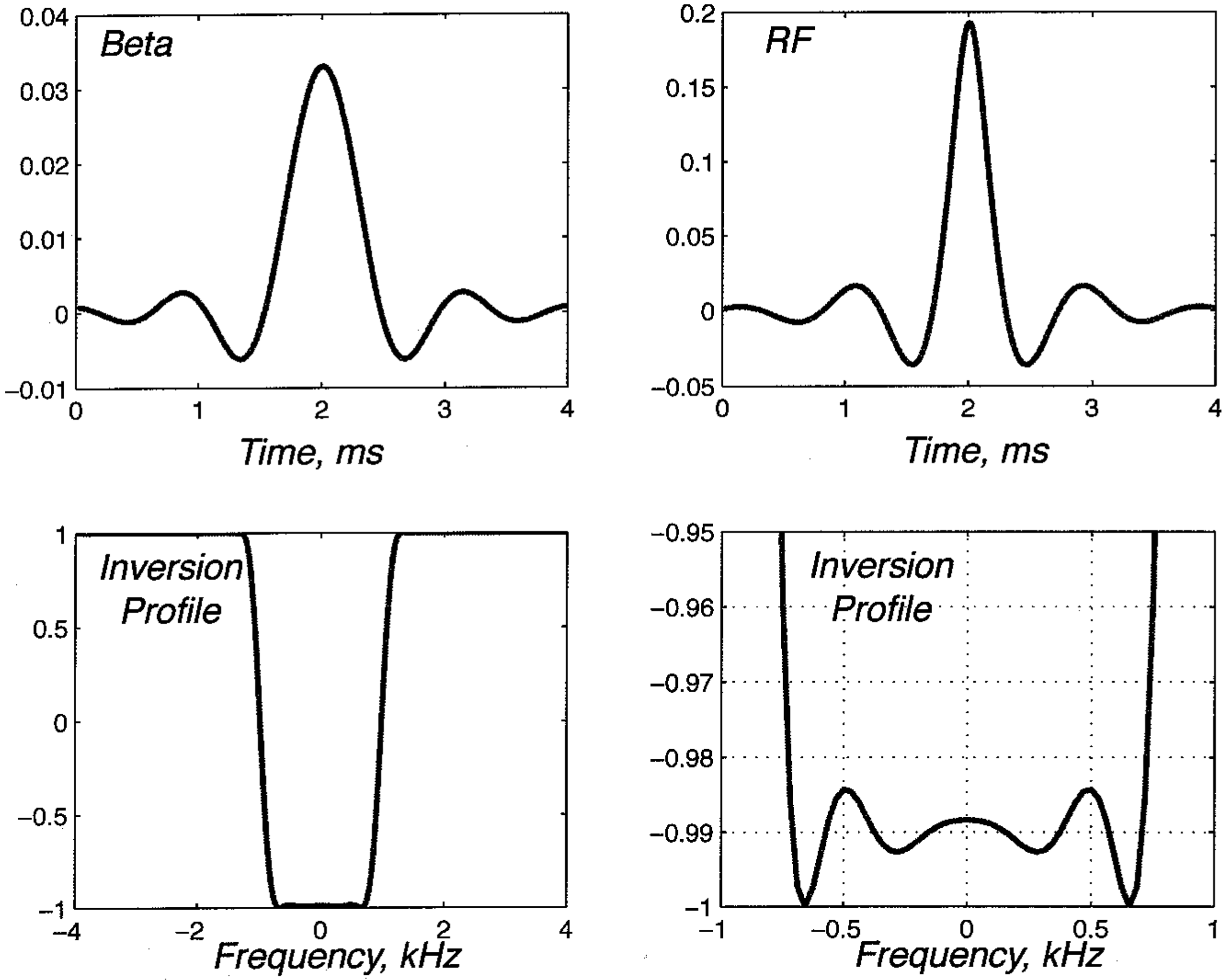
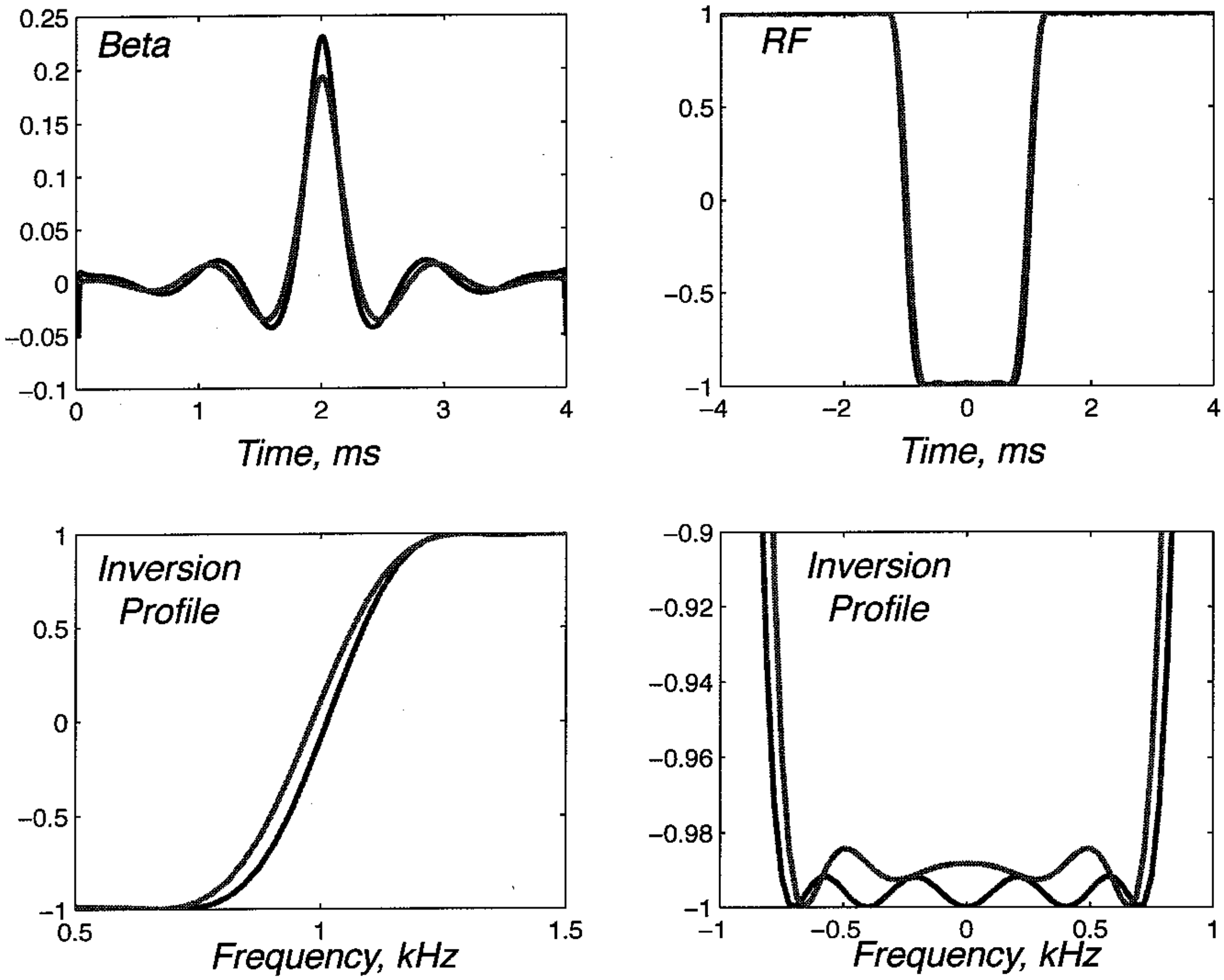
