Sparse MRI

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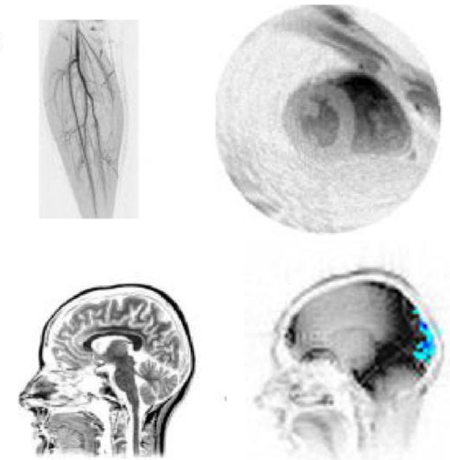
"Randomness is too important to be left to chance* "

*R. Conveyo, Oak Ridge National Laboratory

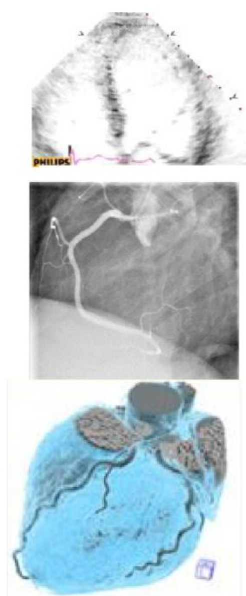
MR Imaging

- No radiation non toxic
- Flexible contrast
- Arbitrary imaging plane
- Many applications



Cons…

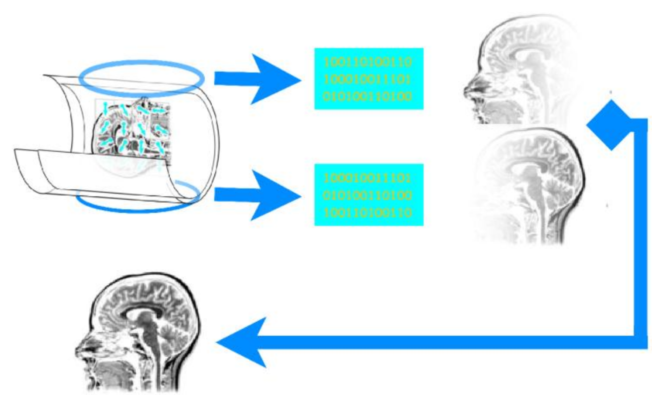
- Inherent slow data collection
  - Limits spatial resolution
  - Limits temporal resolution
  - Artifact in the image
- Possible solution:
  Faster imaging by reducing data
  (by exploiting redundancies)



[1] cardiovascularultrasound.com
[2] siemensehealthcare.com

Redundancy I: Phased Array

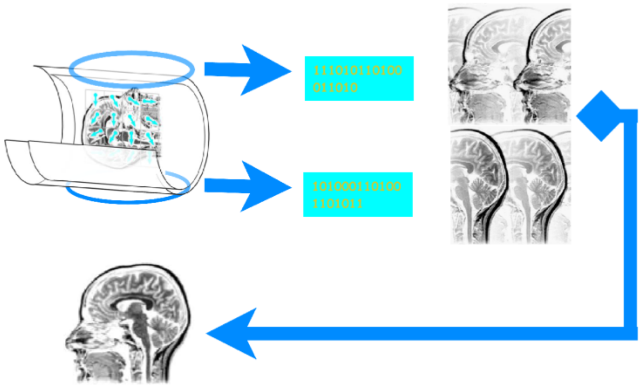
Multiple receive channels
redundant data



Parallel Imaging

Multiple receive channels

reduced data - Parallel Imaging



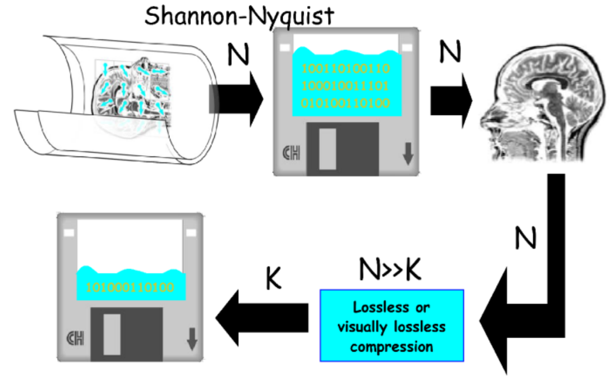
Sodickson et. al., 1997

Pruessmann et. al., 1999

Redundancy II: Compression

Most images are compressible

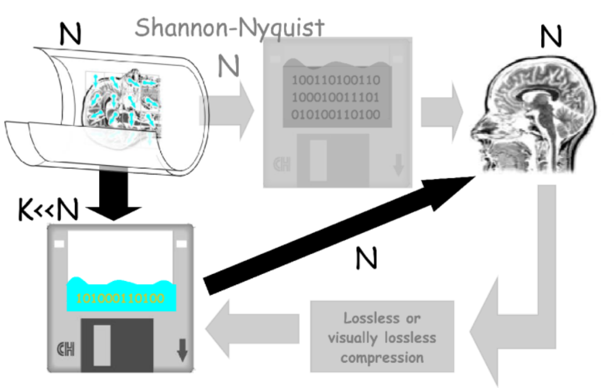
Standard approach: First collect, then compress



Compressed Sensing

Instead: Compressed Sensing (CS)

First Compress, then reconstruct.



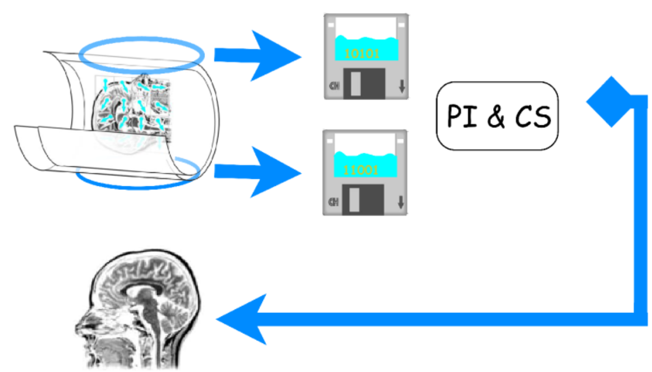
Candes et al. IEEE TIF '06

Donoho IEEE TIF '06

Parallel Imaging + Compressed Sensing

Synergy: multiple receivers + compressibility

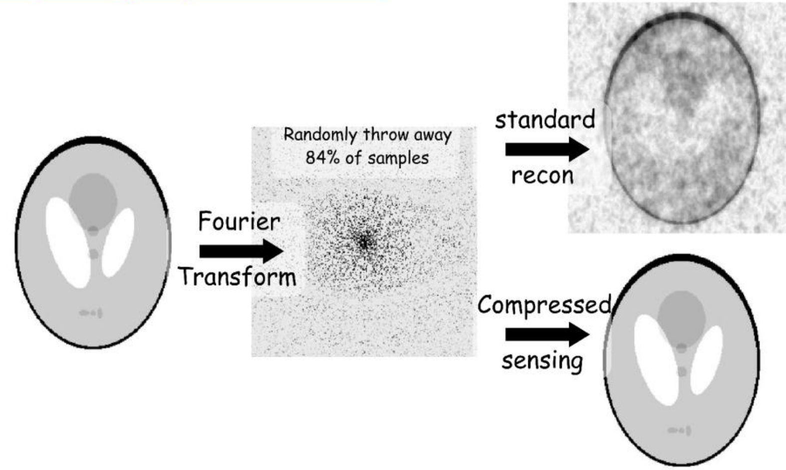
Faster imaging, or better images.



Outline

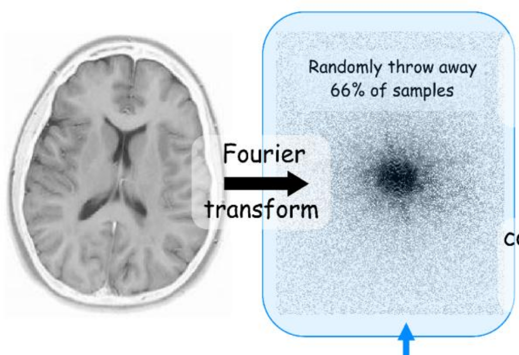
- Compressed review of
  - compressed sensing
  - parallel imaging
- parallel imaging + CS

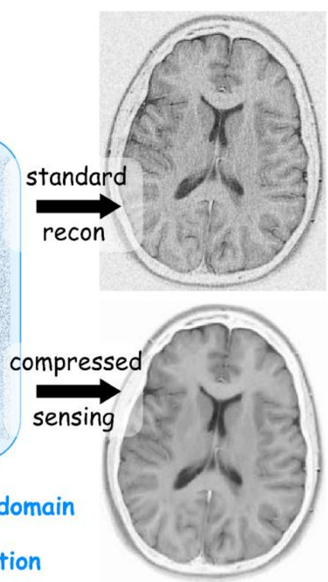
A Surprising Experiment



Candes, Romberg and Tao; 2004

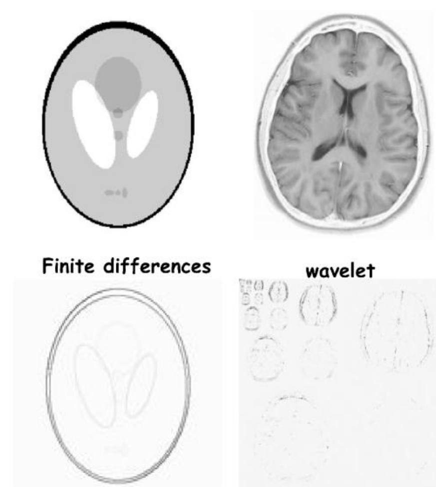
Implications…





- MRI data are obtained in the **frequency domain**
- Potential for significant **scan time reduction**

