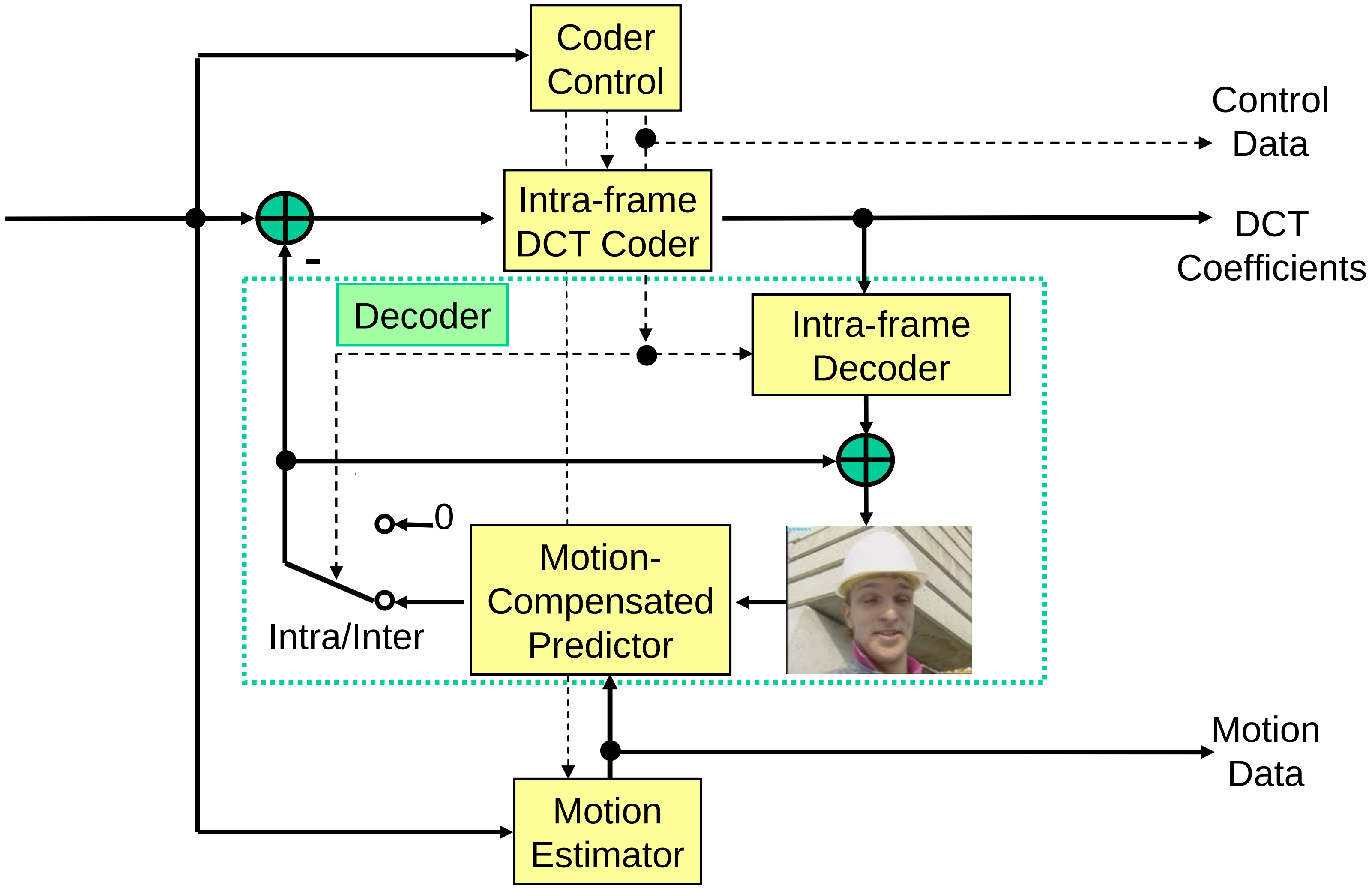
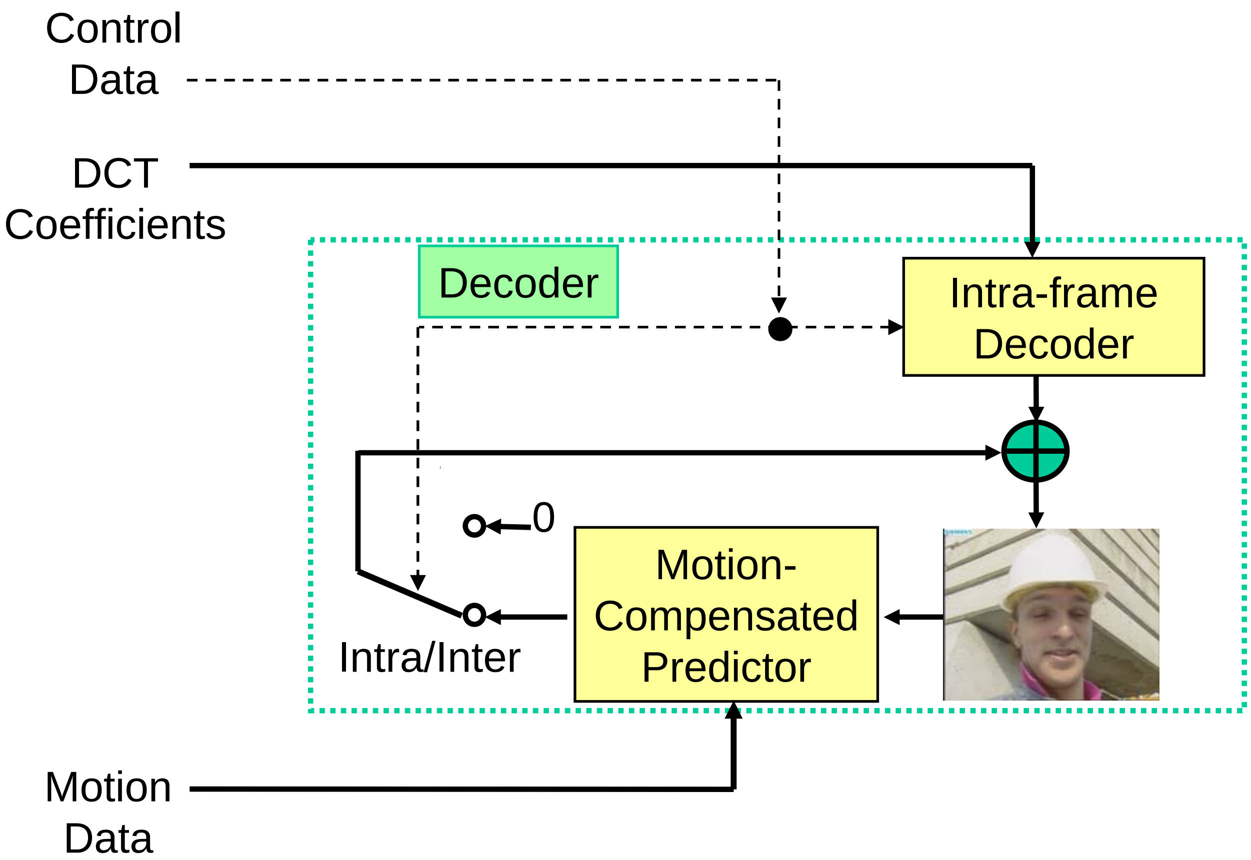
Motion-compensated hybrid coding

- Power spectral density of motion-compensated prediction error
- Rate-distortion analysis of m.c. hybrid coding
- RD performance vs. motion compensation accuracy
- Loop filter
- Rate-constrained motion estimation