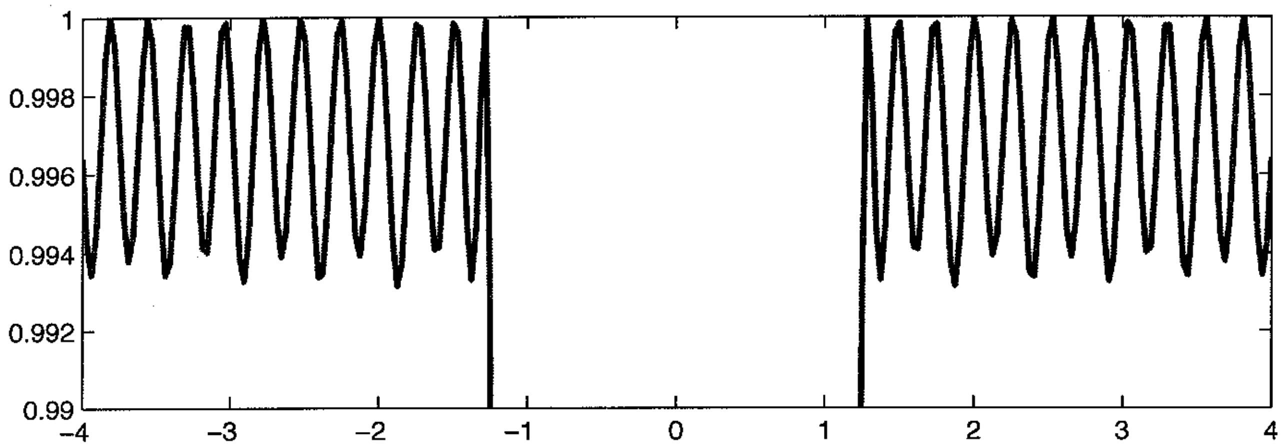
Weighted Least Squared Inversion