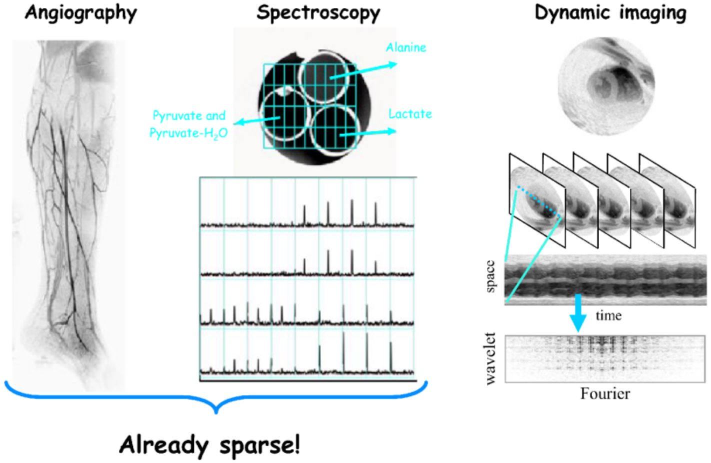
Sparsity



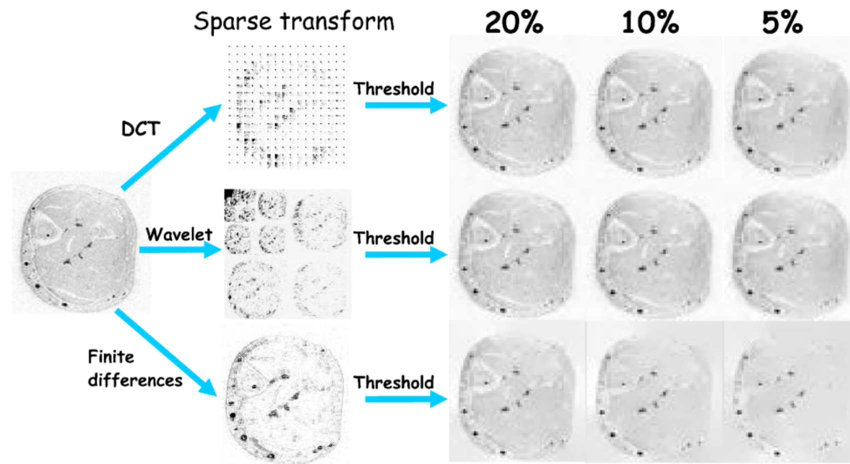
N pixels

K << N
coefficients

Sparsity is everywhere

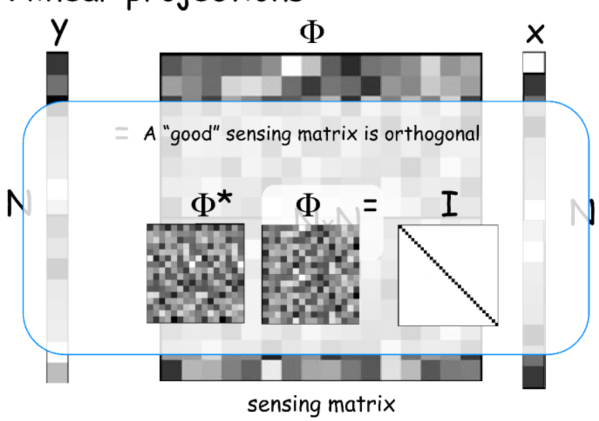


Compressibility



Traditional Sensing

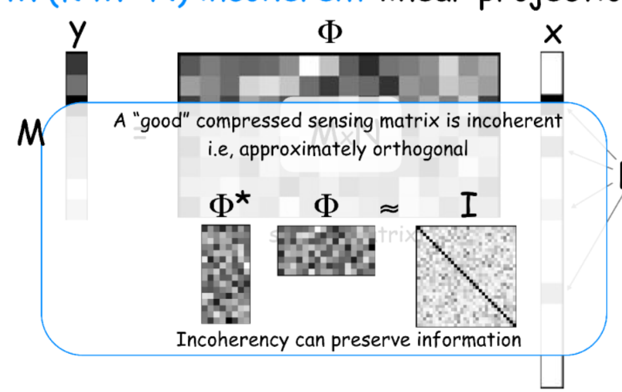
- $x \in \Re^N$ is a signal
- Make N linear projections



Compressed Sensing

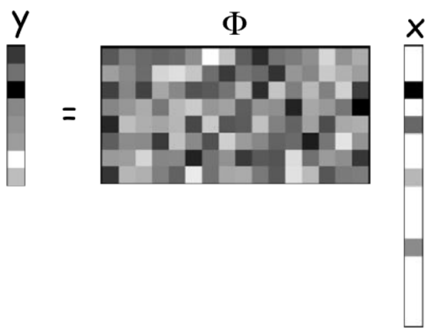
(Candes, Romber, Tao 2006; Donoho 2006)

- $x \in \Re^N$ is a K-sparse signal (K<<N)
- Make M (K<M<<N) incoherent linear projections



CS recovery

- Given $y = \Phi x$, find x — Under-determined
- But there's hope, x is sparse!



CS recovery

- Given $y = \Phi x$, find x — Under-determined
- But there's hope, x is sparse!

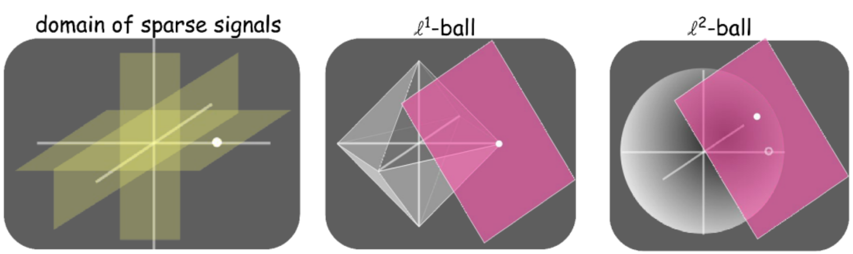
minimize $||x||_1$

s.t. $y = \Phi x$

need $M \approx K \log(N) \ll N$

Solved by linear-programming

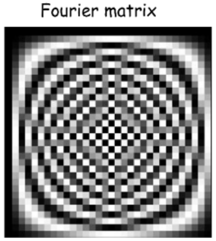
Geometric Interpretation



Practicality of CS

- *Can such sensing system exist in practice?*

Fourier matrix



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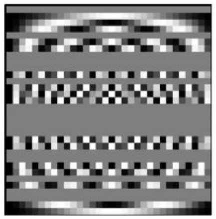

Practicality of CS

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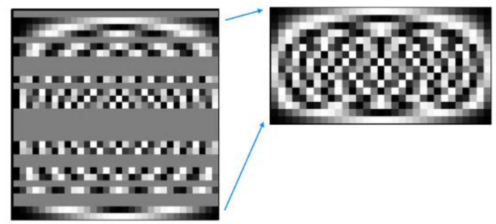
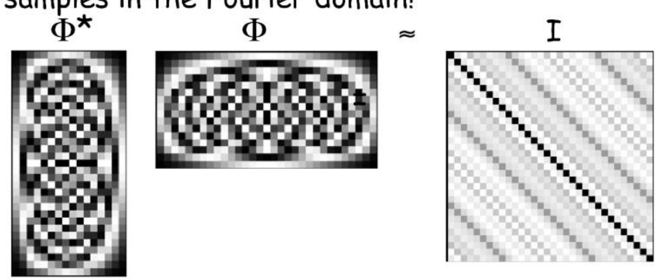
Fourier matrix



Practicality of CS

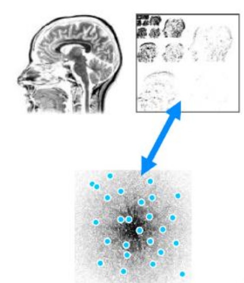
- Can such sensing system exist in practice?
- Randomly undersampled Fourier is incoherent
- MRI samples in the Fourier domain!



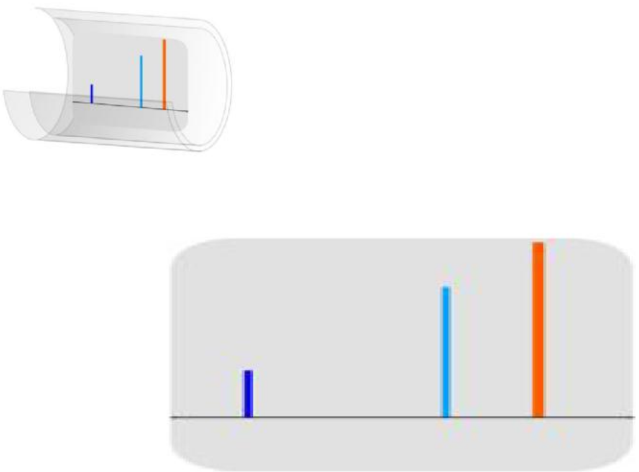
Compressed Sensing

Ingredients:

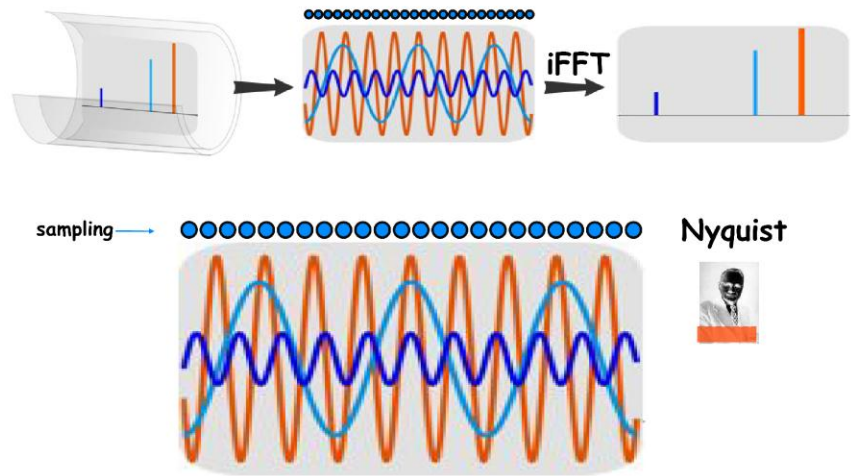
- **Compressible** signals. (K<<N significant coefficients)
- **Incoherent measurements.**
  i.e., incoherent aliasing in the transform domain (randomly under-sampled k-space).
- Recovery by solving a **non-linear** convex optimization.