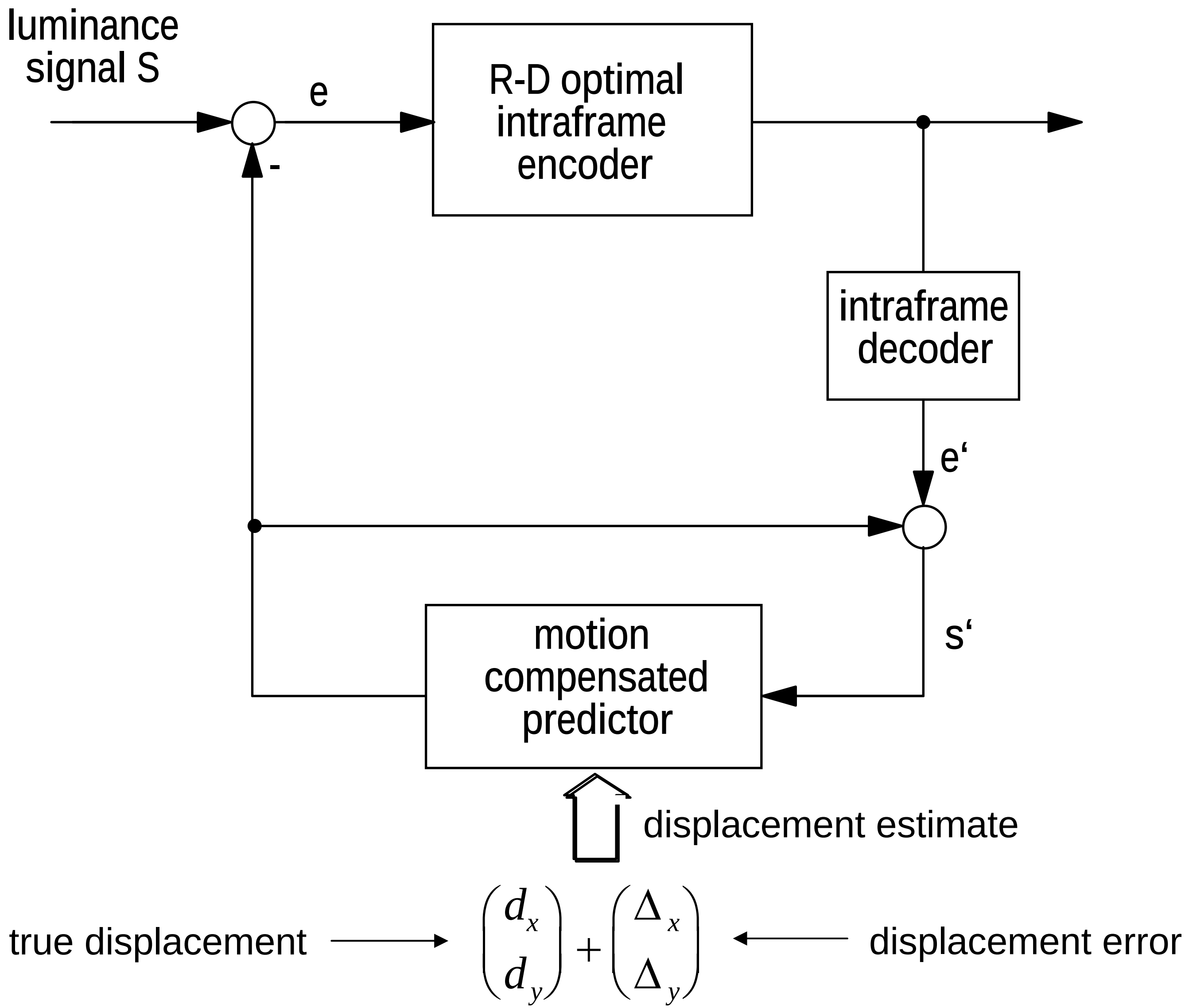
Motion-compensated hybrid coder



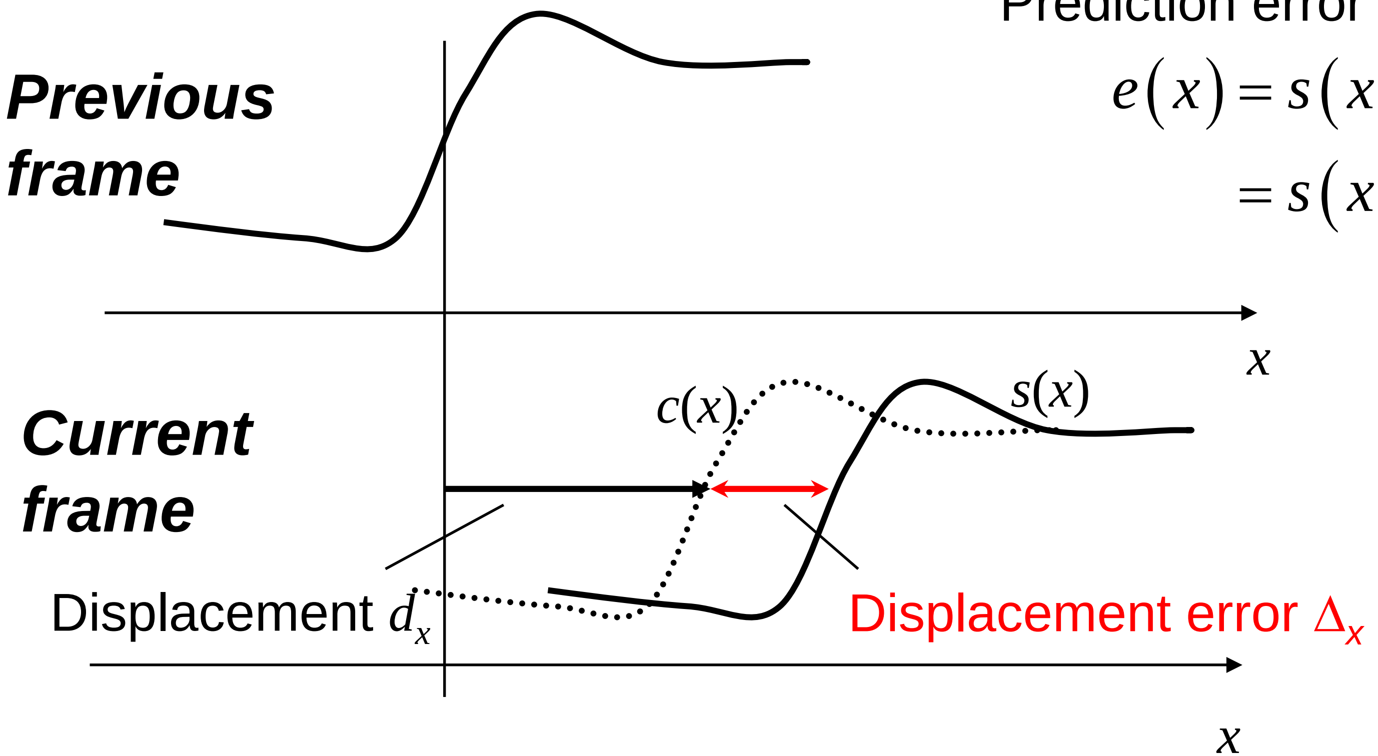
Motion-compensated hybrid decoder



Model for performance analysis of an MCP hybrid coder



Analysis of the motion-compensated prediction error



Motion-compensated signal

$$c(x) = s(x - \Delta_x) - n(x)$$

Prediction error

$$e(x) = s(x) - c(x)$$
$$= s(x) - s(x - \Delta_x) + n(x)$$

Analysis of m.c. prediction error (cont.)

- Motion-compensated prediction error

$$e(x) = s(x) - c(x) = s(x) - s(x - \Delta_x) + n(x) = \left(\delta(x) - \delta(x - \Delta_x)\right) * s(x) + n(x)$$

- Power spectrum of prediction error, assuming constant displacement error Δ_x, statistical independence of s and n

$$\Phi_{ee}(\omega) = \Phi_{ss}(\omega)\left(1 - e^{-j\omega\Delta_x}\right)\left(1 - e^{j\omega\Delta_x}\right) + \Phi_{nn}(\omega)$$
$$= 2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{e^{-j\omega\Delta_x}\right\}\right) + \Phi_{nn}(\omega)$$

- Random displacement error Δ_x, statistically independent from s, n

$$\Phi_{ee}(\omega) = E\left\{2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{e^{-j\omega\Delta_x}\right\}\right) + \Phi_{nn}(\omega)\right\}$$
$$= 2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{E\left\{e^{-j\omega\Delta_x}\right\}\right\}\right) + \Phi_{nn}(\omega)$$
$$= 2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{P(\omega)\right\}\right) + \Phi_{nn}(\omega)$$

Analysis of m.c. prediction error (cont.)