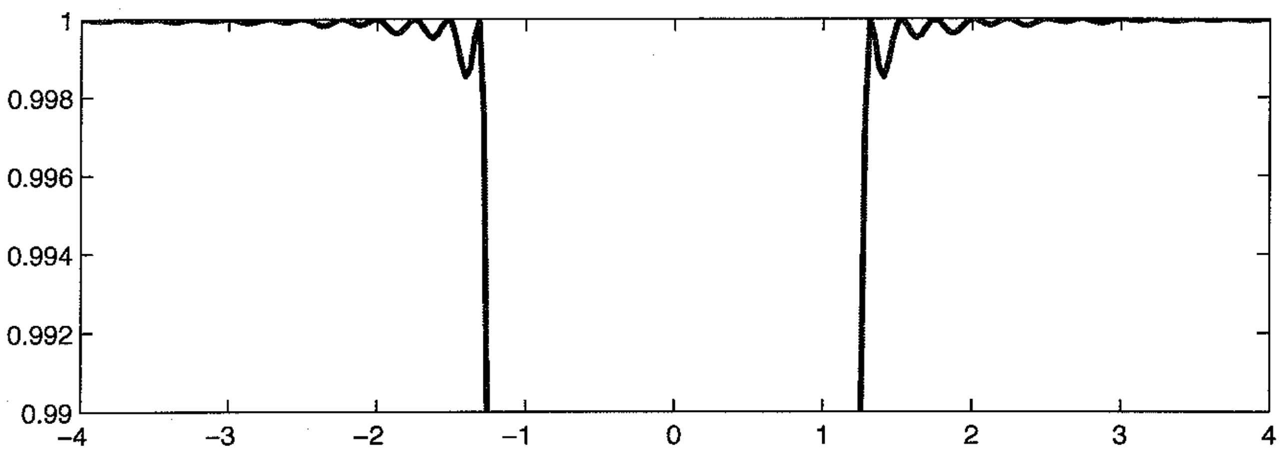


Comparison Between PM and LS Inversions



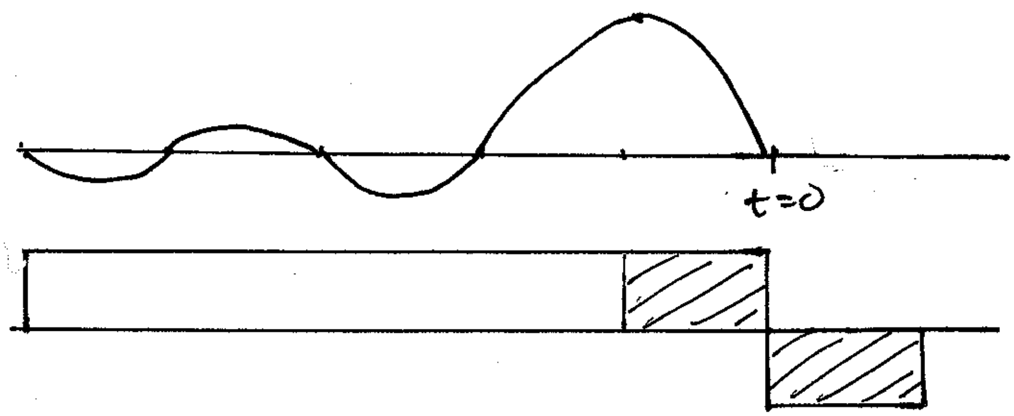
Comparison Between PM and LS Inversions





MINIMUM / MAXIMUM PHASE PULSES

EXCITATION AS LATE AS POSSIBLE



BENEFITS

SHARPER PROFILE

LESS REFOCUSING

SHORTER ECHO TIME

USES

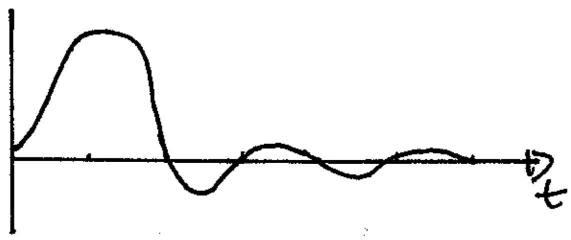
SLAB SELECT PULSE

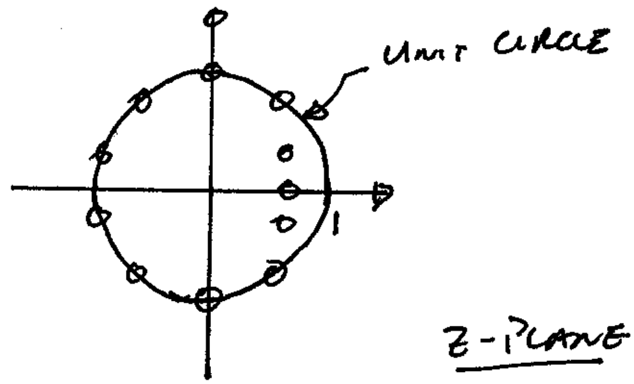
SATURATION PULSES

SHORT ECHO TIME EXCITATIONS

INVERSION PULSES

MINIMUM PHASE <u>CAUSAL</u> SIGNAL / FILTER





CAUSAL MINIMUM PHASE SIGNAL