Intuitive example of CS



Intuitive example of CS



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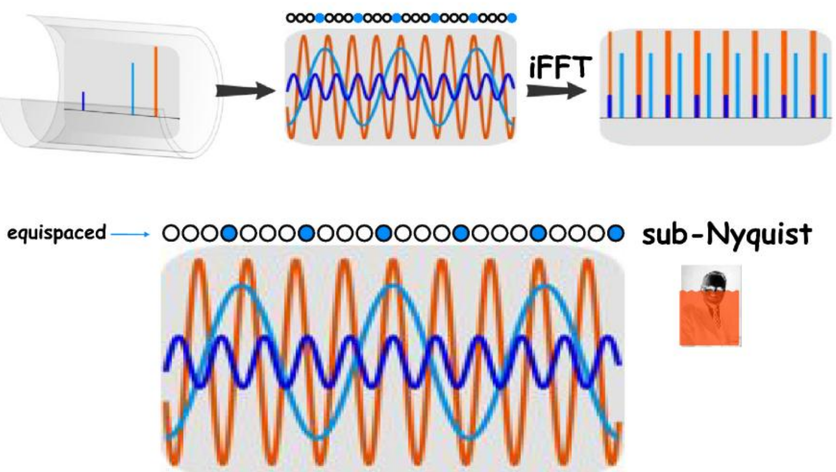
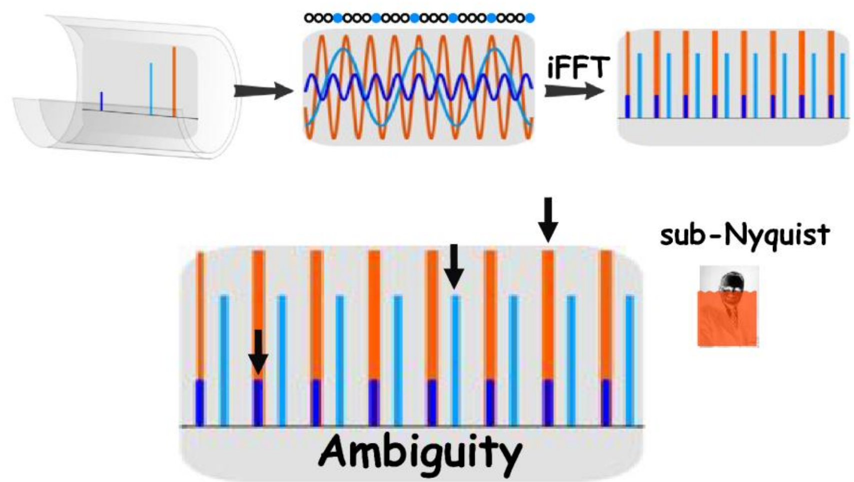
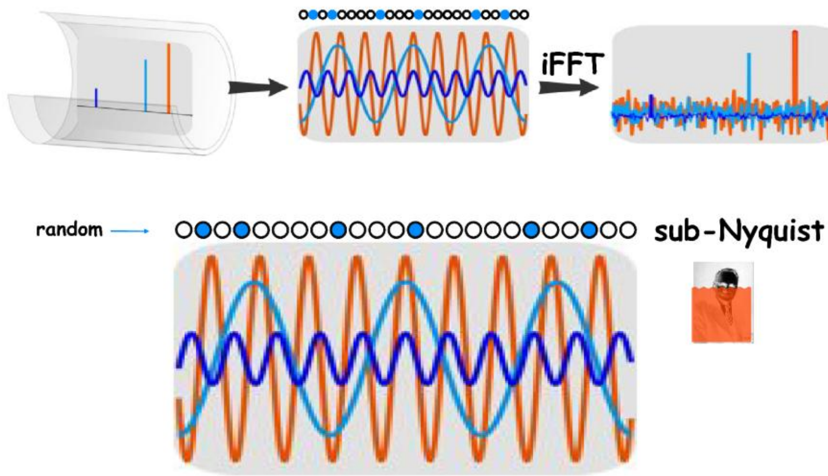


Intuitive example of CS



Intuitive example of CS



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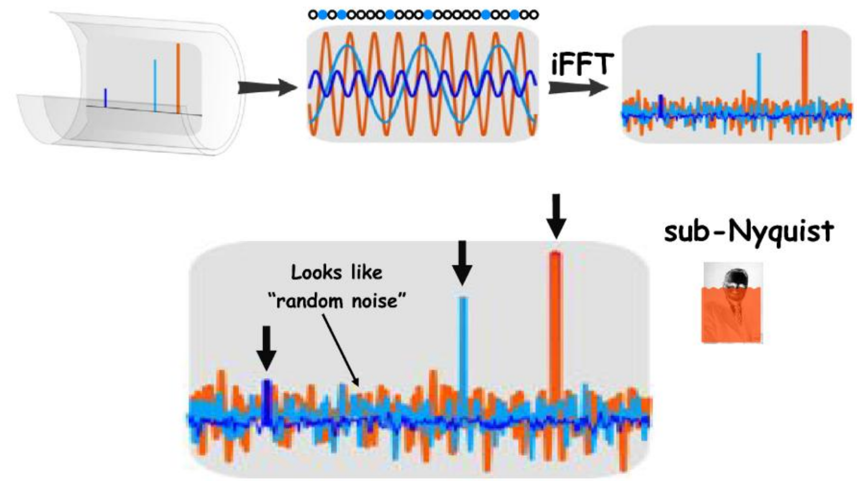
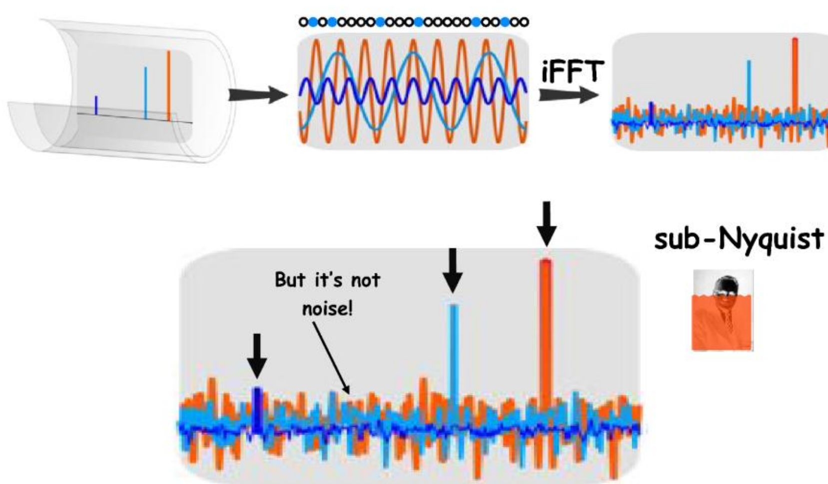
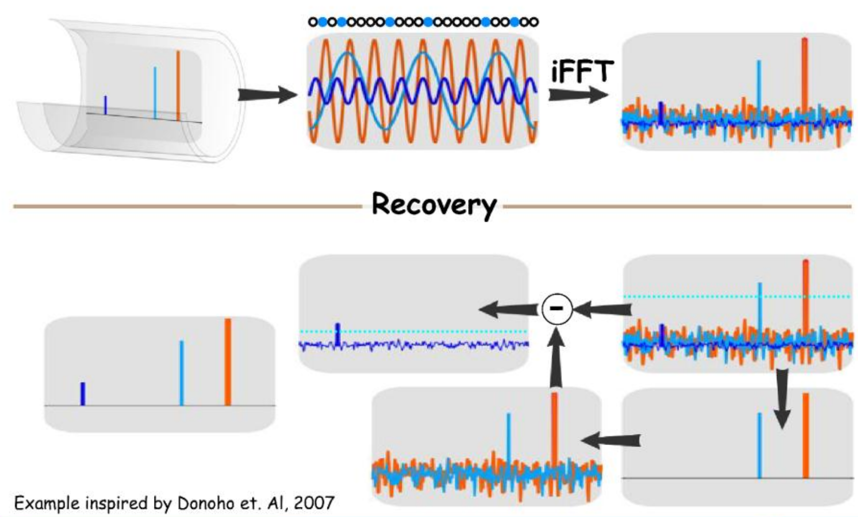