- What is $P(\omega)$?

$$P(\omega) = E\left\{e^{-j\omega\Delta_x}\right\}$$

$$= \int_{-\infty}^{\infty} p_{\Delta_x}(\Delta) e^{-j\omega\Delta} d\Delta = F\left\{p_{\Delta_x}(\Delta_x)\right\}$$

Fourier transform of the displacement error pdf!

- Same as characteristic function of displacement error, except for sign
- Extension to 2-d

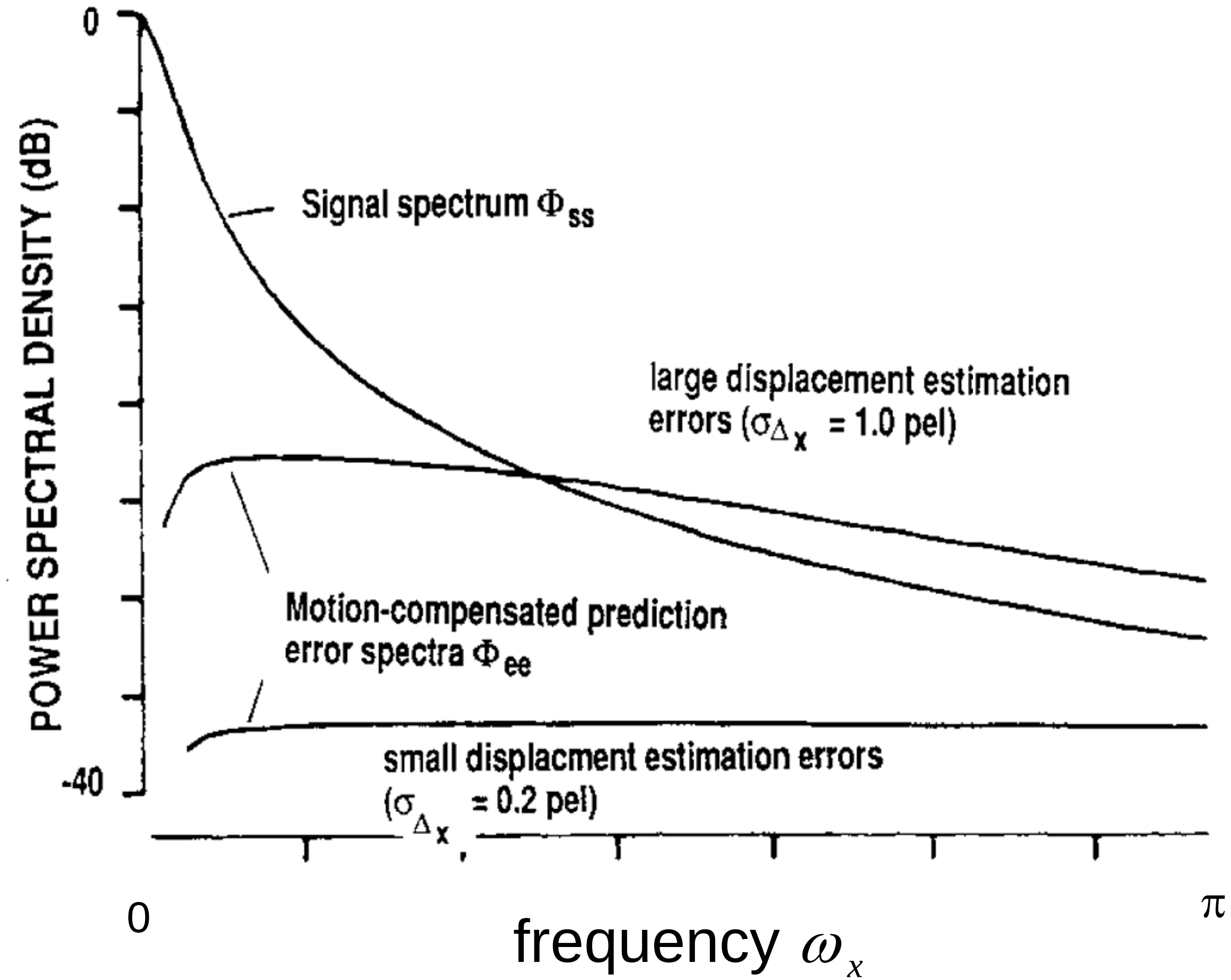
$$\Phi_{ee}(\omega_x,\omega_y) = 2\Phi_{ss}(\omega_x,\omega_y)\left(1-\mathrm{Re}\left\{P(\omega_x,\omega_y)\right\}\right)+\Phi_{nn}(\omega_x,\omega_y)$$

power spectrum of luminance signal

Fourier transform of the displacement error pdf $p(\Delta_x,\Delta_y)$

noise spectrum

Power spectrum of motion-compensated prediction error



R-D function for MCP with integer-pixel accuracy

- $(\Delta_x, \Delta_y)^T$ assumed uniformly distributed between

$$\Delta_x = \pm \frac{1}{2} \text{ pel}$$

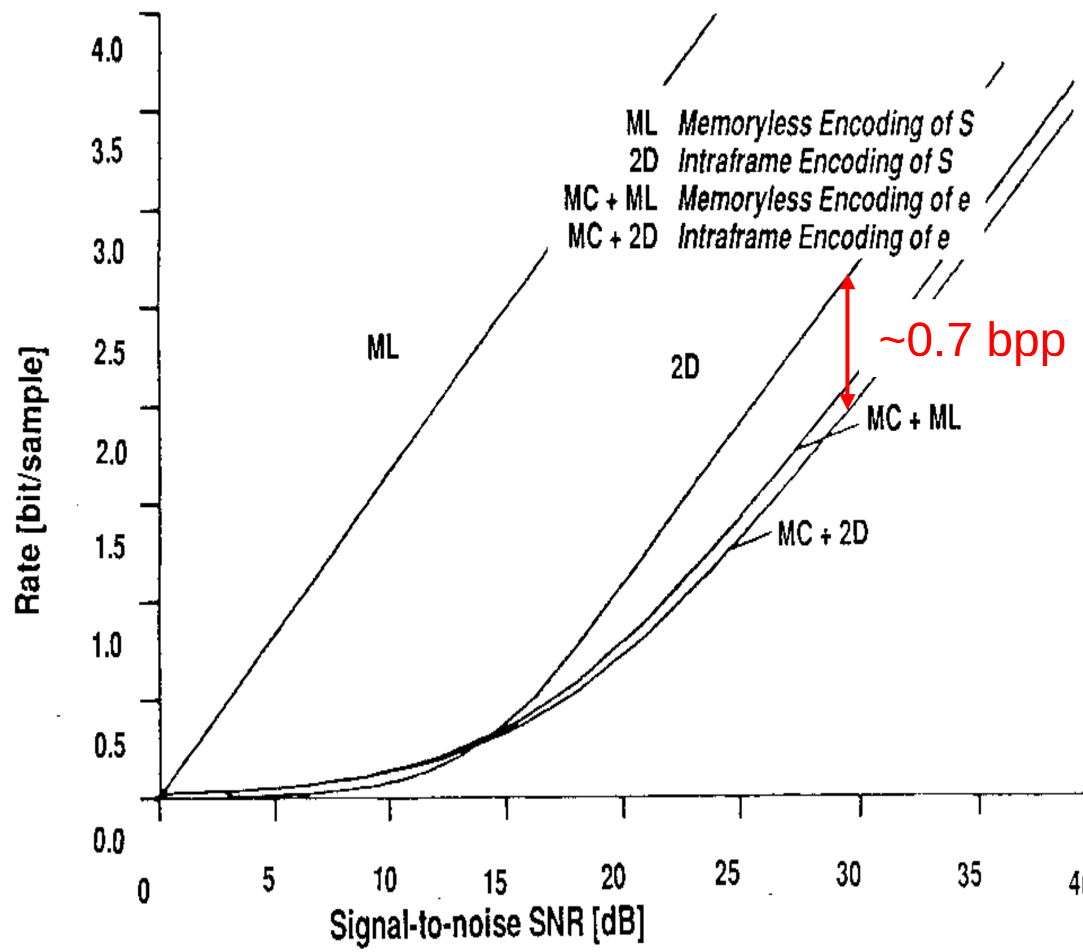
$$\Delta_y = \pm \frac{1}{2} \text{ line}$$