SIGNAL CONCENTRATED AT BEGINNING

PASSBAND ZEROS INSIDE UNIT CIRCLE

MINIMUM PHASE RF PULSE

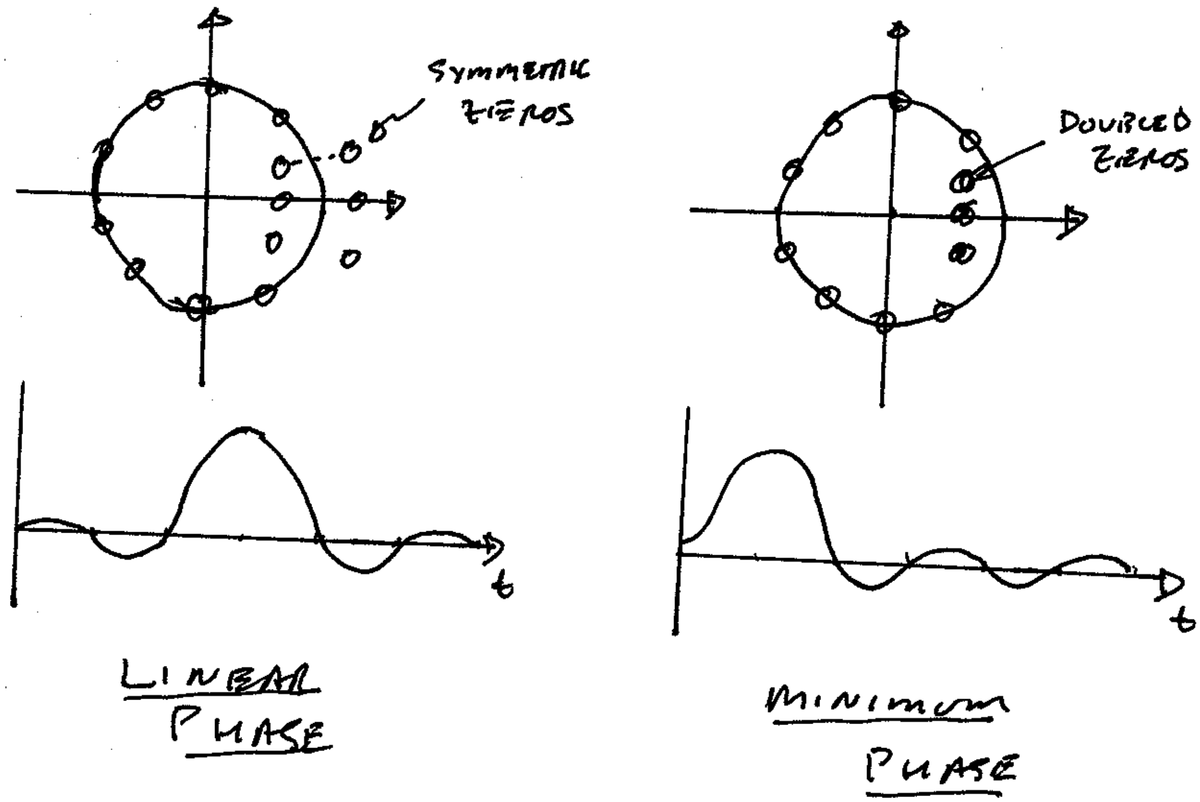
SIGNAL CONCENTRATED AT END (ORIGIN)

PASSBAND ZEROS OUTSIDE UNIT CIRCLE

CAUSAL DESIGN MUCH MORE FAMILIAR

DESIGN CAUSAL FILTERS, REVERSE FOR RF PULSE.

ANY SIGNAL HAS A MINIMUM PHASE SIGNAL WITH SAME MAGNITUDE



THESE BOTH HAVE SAME MAGNITUDE PROFILE!
NO GAIN IN SELECTIVITY.

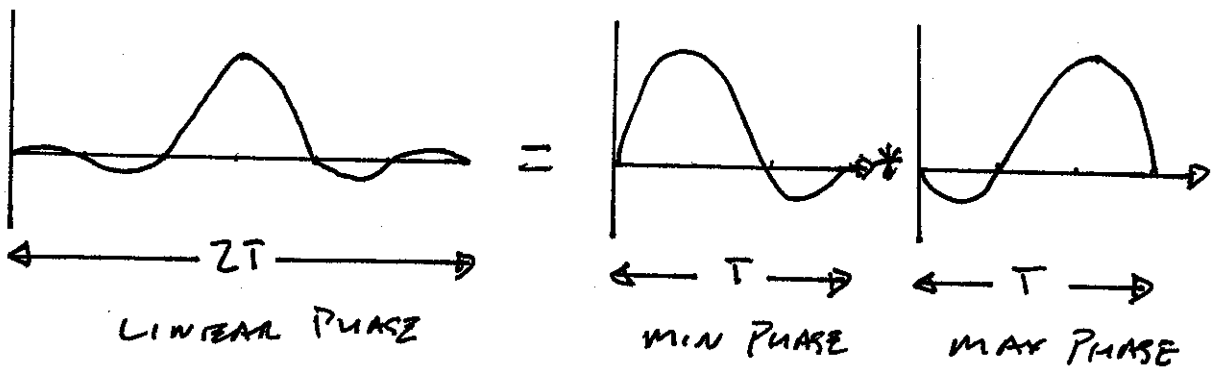
ONLY REALLY WANT SINGLE ZEROS INSIDE UNIT CIRCLE

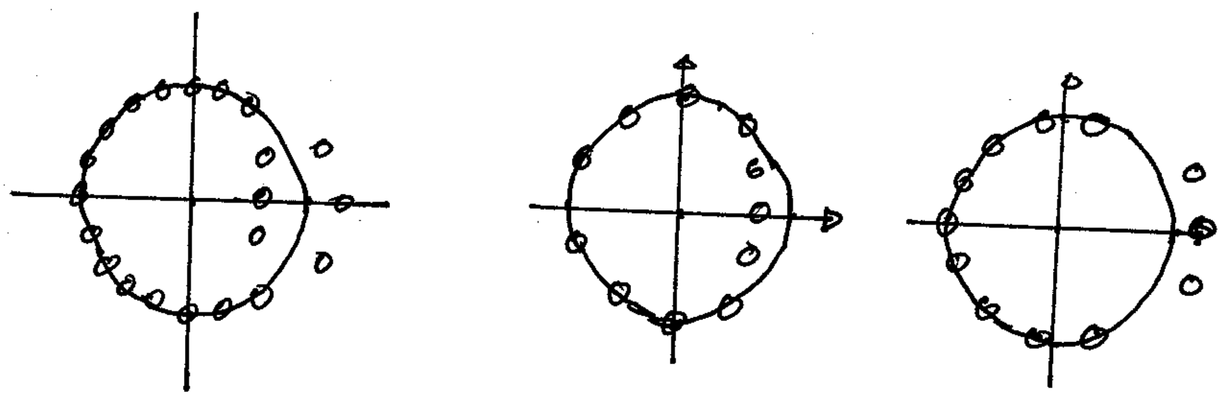
<u>BASIC IDEA</u>:

DESIGN A SPECIAL LINEAR PHASE PULSE

FACTOR INTO MINIMUM PHASE COMPONENT

LINEAR PHASE FILTER IS A CONVOLUTION OF A MINIMUM AND A MAXIMUM PHASE FILTER

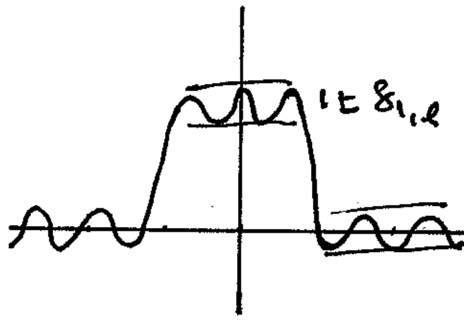


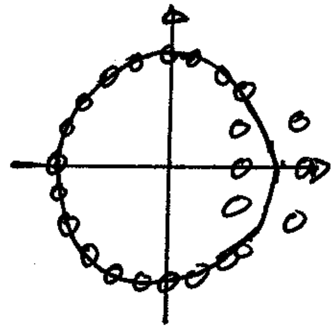


WE WANT TO DESIGN LINEAR PHASE FILTER TO BE EASY TO FACTOR

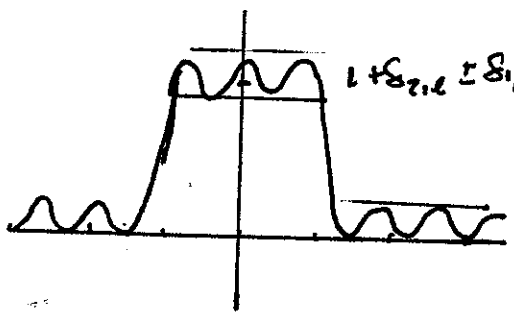
EQUAL-RIPPLE (PARKS-McCLELLAN) FILTER

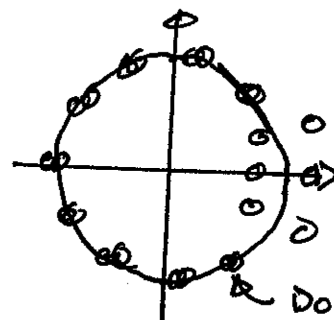
START WITH A PM FILTER





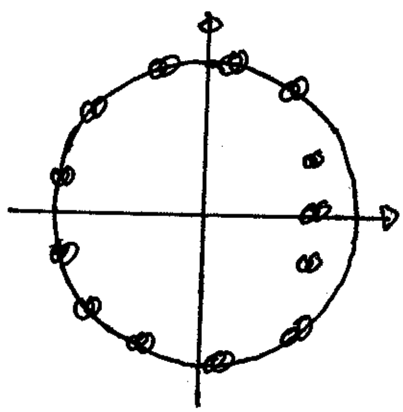
ADD A BIAS OF $\delta_{2,\ell}$





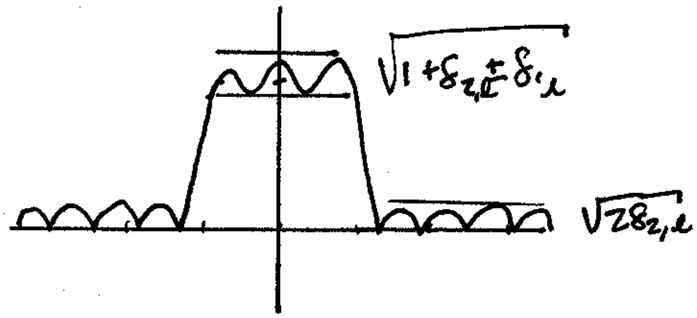
THIS MAGNITUDE PROFILE IS THE SAME AS

PERFECT SQUARE!

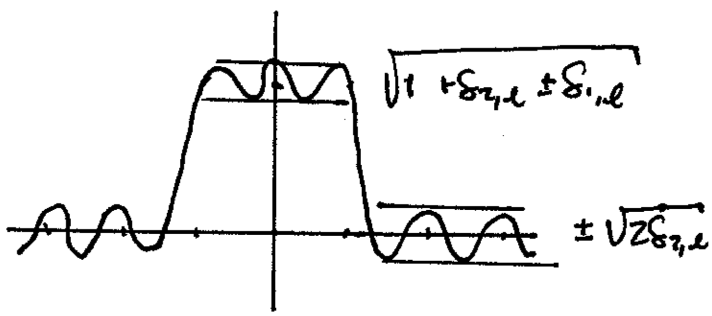


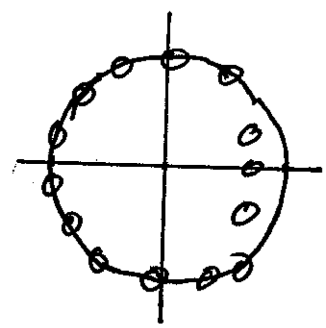
ALL ZEROS DOUBLED

TAKE SQUARE ROOT OF PROFILE



USE HILBERT TRANSFORM RELATIONSHIP TO
FIND PHASE





EQUAL RIPPLE, MINIMUM PHASE PULSE

SHARPEST TRANSITION.

HOW DO WE DESIGN THE ORIGINAL
LINEAR PHASE FILTER TO GIVE A
SPECIFIED MINIMUM PHASE PROFILE?