Intuitive example of CS

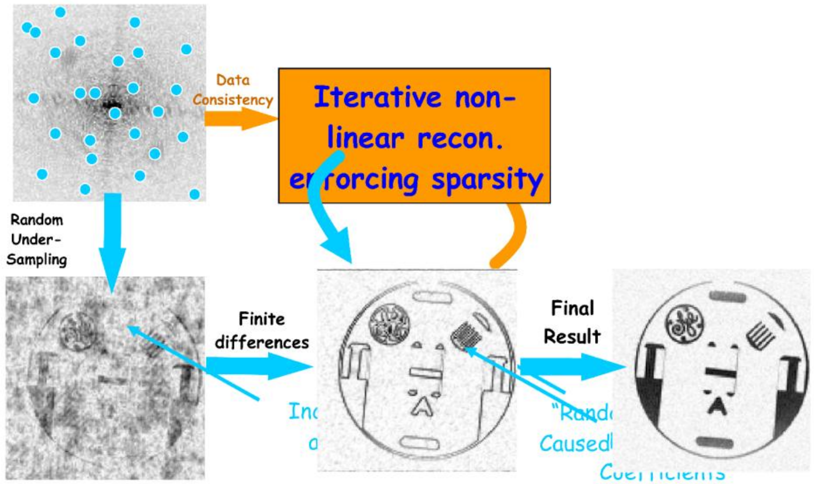


Intuitive example of CS

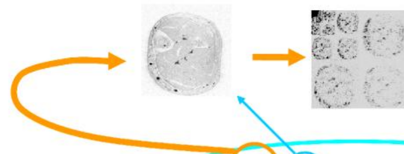


Example inspired by Donoho et. Al, 2007

Sparse Reconstruction



Sparse Reconstruction

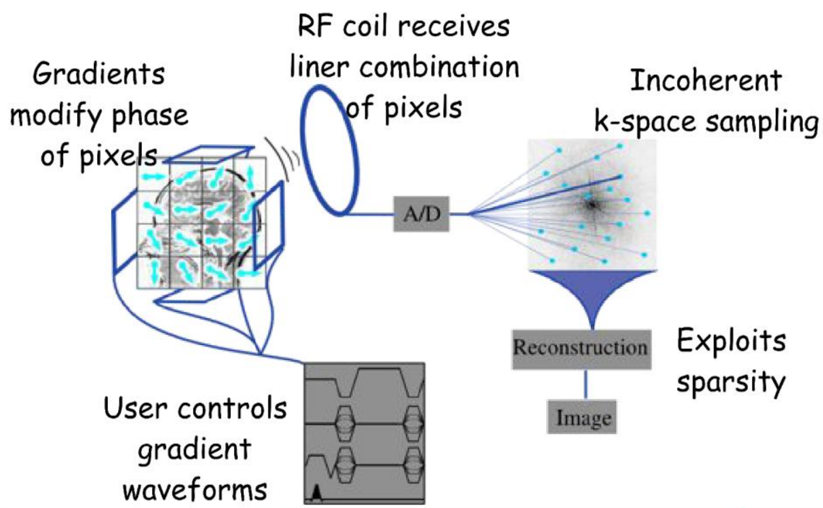


- Solve: min. $||\Psi m||_1$
  s.t. $||F_u m - y||_2 \leq \varepsilon$

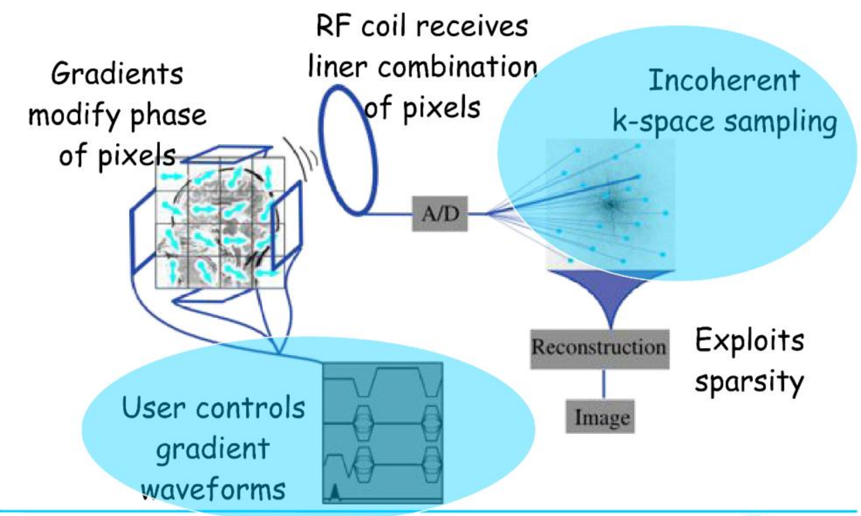
Enforces Sparsity

Enforces Data Consistency

MRI - a natural CS hardware



MRI - a natural CS hardware



Incoherent Sampling

"Randomness is too important to be left to chance"*

- Metric of incoherency
  - Point Spread Function (PSF)
  - Transform Point Spread Function (TPSF)

- Practical incoherent sampling schemes.

*Robert R. Coveyou, Oak Ridge National Laboratory

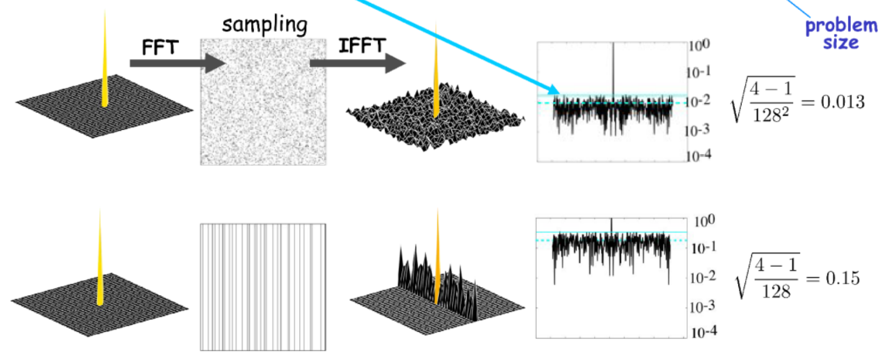
Point Spread Function (PSF)

- Natural measure of incoherence
- Good analytic lower-bound estimate
- Criteria: peak side-lobe

undersampling

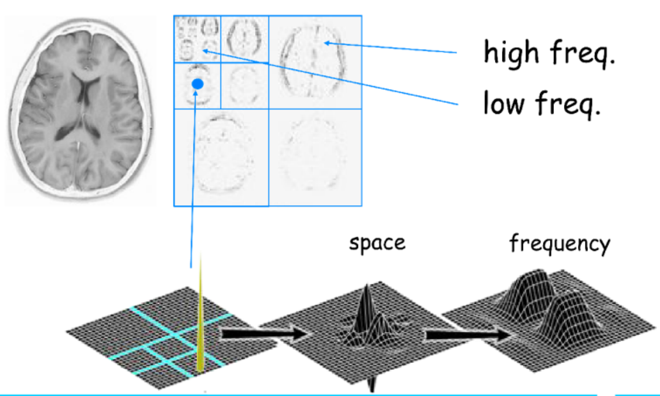
$$\sigma = \sqrt{\frac{p-1}{D}}$$

problem size



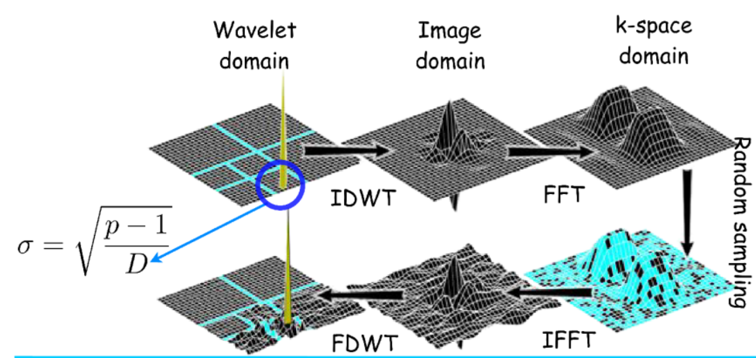
The wavelet transform

- Wavelets are band pass filters
- Wavelet coefficients have both spatial and spectral information



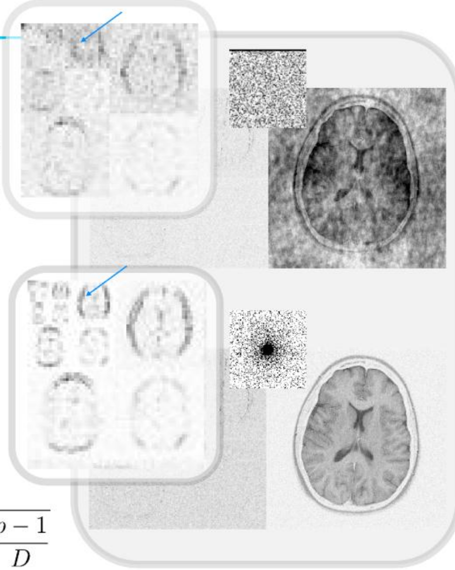
Transform Point Spread Function (TPSF)

- Transform incoherency?
- Transform Spread Function (TPSF)
  - Similar analytic indicator
  - Look at peak side-lobe



Variable density sampling

- k-space is **not** uniform
- Coarse-scale - not sparse
- Coherent low-res aliasing



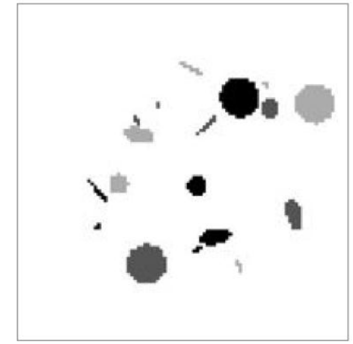
- Correct with variable density
  - Equalizes aliasing
  - Improve incoherence
  - Faster convergence

$$\sigma = \sqrt{\frac{p-1}{D}}$$

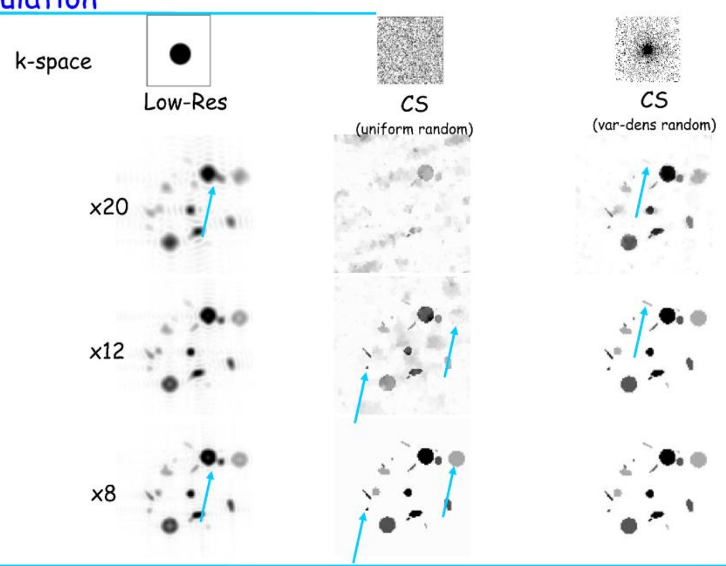
Simulation

- 3 intensities
- 3 feature sizes
- Size: 100x100
- 5.75% pixels
- 4.25% finite-differences

Target: recon. artifacts with random under-sampling.



Simulation

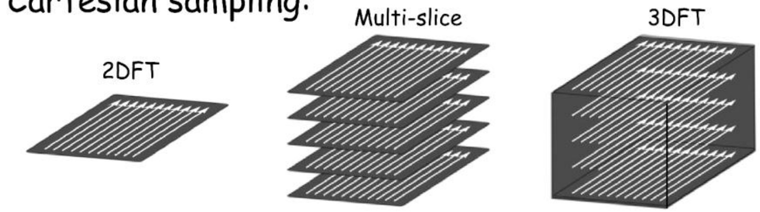


Practical Incoherent Sampling Schemes

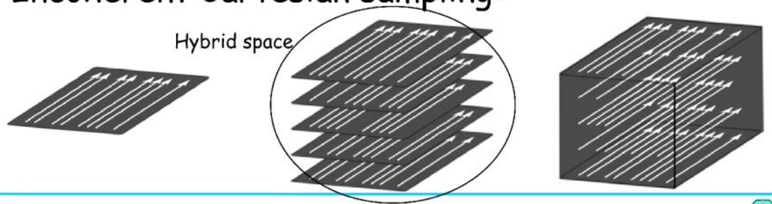
- “Pure random” sampling is impractical in MRI.
- Instead, design “effectively random” sampling.
  - Incoherent PSF/TPSF.
  - Efficient for hardware and application
  - Robust
- Tailor trajectory for application (Cartesian, spiral…)
- Randomly perturb to be “effectively random”.