- Gaussian signal model

$$\Phi_{ss}(\omega_x, \omega_y) = A\left(1 + \frac{\omega_x^2 + \omega_y^2}{\omega_0^2}\right)^{-\frac{3}{2}}$$

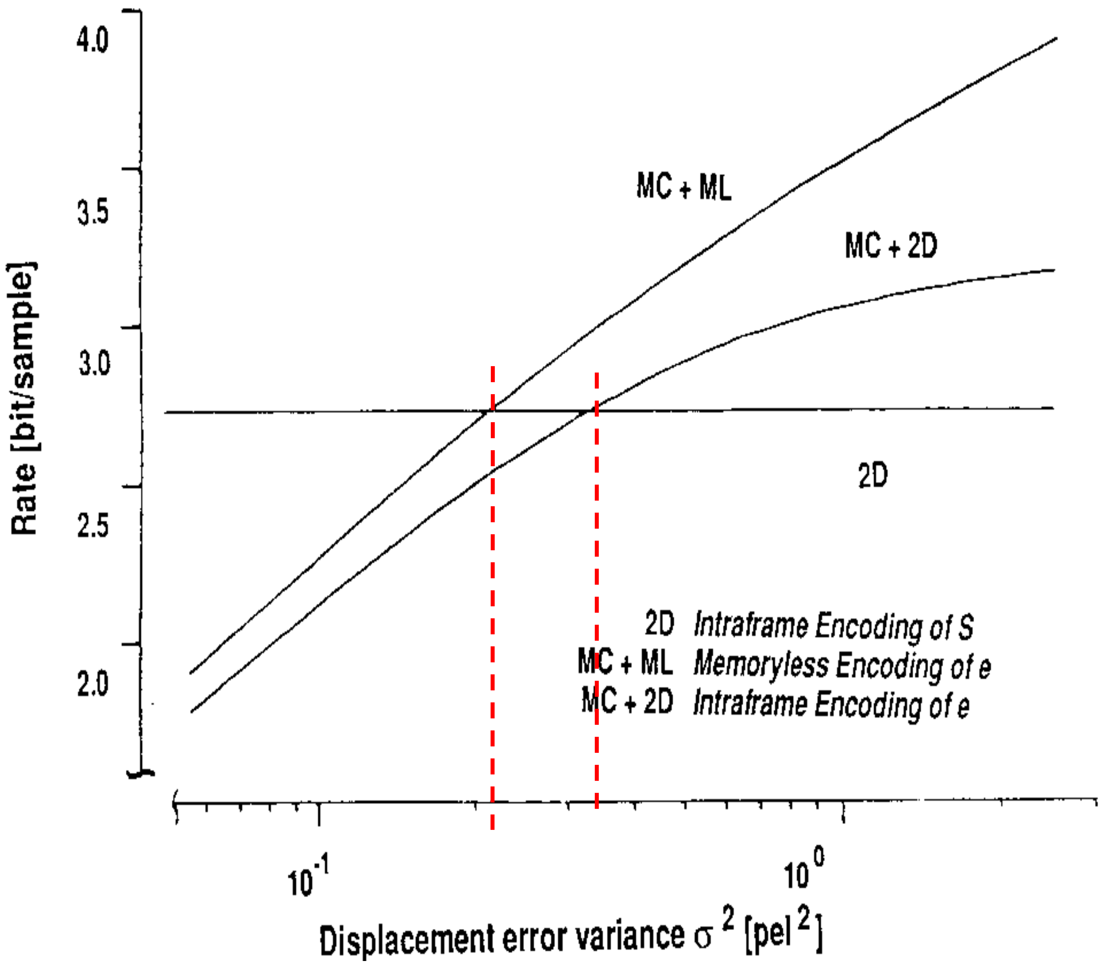
- Typical parameters for CIF resolution (352 x 288 pixels)

Minimum bit-rate for given SNR

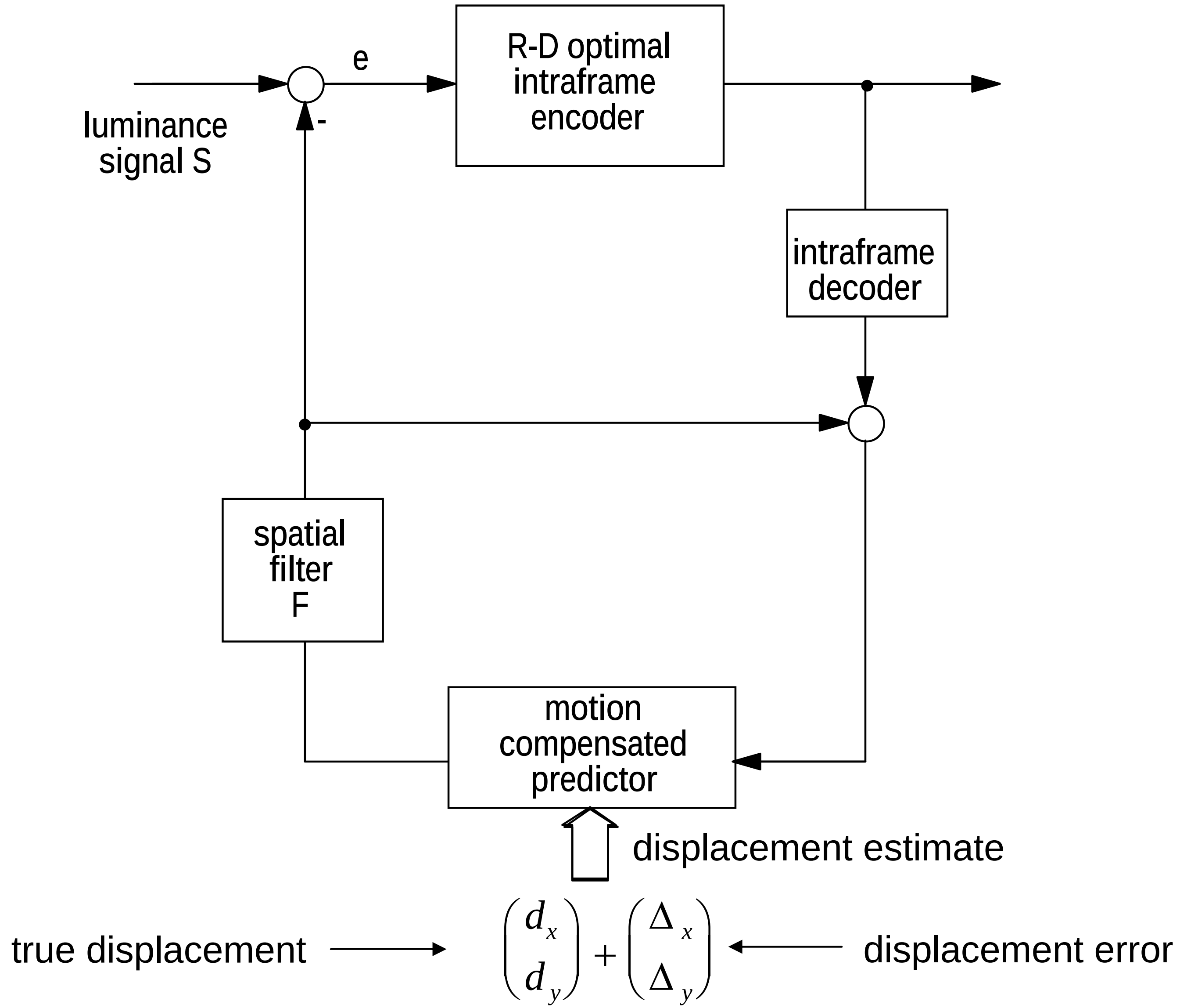


Required accuracy of motion compensation

- $p(\Delta_x, \Delta_y)$ isotropic Gaussian pdf with variance σ^2
- $$\Phi_{ss}(\omega_x, \omega_y) = A\left(1 + \frac{\omega_x^2 + \omega_y^2}{\omega_0^2}\right)^{-\frac{3}{2}}$$
- Typical parameters for CIF resolution (352 x 288 pixels)
- Minimum bit-rate for SNR = 30 dB



