# Oct 23

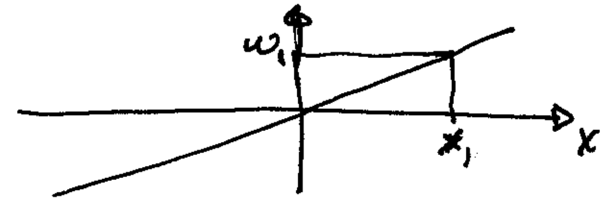
## Off-Resonance Correction

- Off-Resonance Effects
- Field maps
- Linear Term Correction
- Map Based Correction
- Auto Focus

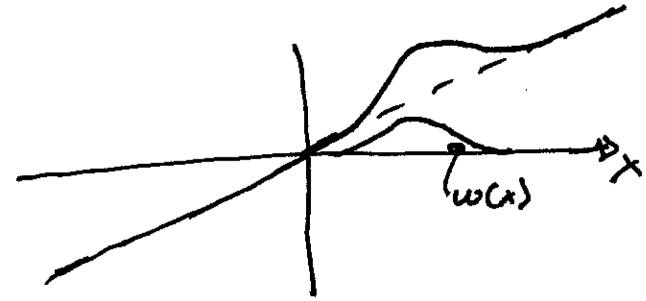


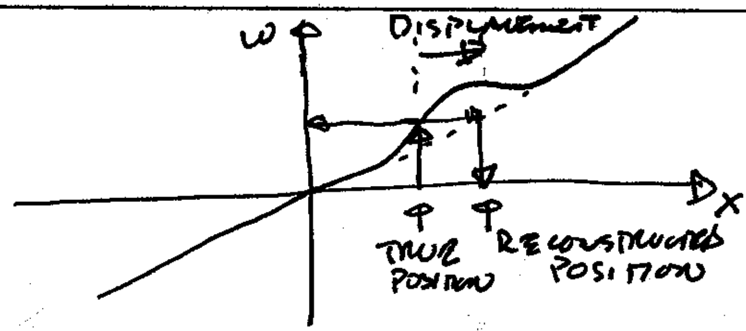
## Spin Warp (EPI)

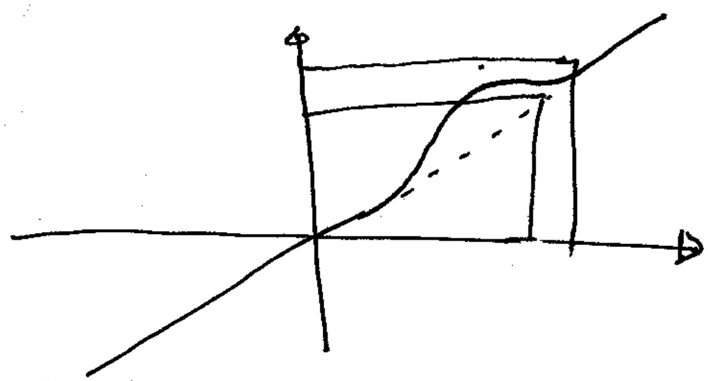
Frequency ⟺ Position

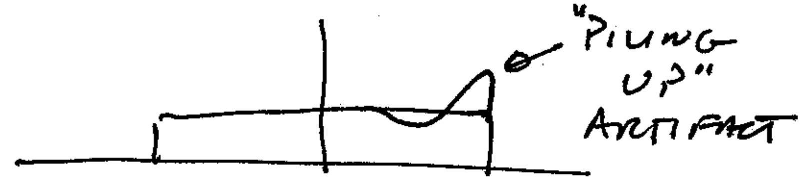



With off-resonance, not so clear





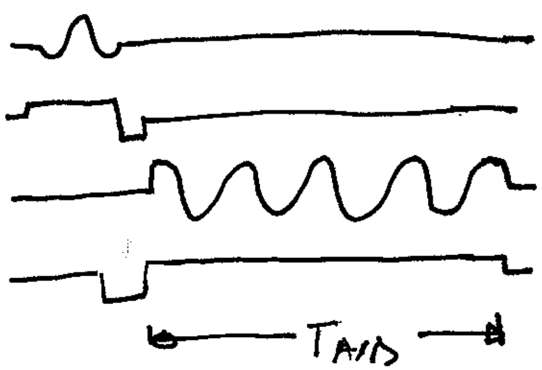

Displacement in pixels is

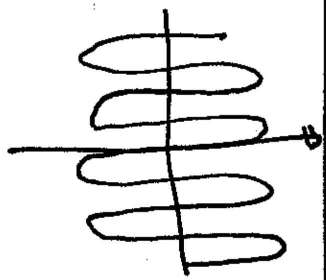
$$\Delta x = \#\text{cycles over readout}$$
$$= (\Delta f)(T_{AD})$$

