

Homework Set #8

1. **Investment growth rate.**

Let

$$X = \begin{cases} (1, a), & \text{with probability } 1/2 \\ (1, 1/a), & \text{with probability } 1/2 \end{cases},$$

where $a > 1$. This vector \mathbf{X} represents a stock market vector of cash vs. a hot stock.

Let

$$W(\mathbf{b}, F) = E \log \mathbf{b}^t \mathbf{X}$$

and

$$W^* = \max_{\mathbf{b}} W(\mathbf{b}, F)$$

be the growth rate.

- (a) Find the log optimal portfolio \mathbf{b}^* .
- (b) Find the growth rate W^* .
- (c) Find the asymptotic behavior of

$$S_n = \prod_{i=1}^n \mathbf{b}^t \mathbf{X}_i$$

for all \mathbf{b} .