Model of MCP hybrid coder with loop filter



Motion-compensated prediction error with loop filter

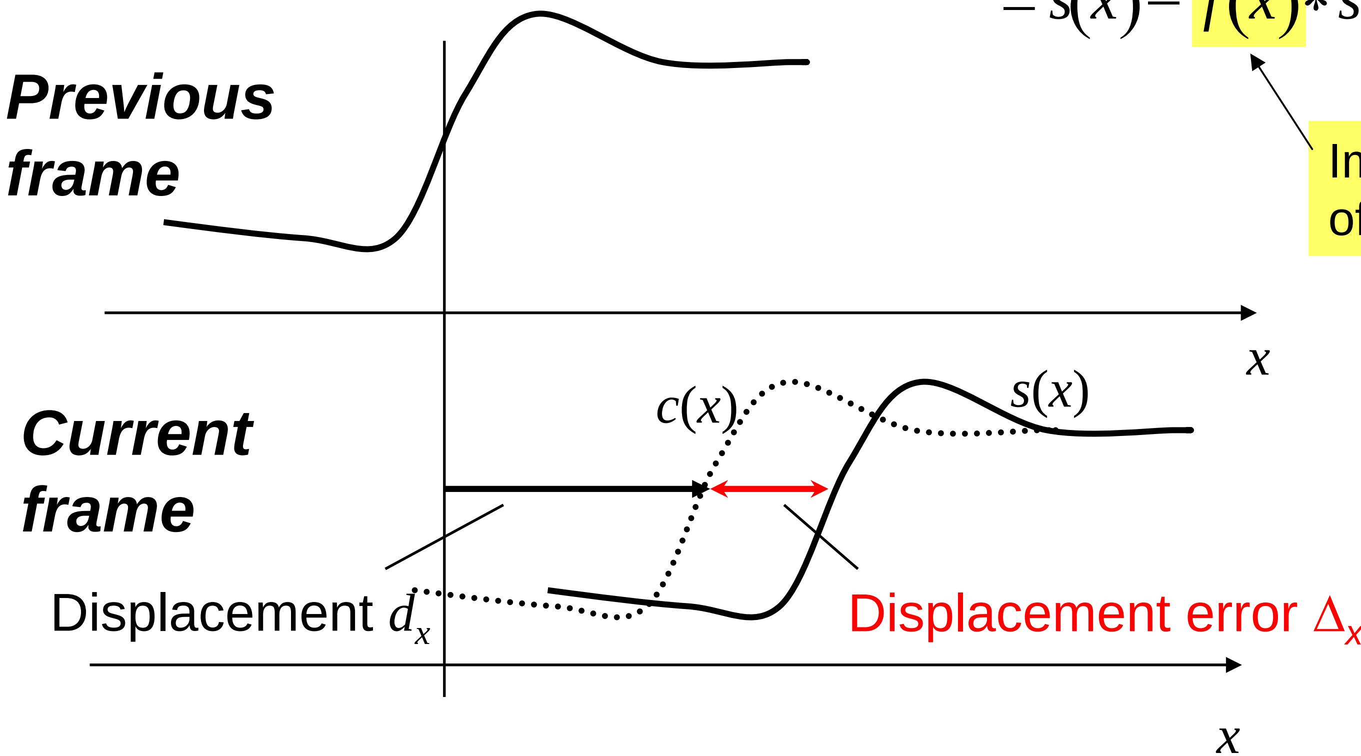
Motion-compensated signal

$$c(x) = s(x - \Delta_x) - n(x)$$

Prediction error

$$e(x) = s(x) - f(x) * c(x)$$
$$= s(x) - f(x) * s(x - \Delta_x) + f(x) * n(x)$$

Impulse response of loop filter



Spatial power spectrum of m.c. prediction error with loop filter

$$\Phi_{ee}(\Lambda) = \Phi_{ss}(\Lambda)\left(1 + |F(\Lambda)|^2 - 2\,\mathrm{Re}\left\{F(\Lambda)P(\Lambda)\right\}\right) + \Phi_{nn}(\Lambda)\,|F(\Lambda)|^2$$

$P(\Lambda)$	2-D Fourier transform of displacement error pdf
$F(\Lambda)$	2-D Fourier transform of $f(x,y)$
Φ_{uu}	spatial spectral power density of signal u
Λ	vector of spatial frequencies (ω_x, ω_y)
$n(x,y)$	noise

Optimum loop filter

- Wiener filter minimizes prediction error variance

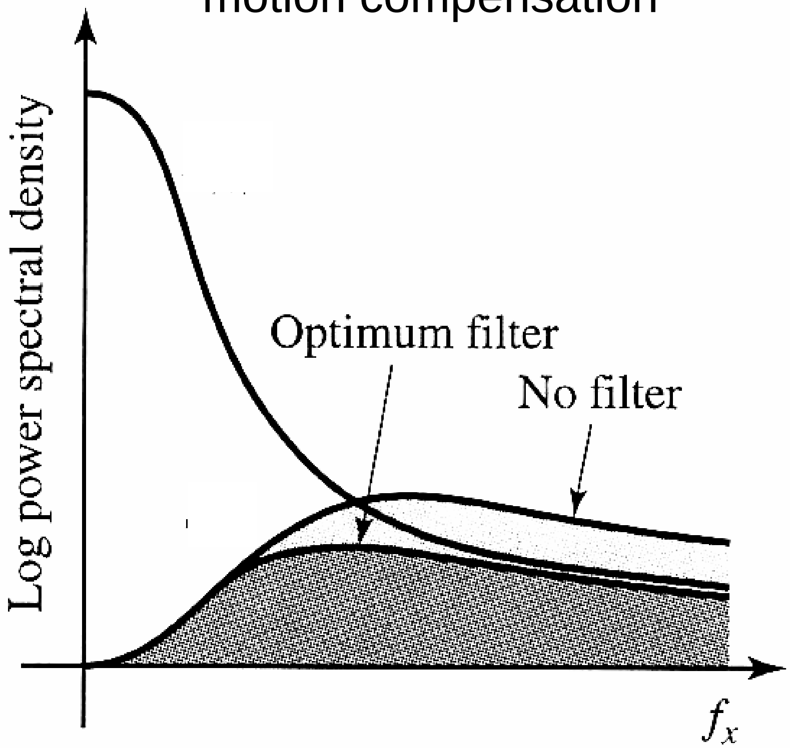
$$F_{\text{opt}}(\Lambda) = \underbrace{P^*(\Lambda)}_{\text{accounts for accuracy of motion compensation}} \cdot \frac{\Phi_{ss}(\Lambda)}{\underbrace{\Phi_{ss}(\Lambda) + \Phi_{nn}(\Lambda)}_{\text{accounts for noise}}}$$

