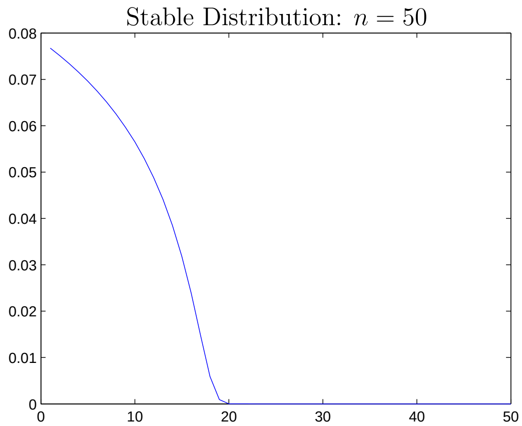
EE376B
Information Theory
Prof. T. Cover

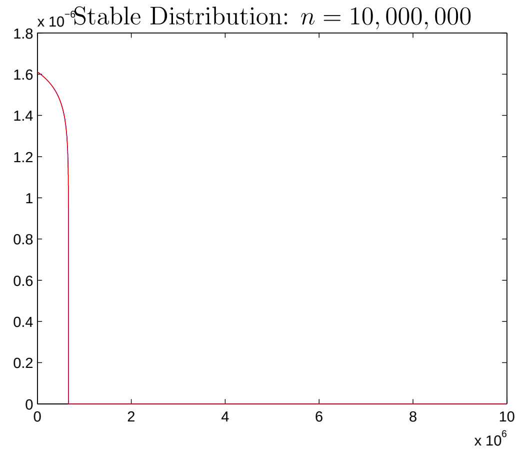
Handout #15
Thursday, May 14, 2009

Game: Pick the smallest unique positive integer

In the course of studying the multiple access channel (interference), we played a game in the class where everyone chose a positive integer, and the winner would be the one who chose the smallest unique integer. Many people chose the number 1. The winner chose the number 4. However, no one chose the number 2.

One might wonder what strategy is best for playing this game. Here we identify a stable strategy. If everyone played according to this strategy then no one could improve by deviating from it.





It appears that the support of the distribution scales roughly as $n/\ln(n)$.

By symmetry, every player will have a $1/n$ chance of winning under any symmetric stable distribution. It's possible that a heterogeneous collection of strategies might also be stable, but for simplicity we keep everything symmetric. Assuming that $n-1$ players are playing independently according to $p(x)$, we can calculate the probability of the nth player winning for each integer choice. This probability should be $1/n$ for all active pure strategies, so we find the $p(x)$ that induces this result.

The calculation of $p(x)$ is simplified if we make one approximation. It is easy to calculate the probability that exactly one person (or zero) chooses any particular integer. The simplification is that we assume these are independent from one integer to the next. This is close to accurate, but becomes worse and worse when we calculate joint probabilities for more and more integer bins together. So this may affect the calculation of $p(x)$ for large x.

With this simplification, the calculation of $p(x)$ becomes a simple induction, starting with $x = 1$:

$$\prod_{i=1}^{x-1} \left(1-(n-1)p(i)(1-p(i))^{n-2}\right)(1-p(x))^{n-1} \quad = \quad 1/n.$$