Cartesian incoherent sampling

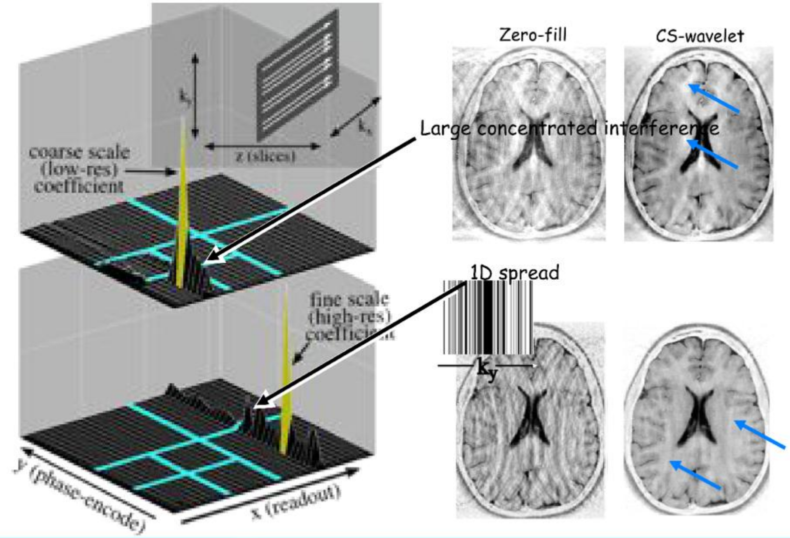
Cartesian sampling:



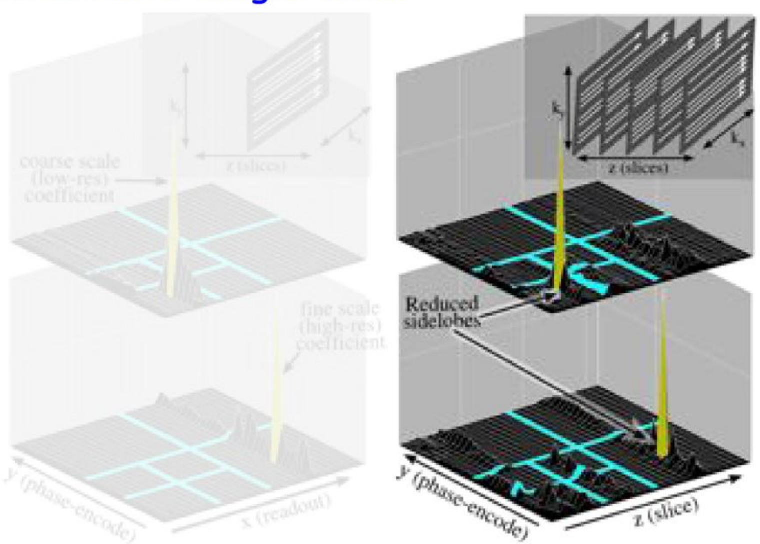
Incoherent Cartesian sampling:



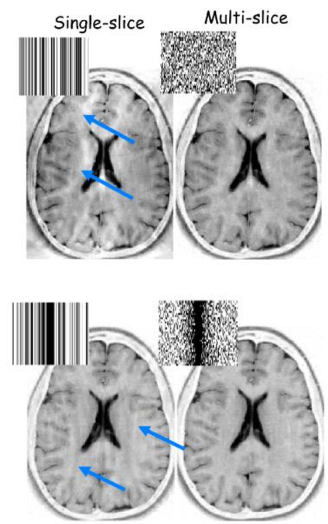
Single-slice 2DFT

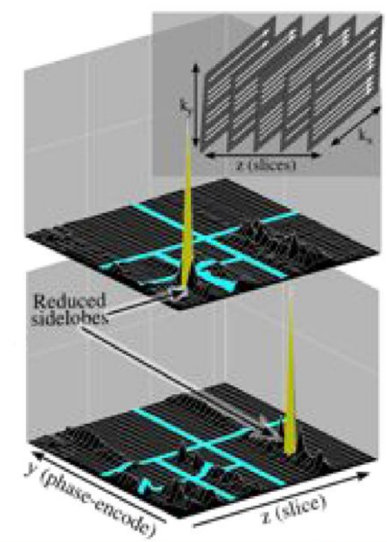


Multi-slice vs Single-slice



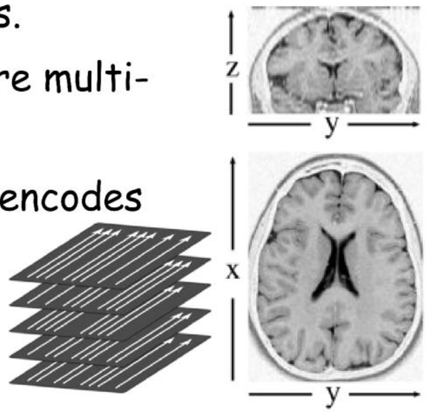
Multi-slice vs Single-slice



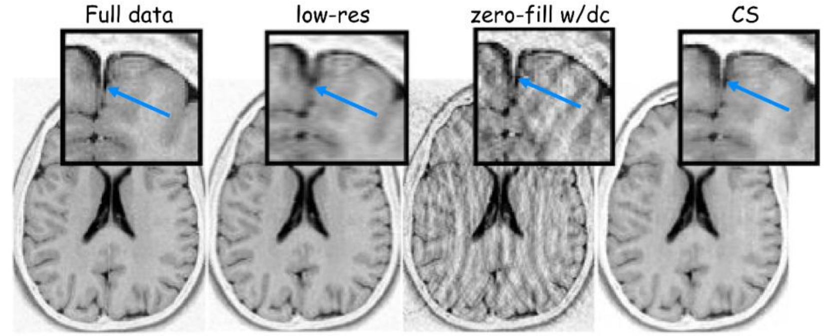


Multi-slice FSE brain

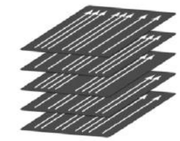
- Head scans are the most common MRI exams.
- Most brain scans are multi-slice.
- Use 80/192 phase-encodes x2.4



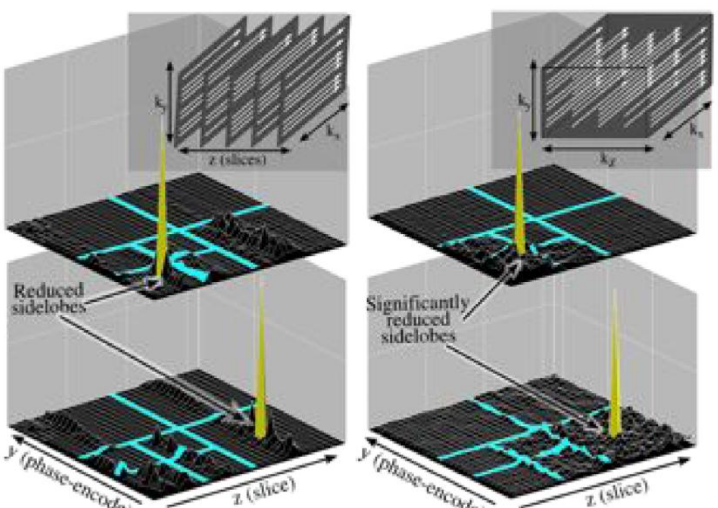
Multi-slice Brain Imaging



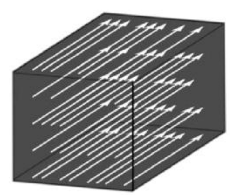
- Scan reduction: x2.4
- Transform: wavelet