- Resulting minimum prediction error spectrum

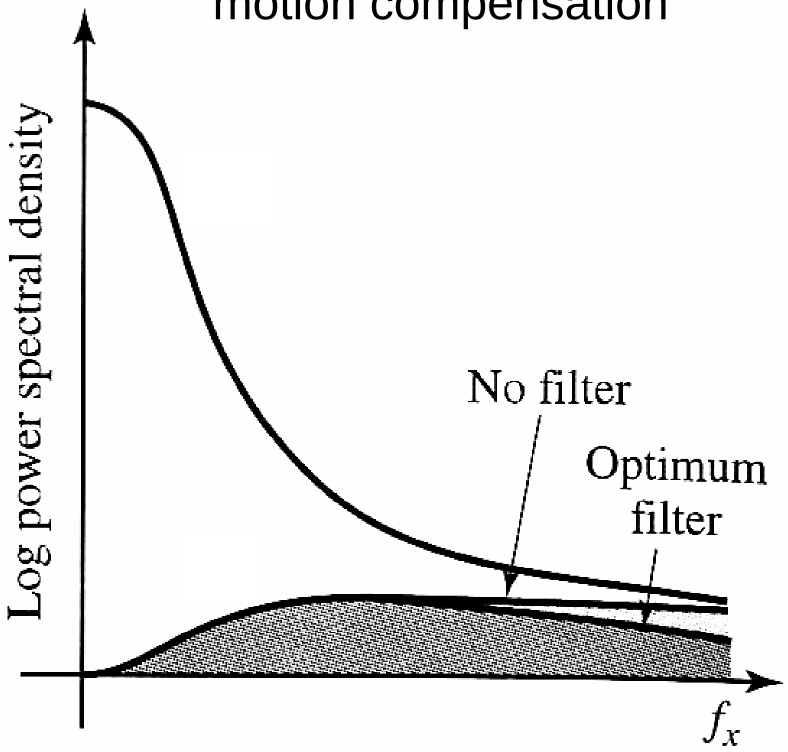
$$\Phi_{ee}(\Lambda) = \Phi_{ss}(\Lambda)\left(1 - |P(\Lambda)|^2 \frac{\Phi_{ss}(\Lambda)}{\Phi_{ss}(\Lambda) + \Phi_{nn}(\Lambda)}\right)$$

Effect of loop filter

Moderately accurate
motion compensation

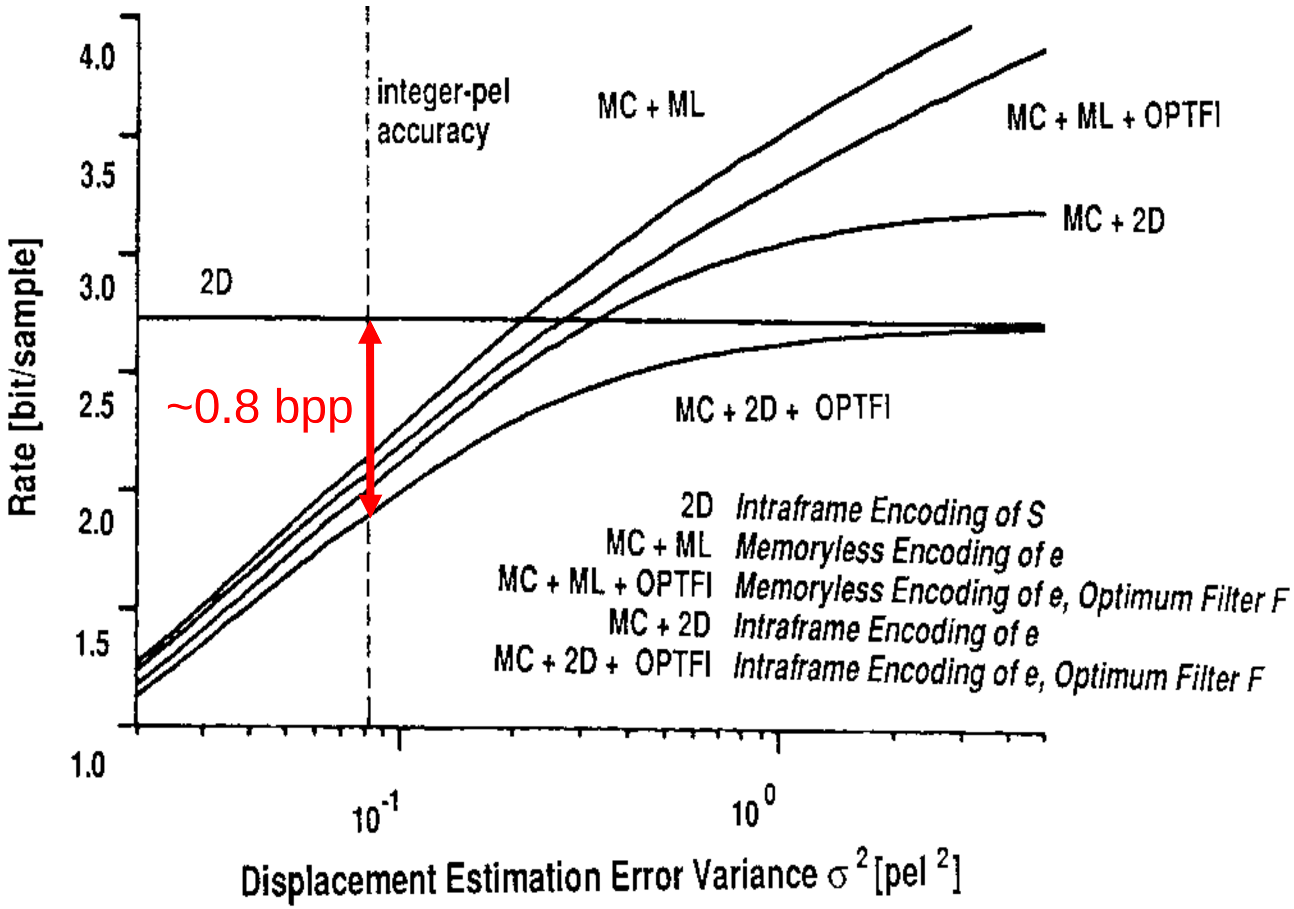


Very accurate
motion compensation



Required accuracy of motion compensation with loop filter

- $p(\Delta_x, \Delta_y)$ isotropic Gaussian pdf with variance σ^2
- Minimum bit-rate for SNR = 30 dB



Practical optimum loop filter design

- Not practical for loop filter design

$$F_{\text{opt}}(\Lambda) = \underbrace{P^*(\Lambda)} \cdot \frac{\Phi_{ss}(\Lambda)}{\underbrace{\Phi_{ss}(\Lambda) + \Phi_{nn}(\Lambda)}}$$

Motion compensation accuracy not known

"Noise" psd not known

- To determine Wiener filter from measurements:

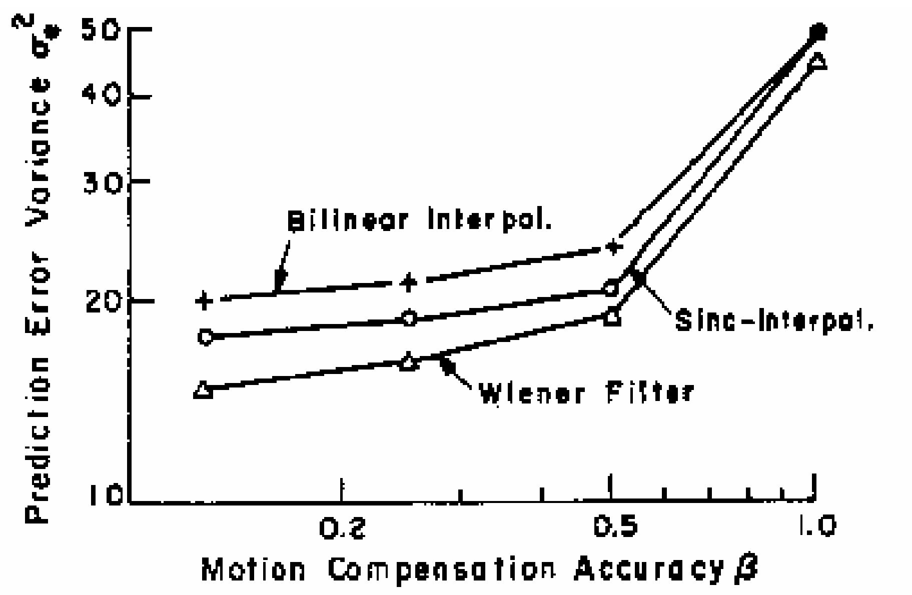
$$F_{\text{opt}}(\Lambda) = \frac{\Phi_{sc}(\Lambda)}{\Phi_{cc}(\Lambda)}$$

cross spectrum between $s(x,y)$ and the motion-compensated signal $c(x,y) = r(x - \hat{d}_x, y - \hat{d}_y)$