PASSBAND RIPPLE

$$\sqrt{1+\delta_{2,l} \pm \delta_{1,l}} \simeq 1+\delta_{2,l} \pm \tfrac{1}{2}\delta_{1,l}$$

$$\delta_{1,m} \simeq \tfrac{1}{2}\delta_{1,l}$$

$$\underline{\delta_{1,l} = 2\,\delta_{1,m}}$$

STOPBAND RIPPLE

$$\delta_{2,m} = \sqrt{2\,\delta_{2,l}}$$

$$\underline{\delta_{2,l} = \delta_{2,m}^{2}/2}$$

DESIGN RELATION FOR LINEAR PHASE FILTER

$$(2T)(\Delta F) = D_\infty(\delta_{1,l}, \delta_{2,l})$$

$$\underbrace{(2T)}_{\text{LENGTH OF LINEAR PHASE FILTER}}(\Delta F) = D_\infty(2\delta_{1,m}, \delta_{2,m}^{2}/2)$$

WHERE

T - LENGTH OF MINIMUM PHASE FILTER

THEN

$$T\,\Delta F = \tfrac{1}{2} D_\infty(2\delta_{1,m}, \delta_{2,m}^{2}/2)$$

$$= D_{\infty,m}(\delta_1, \delta_2)$$

WHERE

$$D_{\infty, m}(\delta_1, \delta_2) = \frac{1}{2} D_\infty(2\delta_1, \delta_2^2/2)$$

RECALL

$$\Delta F = \frac{D_{\infty, m}(\delta_1, \delta_2)}{T}$$

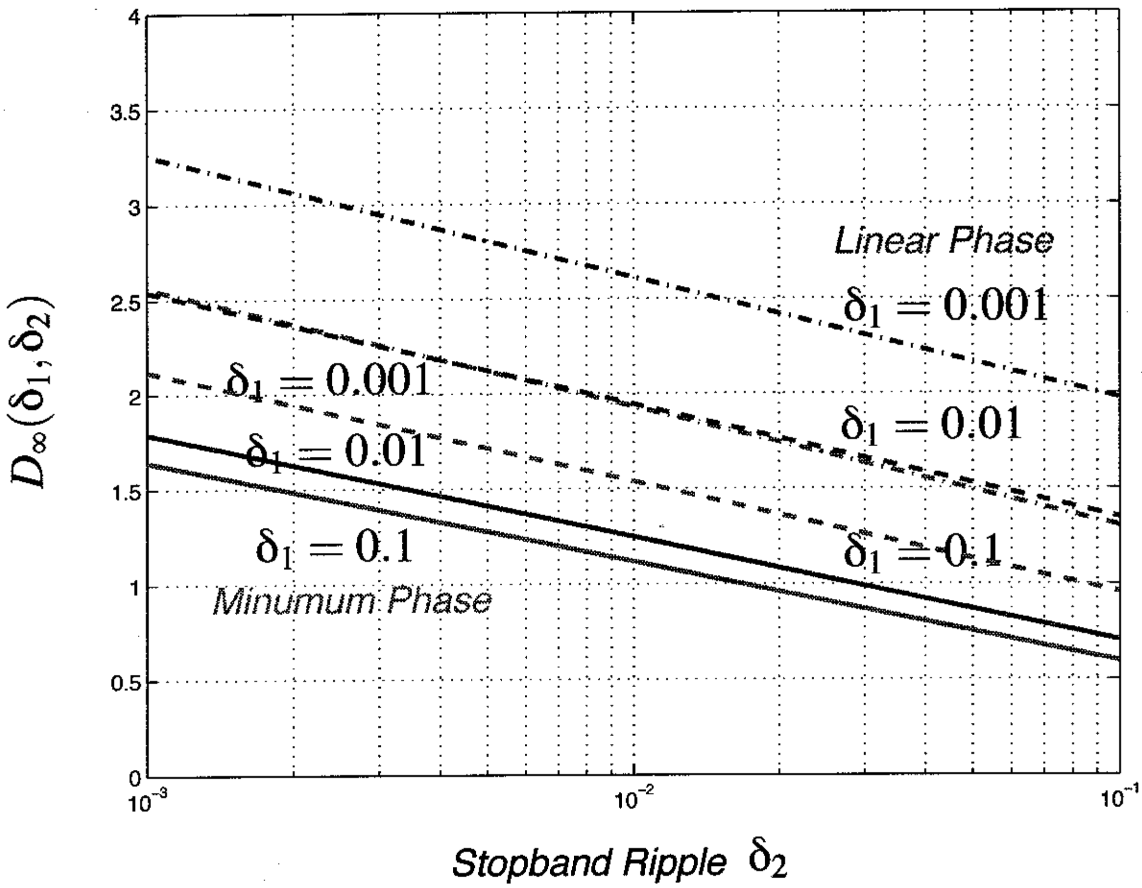
SO FOR A GIVEN T, A MINIMUM PHASE FILTER CAN HAVE HALF THE TRANSITION WIDTH OF LINEAR PHASE FILTER!

IN PRACTICE, THIS IS LESS.