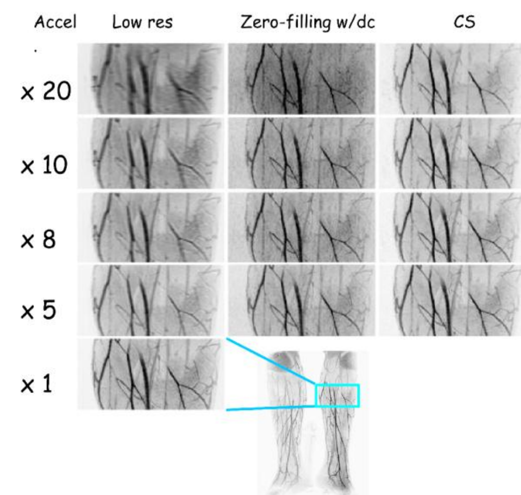


Multi-slice vs 3D

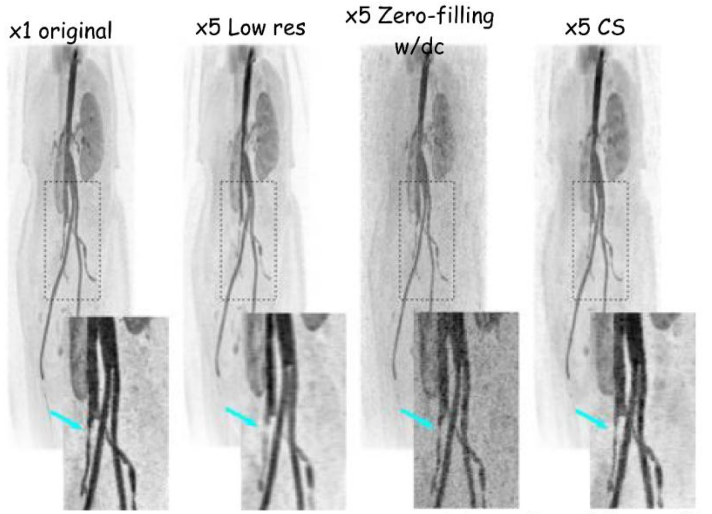


3DFT Angiography





3D Angiography - 1st Pass



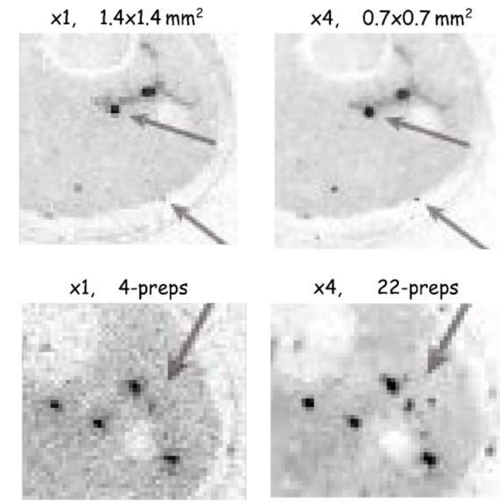
Data courtesy of Marcus Alley

Flow independent angiography

Cukur et al, ISMRM'08

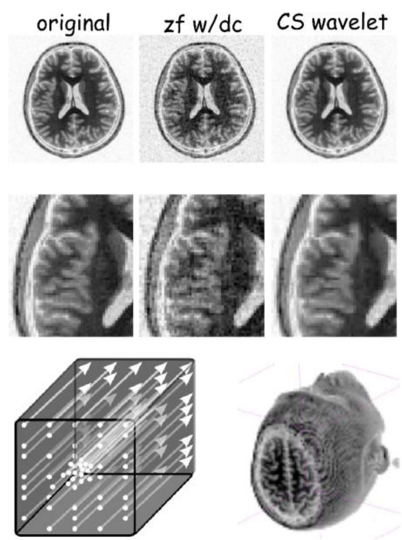
- Hi-res ↑ sparsity
- T_2 Prep pulses ↑ sparsity

Transform: finite-differences (TV)



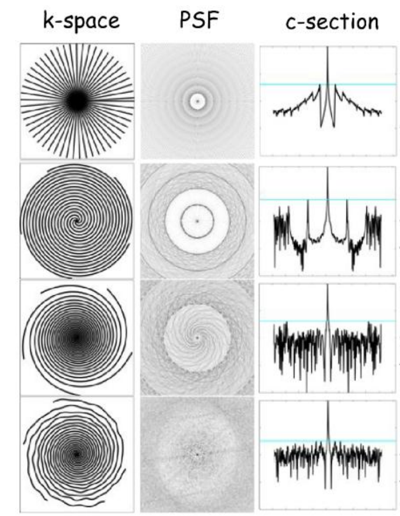
3DFT Brain

- Scan time reduction: 2.4
- Transform: wavelet



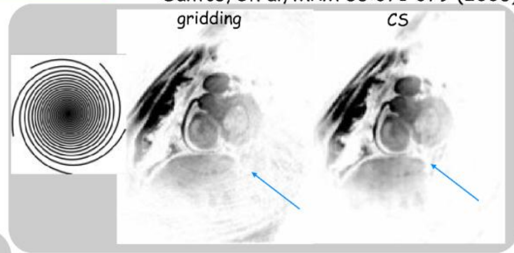
Non-cartesian sampling

- More degrees of freedom.
- Not as incoherent as random 2D sampling
  - But very close!

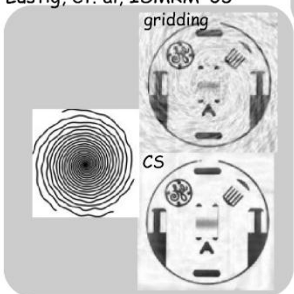


Non Cartesian CS

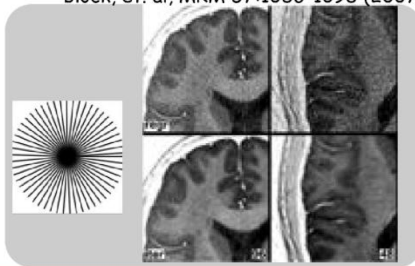
Santos, et. al, MRM 55:371-379 (2006)



Lustig, et. al, ISMRM '05

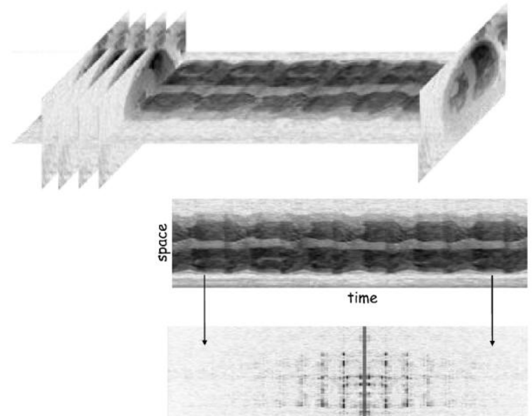


block, et. al, MRM 57:1086-1098 (2007)



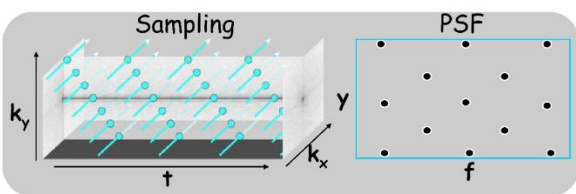
k-t SPARSE: Dynamic Imaging

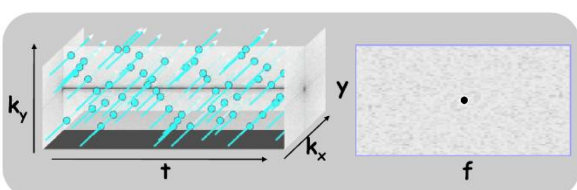
- Smooth& periodic signals have a sparse representation.



Dynamic Incoherent Sampling

- Random line ordering randomly samples k-t space.
- PSF is incoherent

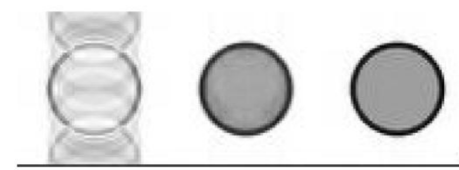




RT-dynamic cardiac

- Sparse in temporal frequency
- Aim for better temporal resolution

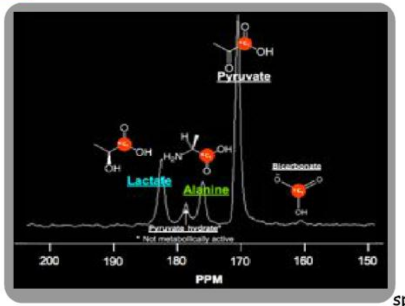
Sliding window k-t SPARSE True object

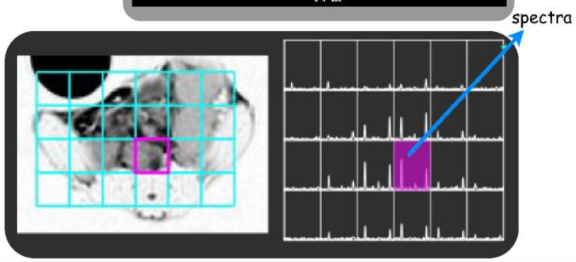


Sliding window k-t SPARSE

Spectroscopic Imaging

- Different metabolites, different spectrum
- Want spatial localization of metabolic activity
- 4D signal
- Very sparse
- Often low-SNR





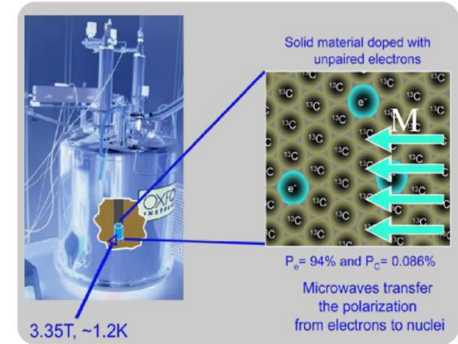
Hyperpolarization

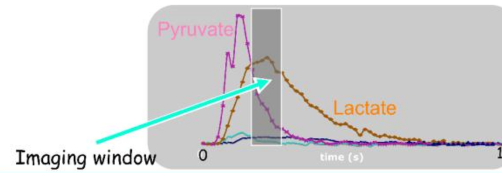
- Hyperpolarization ⇒

  >10,000 boost in signal
- Returns to equilibrium in ~1.5min
- Image metabolizm:

  Pyruvate ⇔ Alanin

  Pyruvate ⇔ Lactate
- Elevated lactate indicates cancer





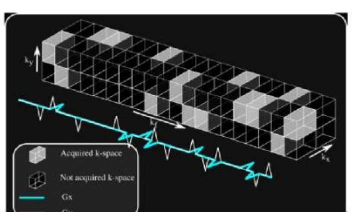
Hyperpolarized ^{13}C spectroscopy

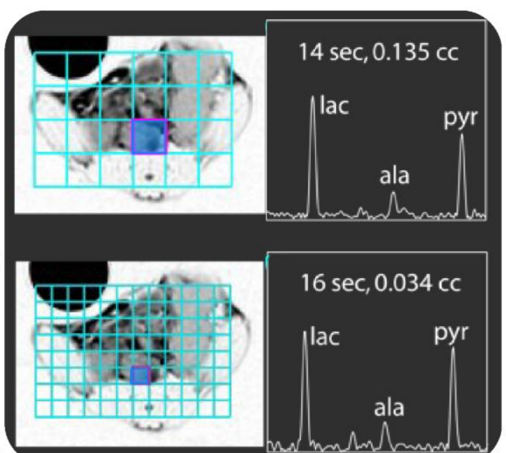
Combination of

- Abundant SNR
- Extreme sparsity
- 4D signal
- Strict encoding time

- Novel blipped EPSI
- Random 3D sampling

Hu et al, JMR 2008





Compressed Sensing:

1. Sparsity/compressibility
2. Incoherent Sampling (random k-space)
3. Non-Linear reconstruction.

Parallel Imaging

Parallel Imaging Methods

Sensitivity Encoding (SENSE)

- Inverse problem
- Explicit sensitivity maps
- Optimal noise performance
- Reconstructs 1 image
- Less robust in practice

