Stanford University

Department of Electrical Engineering

EE384Y, Spring 2006 — Tuesday, May 2, 2006

Midterm

The midterm is closed-book, and you are allowed one reference sheet of your own design.

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1. Maximum and Maximal Size Matchings

Consider the request graph shown in the figure below. An edge between input i and output j indicates that input i has a packet for output j. Matchings between inputs and outputs can only take place along the dashed edges and no input (output) is connected to more than one output (input).

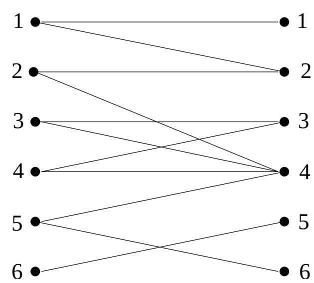

Figure 1: A request graph for a 6×6 switch

(a) Produce a maximum size matching for this graph.

(b) Recall that a maximal match is one where each input is either a part of a match, or all the outputs that it can possibly connect to are already matched to other inputs. Produce a maximal match that is *not* a maximum match.

(c) Notice that the maximal match you produced has at least half the number of edges of the maximum match. Can you produce a maximal match where the number of edges is strictly less than half the number of edges in a maximum size match? If not, prove the following statement: Every maximal match is at least half the size of a maximum size match.

2. The Random Switch Schedule.

Consider a 2×2 IQ switch with VOQs at which packets arrive in a Bernoulli IID fashion according to the rate matrix

$$\Lambda = \begin{bmatrix} 0.1 & 0.6 \\ 0.7 & 0.2 \end{bmatrix}.$$

A scheduler uses the Random matching algorithm to transfer packets: In each time slot, the scheduler chooses the matching M_1 with probability α and the matching M_2 with probability $1 - \alpha$, independently from time slot to time slot, where

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(a) What range of values for α ensure the stability of the switch?

(b) What value of α in this range minimizes $\sum_{ij} E(Q_{ij})$, where Q_{ij} is the equilibrium occupancy of VOQ$_{ij}$? (If your answer involves an equation in terms of α you are not expected to solve it.)

3. Many Queues with a Single Server.

Consider a system of N queues, each with a finite capacity of k, served by 1 common server. Time is slotted. In each time slot at most s packets can arrive. Each arriving packet can join any of the N queues (i.e. a queue can have multiple arrivals). The server can serve only one queue per time slot. When a queue is served, all of the packets in it are drained completely; that is, the queue becomes empty at the end of service.

The server has to decide which queue to serve among all possible N queues. This decision policy affects the queue sizes.

(a) Suppose $N = 2$, $k = 3$ and $s = 2$. What service policy ensures that the queues never overflow? State the policy and show that it never results in an overflow. (Note: Even though the number of arriving packets/time slot is limited to 2, they can each join any queue: both to the same queue or to different queues. You need to prove that your service policy never causes overflows.)

(b) Now let $N = 2$, $k = 9$ and $s = 5$. Repeat part (a).

(c) Let $N = 4$, $k = 4$ and $s = 2$. Repeat part (a).