TYPICAL NUMBERS ARE 70-90%

INCREASES WITH T (BW)

$$D_\infty(\delta_1,\delta_2) \quad vs \quad D_{\infty,m}(\delta_1,\delta_2)$$



$$D_{\infty,m}(\delta_1,\delta_2) = \frac{1}{2} D_\infty\left(2\delta_1, \delta_2^2/2\right)$$

TYPICAL TRADEOFFS

FOR ANY δ_2, WE CAN IMPROVE δ_1 FROM 0.01 TO 0.001

FACTOR OF TEN IN PASSBAND RIPPLE

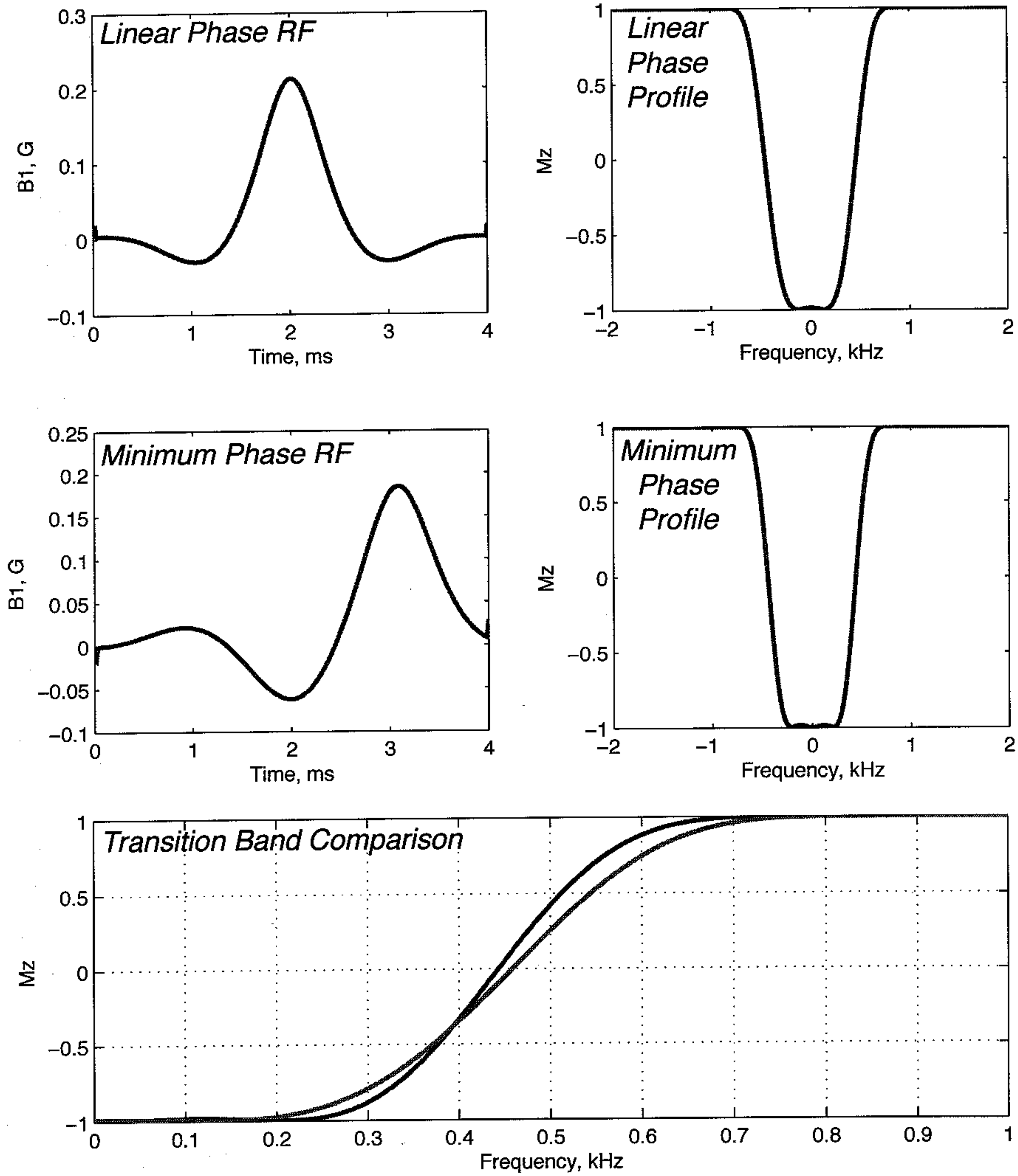
SIMILARLY, FIX δ_1 AND IMPROVE STOPBAND RIPPLE BY FATOR OF TEN

FIX δ_1 AND δ_2 AND REDUCE TRANSITION WIDTH

IF $\delta_2 = 0.001$, $\delta_1 = 0.001$, D_∞ GOES FROM 2.6 TO 2. W REDUCED TO 75%.

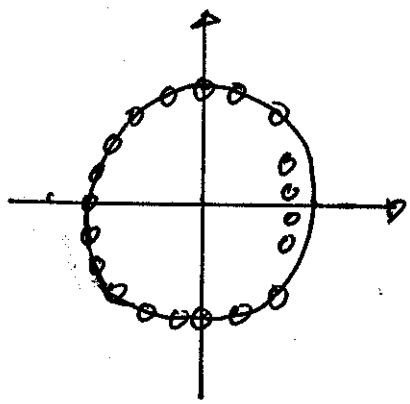
Linear vs Minimum Phase Inversion Pulses