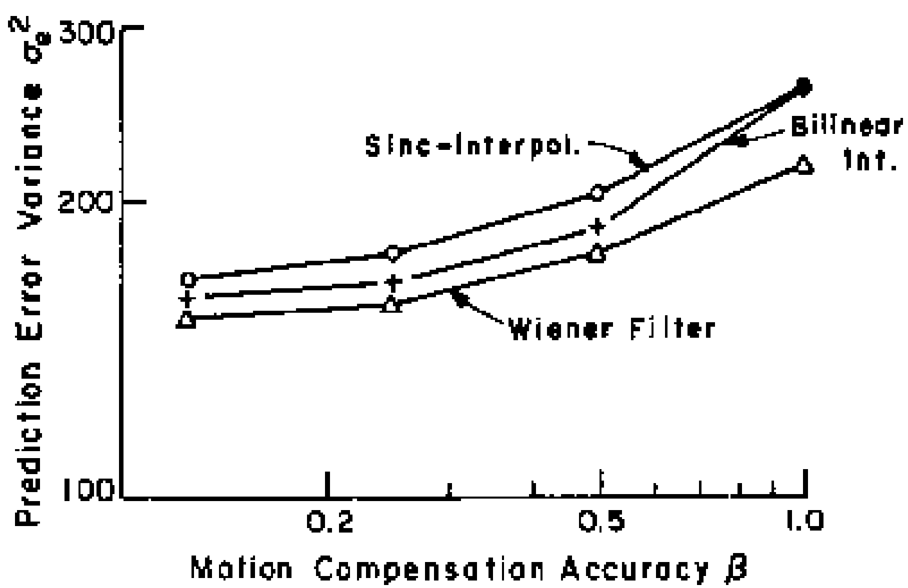
Experimental evaluation of fractional-pixel motion compensation

- ITU-R 601 TV signals, 13.5 MHz sampling rate, interlaced, blockwise motion compensation with blocksize16x16