Pruessmann et. al., 1999

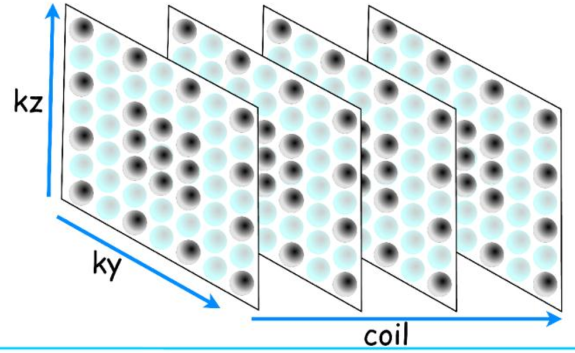
Autocalibrating (GRAPPA)

- Interpolation formulation
- Implicit sensitivity info.
- Not optimal
- Reconstructs individual coil images
- Robust in practice

Griswold et. al., 2002

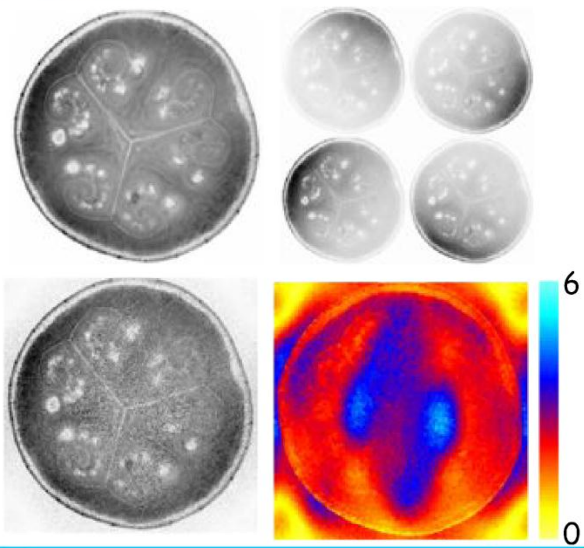
Parallel Imaging as Interpolation

- Generalized sampling theory
- k-space vs. coil sampling domain
- Involves noise amplification

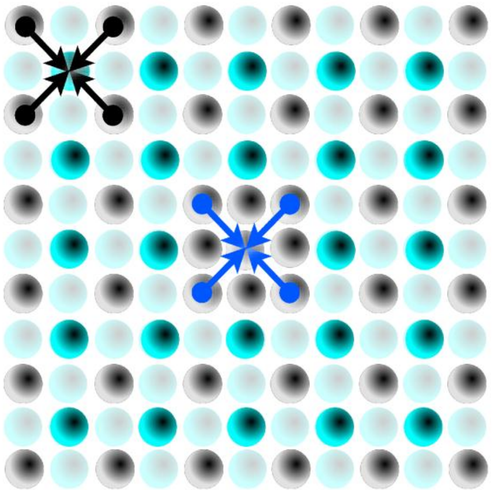


Noise Amplification - g factor

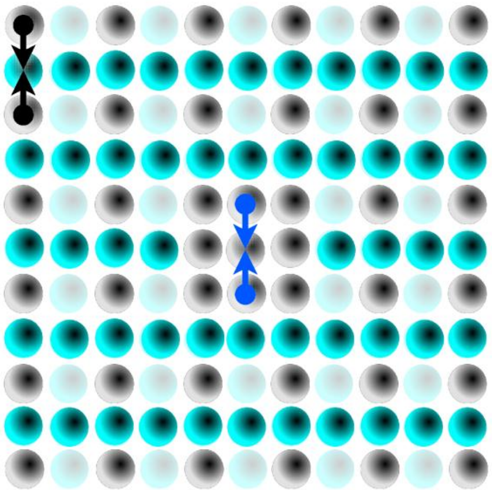
- Sensitivities not orthogonal
- Noise is amplified
- Worse when acceleration close to #coils



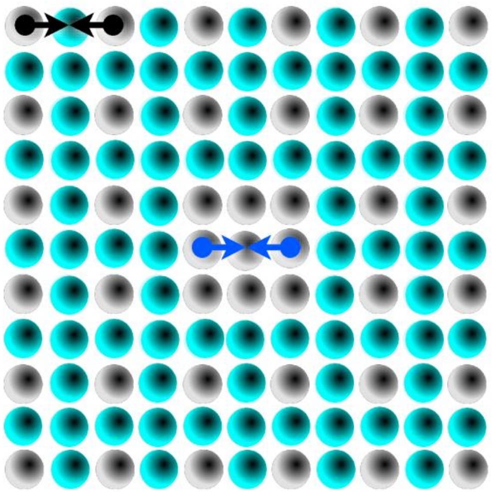
GRAPPA/ARC



GRAPPA/ARC



GRAPPA/ARC



Parallel Imaging

1. Multiple Channels
2. Acceleration limited by noise amplification
3. Rule of thumb: acceleration = 1/2 #coils

Parallel Imaging +

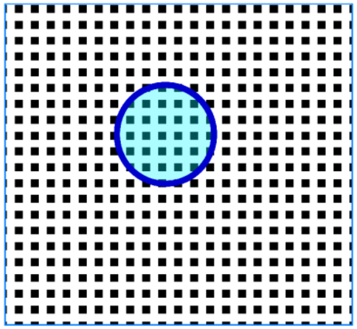
Compressed Sensing

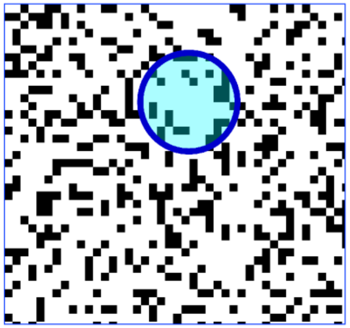
Tools

- New incoherent sampling
- New reconstruction
- Joint sparsity of multiple coil images

Sampling with parallel imaging

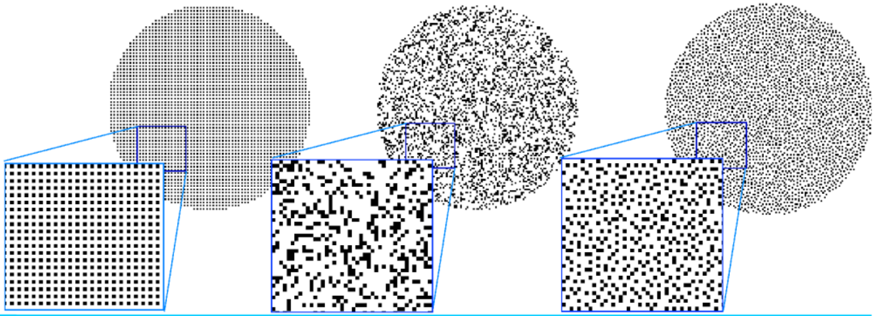
- Coil information is local in k-space
- Uniform sampling is not incoherent
- Random sampling has too many "holes"





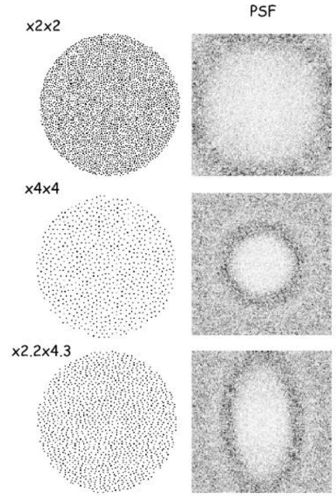
Incoherent Sampling

- Coil information is local in k-space
- Uniform sampling is not random
- Random sampling has too many "holes"
- Poisson-disk sampling is uniform and random



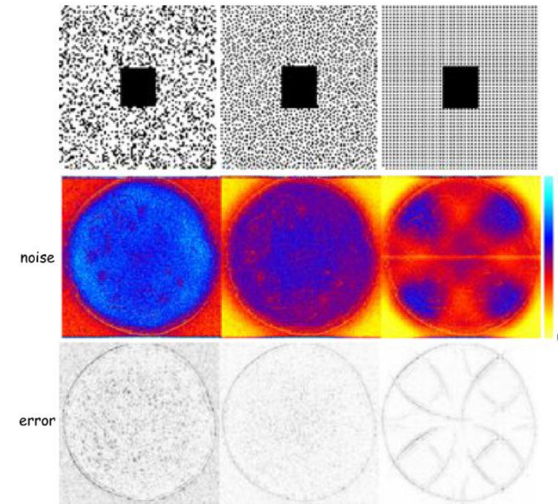
Poisson-disk Sampling

- Incoherent
- Fractional accelration
- Unisotropic acceleration
- Can reconstruct with traditional GRAPPA

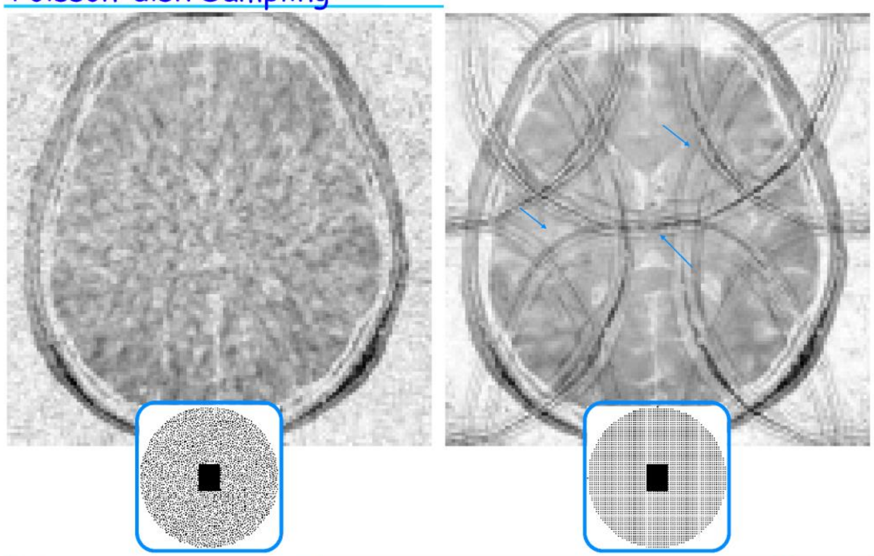


Poison Vs random Vs uniform

- 100 scans
- GRAPPA/ARC recon.