$T(BW) = 4,\ \delta_1 = 0.01,\ \delta_2 = 0.0001$

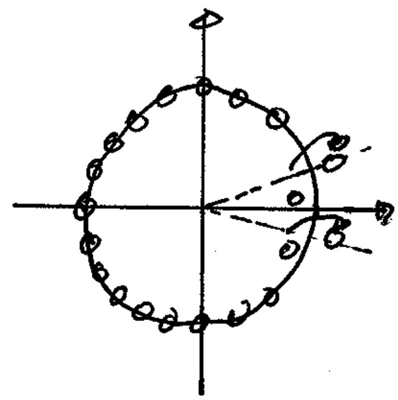


OTHER PHASE PROFILES

ONCE WE HAVE A MINIMUM PHASE DESIGN, THERE ARE MANY OTHER PHASE PROFILES THAT HAVE THE SAME MAGNITUDE PROFILE



MINIMUM PHASE



NON-LINEAR PHASE

EACH PASSBAND ZERO MAY BE FLIPPED OUTSIDE UNIT CIRCLE

THERE ARE ABOUT $T(BW)$ PASSBAND ZEROS

$$N_P \cong T(BW)$$

SO THERE ARE

$$2^{N_P}$$

POSSIBLE PHASE PROFILES

IF PROFILE PHASE IS NOT A CONCERN (SAT PULSES, INVERSION PULSES) WE CAN CHOOSE PHASE TO OPTIMIZE SOME OTHER PARAMETER

$\Rightarrow$ PEAK RF AMPLITUDE

<u>DESIGN PROCEDURE</u>

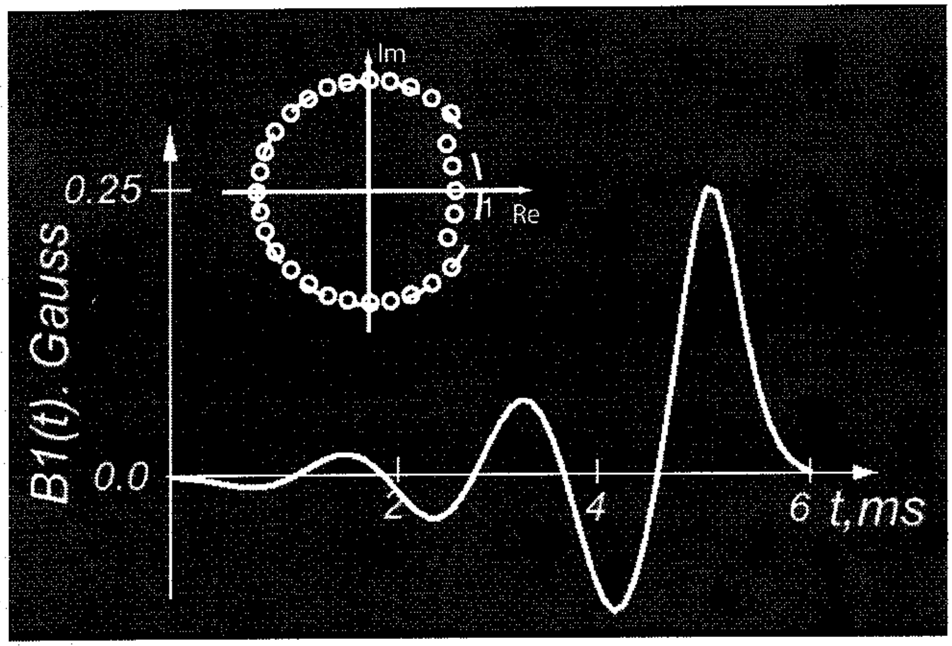
1) DESIGN MINIMUM PHASE $B_N(z)$
2) FACTOR (ROOTS.m IN MATLAB)
3) CHECK EACH COMBINATION OF ROOT FLIPS
    a) CALCULATE $B_N(z)$
    b) DESIGN RF PULSE
4) CHOOSE SOLUTION WITH MINIMUM PEAK $B_1(t)$.

CURRENTLY WORKS FOR 18 PASSBAND ZEROS, IN TENS OF SECONDS OF CPU TIME.

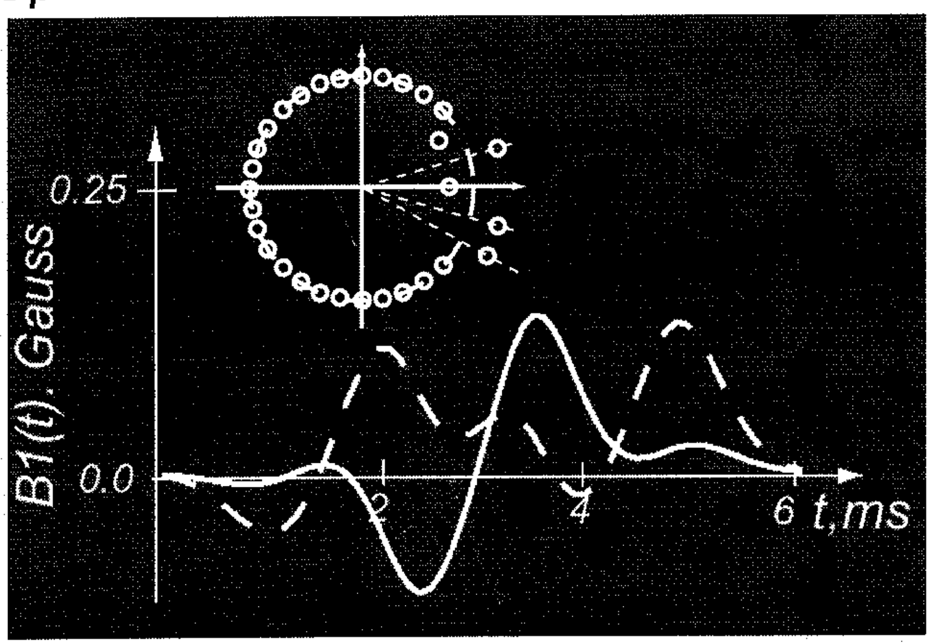
OTHER APPROACHES FOR HIGHER ORDERS.

Non-Linear Phase Inversion Pulses

Minimum Phase Inversion



Optimized Phase Inversion



Peak Amplitude reduced by a factor of 2,
Peak power by a factor of 4