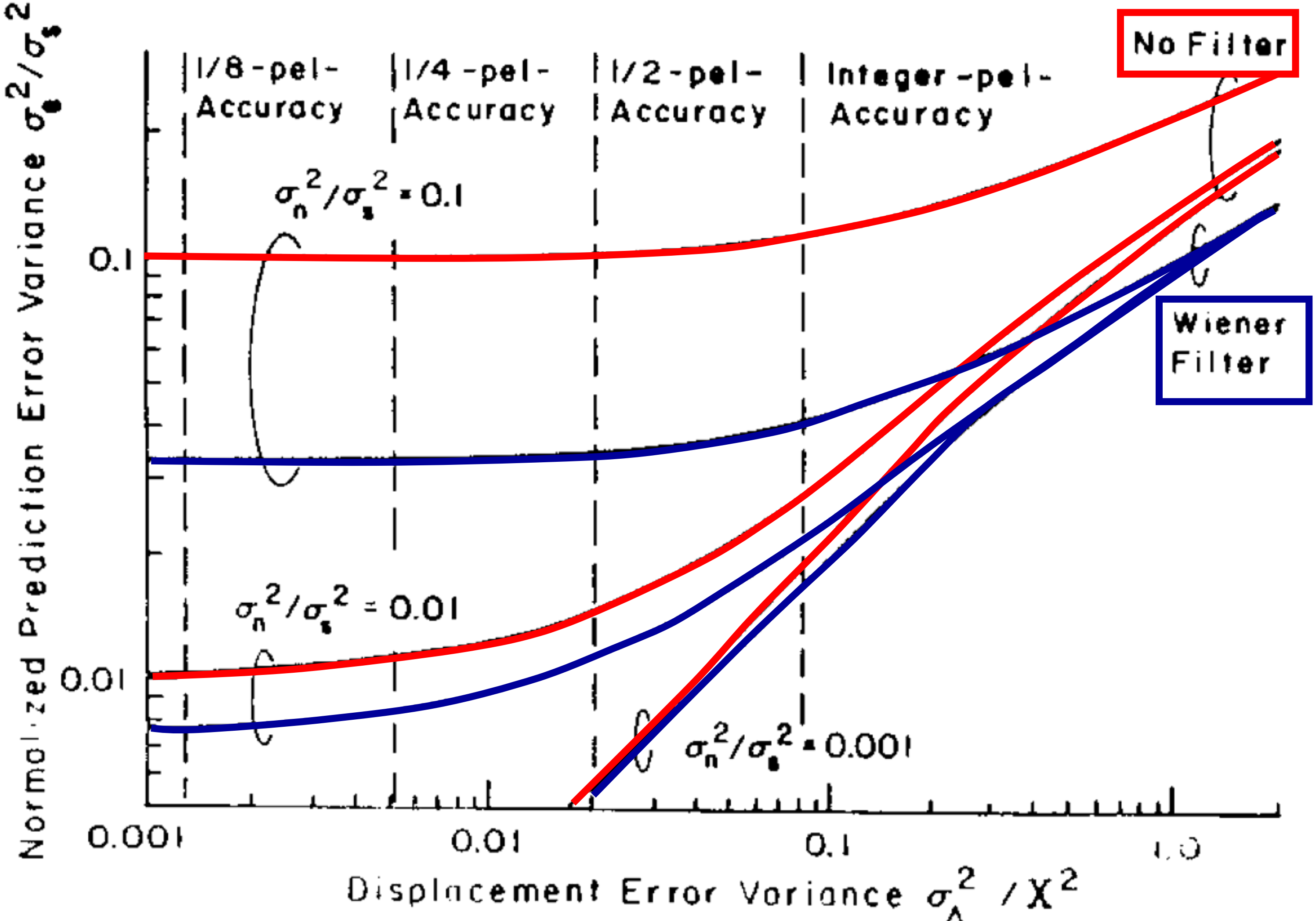
Zoom



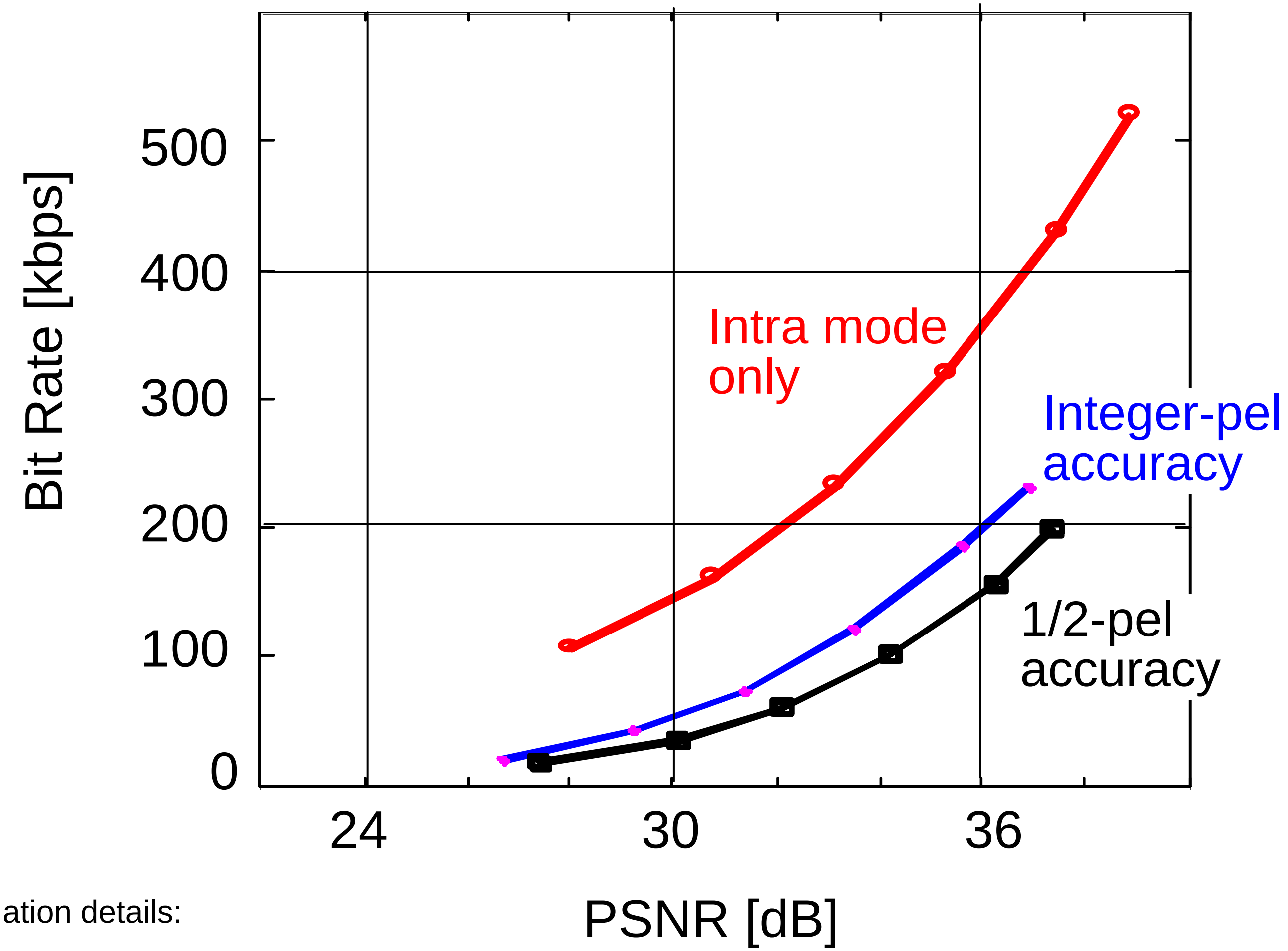
Voiture



Influence of noise on the performance of MCP



Motion Compensation Performance in H.263



Simulation details:

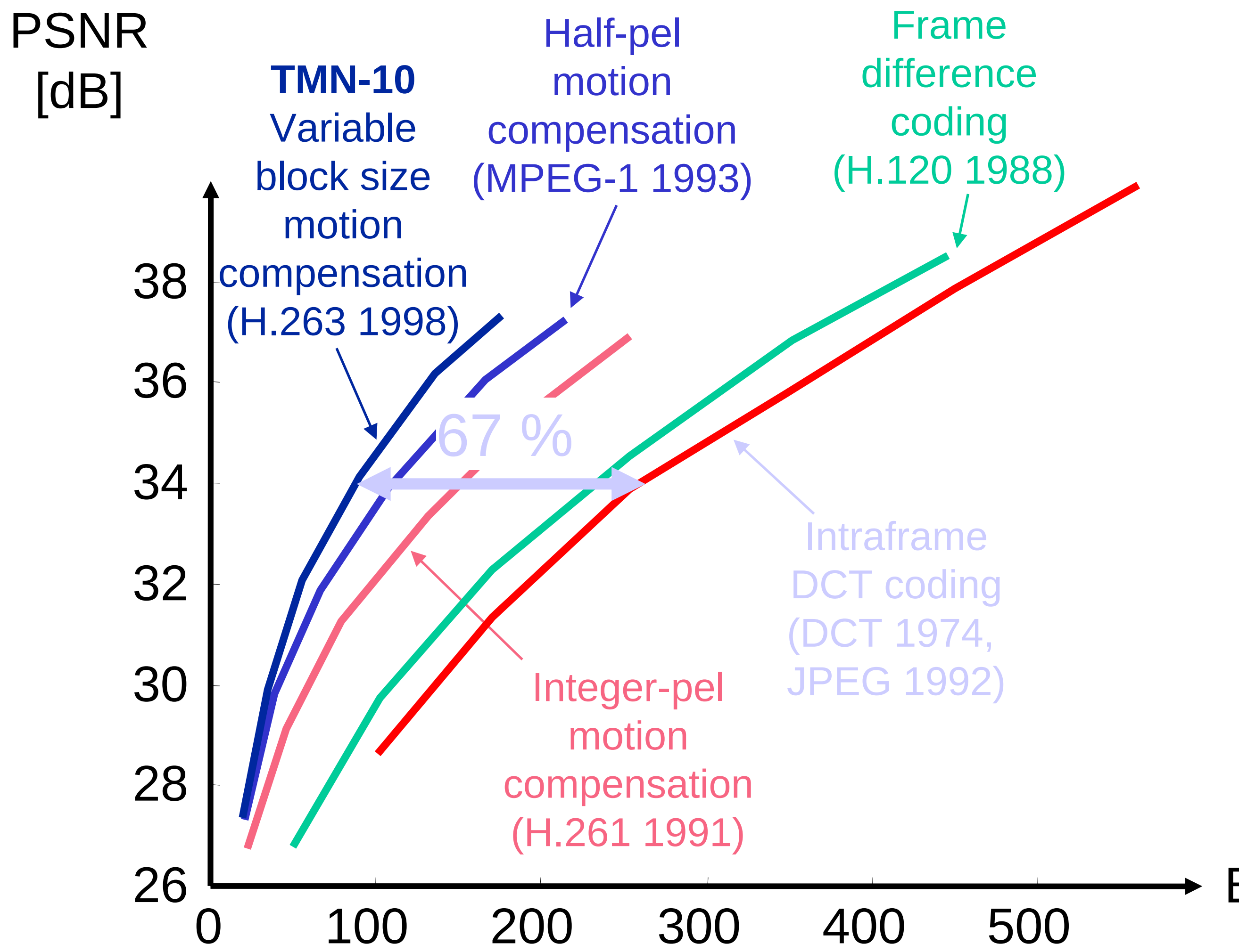
Foreman, QCIF, SKIP=2
Q=4,5,7,10,15,25

History of motion-compensated coding

Complexity increases ↓

- *Intraframe coding:* only spatial correlation exploited
  - ➔ DCT [Ahmed, Natarajan, Rao 1974], JPEG [1992]
- *Conditional replenishment*
  - ➔ H.120 [1984] (*DPCM, scalar quantization*)
- *Frame difference coding*
  - ➔ H.120 Version 2 [1988]
- *Motion compensation: integer-pel accurate displacements*
  - ➔ H.261 [1991]
- *Half-pel accurate motion compensation*
  - ➔ MPEG-1 [1993], MPEG-2/H.262 [1994]
- *Variable block-size motion compensation*
  - ➔ H.263 [1996], MPEG-4 [1999]

Efficiency of motion-compensated coding



Foreman
10 Hz, QCIF
100 frames encoded

Reading

- B. Girod, “Motion-compensating prediction with fractional-pel accuracy,” IEEE Trans. Communications, vol. 41, no. 4, pp. 604-612, April 1993.
- G. J. Sullivan and T. Wiegand, “Rate-Distortion Optimization for Video Compression,” IEEE Signal Processing Magazine, vol. 15, no. 6, pp. 74-90, Nov. 1998.