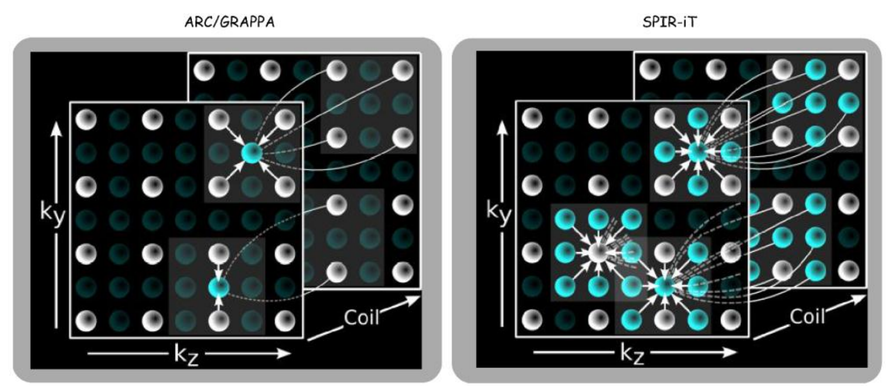


Poisson-disk Sampling



Reconstruction

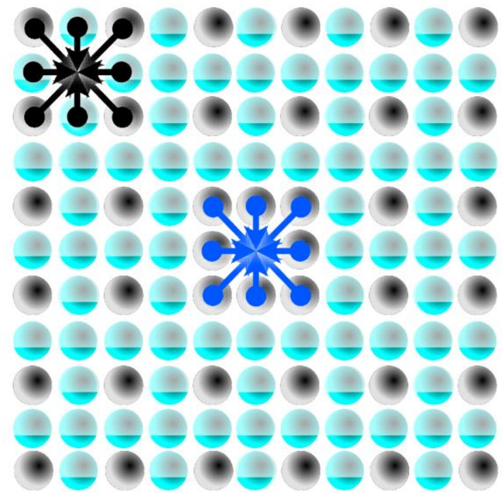
- SPIR-iT:
  iTerative Self-consistent Parallel Imaging Reconstruction



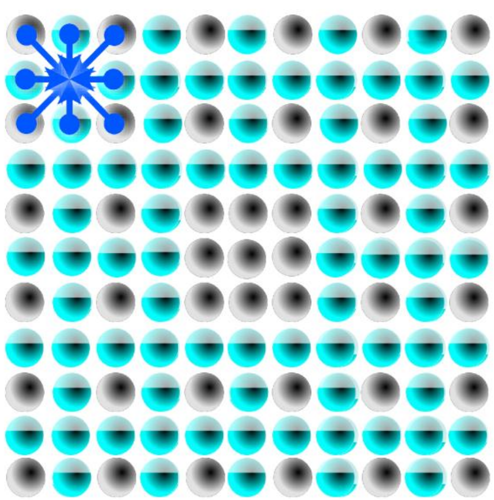
SPIR-iT

- Autocalibrating
- Only 1 calibration kernel
- Iterative
- Optimal data consistency
- Arbitrary trajectories
- Natural fit with CS

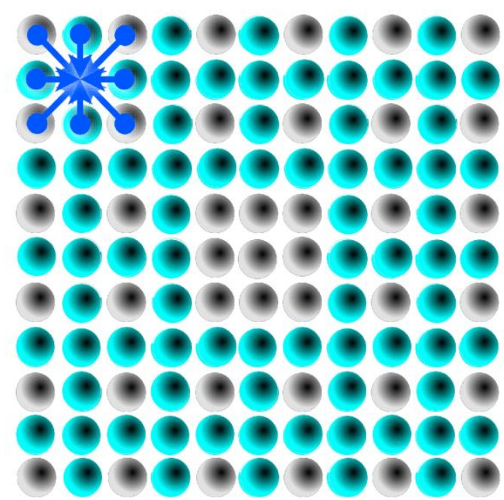
SPIR-iT: Iteration I



SPIR-iT: Iteration II



SPIR-iT: Iteration III



SPIR-iT equation

Calibration consistency

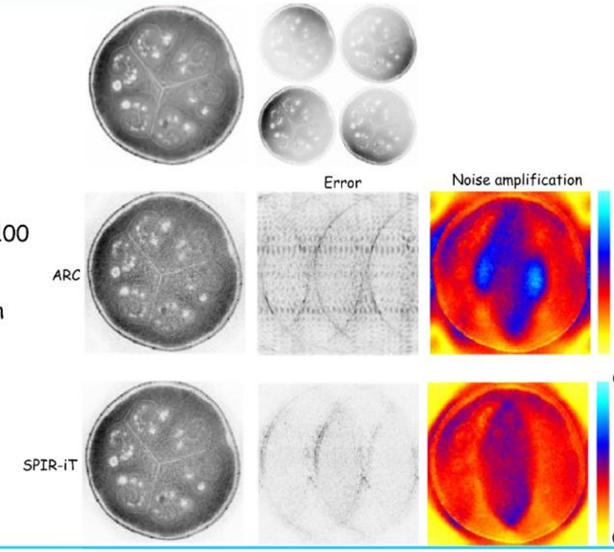
$$Gx = x$$

Acquisition consistency

$$x_{acq} = y$$

SPIR-iT vs ARC/GRAPPA

- statistics from a 100 scans
- x3 1D acceleration
- 4 coils



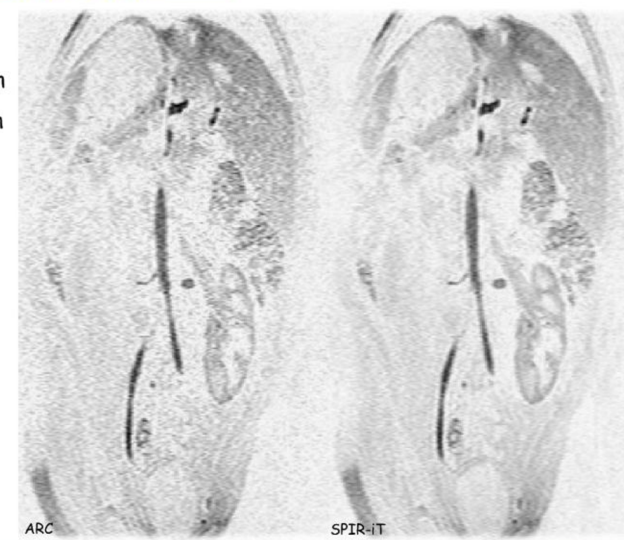
SPIR-iT with CS

$$\text{minimize } ||Gx-x||_2 + ||\Psi F^{-1}x||_1$$

$$\text{s.t.} \qquad x_{acq} = y$$

SPIR-iT with L_1 Wavelet

- 6 yo
- x4 acceleration
- noise reduction



SPIR-iT with Wavelet CS

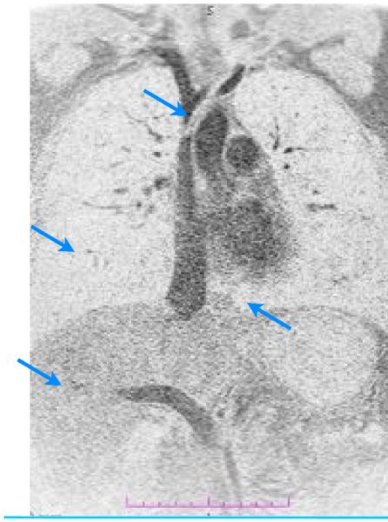
- 4 yo, free breathing, 11 Sec
- x2x2 poisson disc

ARC

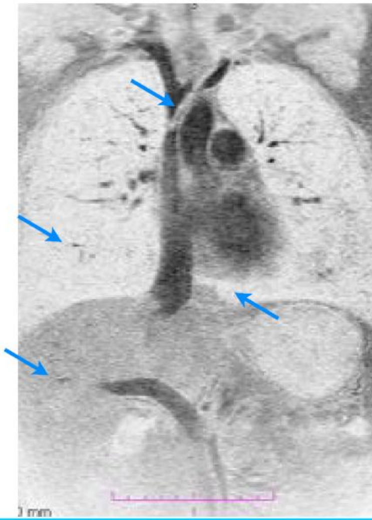
SPIR-iT

SPIR-iT with Wavelet CS

ARC

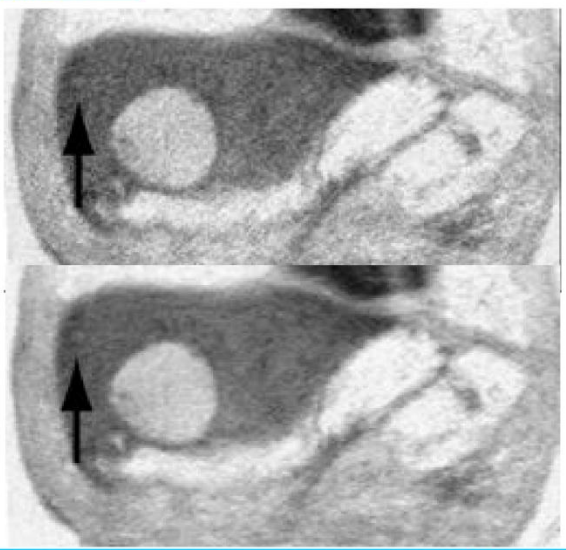


SPIR-iT



SPIR-iT with Wavelet CS

- x5 acceleration
- 8 coils
- denoised
- Subtle features preserved



Summary

- Both compressed sensing and parallel imaging offer high accelerations.
- Both have limitation.
- But, when joined.... synergy!

Collaborators

Stanford

- John Pauly (EE-MRSRL)
- David Donoho (Statistics)
- Juan Santos (EE-MRSRL)
- Tolga Cukur (EE-MRSRL)
- Seung-Jean Kim (EE-ISL)
- Marc Alley (Radiology)
- Shreyas Vasanawala (LPCH/ Radiology)

UCSF:

- Simon Hu (UCSF)
- Daniel Vigniron (UCSF)

GE

- Phil Beatty (ASL west)
- Anja Brau (ASL west)
- Kevin King (ASL)

MRSRL

Resources

- SparseMRI V0.2: matlab code, examples
  http://www.stanford.edu/~mlustig/SparseMRI.html
- Rice University CS page: papers, tutorials, codes, ...
  http://www.dsp.ece.rice.edu/cs/
- IEEE Signal Processing Magazine, special issue on compressive sampling 2008;25(2)
- Blog:
  http://nuit-blanche.blogspot.com/

Thank you!
תודה רבה

MRSRL