Spin-warp, fat-water $\Delta f$ is 230 Hz

$$\Delta x = (230\,\text{Hz})(0.008192\,\text{s})$$
$$= 1.89 \text{ pixels}$$

## EPI

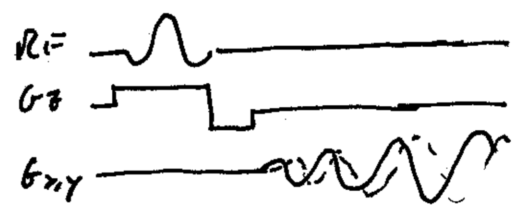


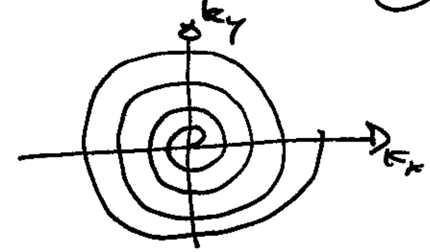



$$\Delta x = (230\,\mathrm{Hz})(0.050\ \mathrm{s})$$
$$= 11.5\ \text{PIXELS}$$

MUCH MORE SEVERE

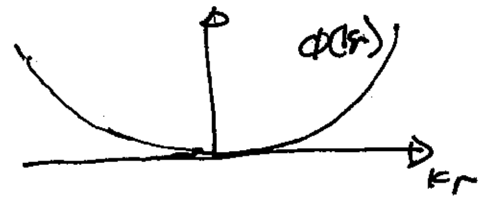
## SPIRAL, PR

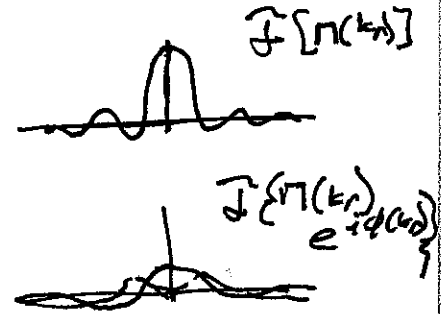



NO CONSTANT AXIS, NO SHIFT

PHASE ACCRUES WITH
TIME
RADIUS

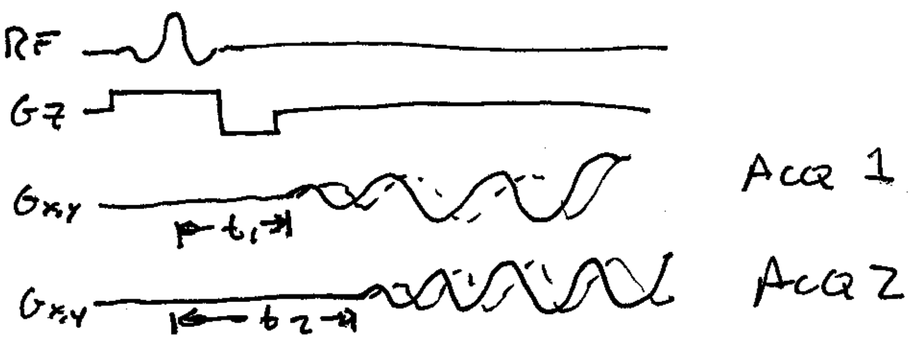



BLURRED

## FIELD MAPS

IMAGES AT TWO ECHO TIMES




$$\Delta\phi(\underline{x}) = \angle\, m_{xy,2}(\underline{x})\, m^{*}_{xy,1}(\underline{x})$$
$$\omega(\underline{x}) = \frac{\Delta\phi(\underline{x})}{\Delta t} \qquad \Delta t = t_2 - t_1$$
$$f(\underline{x}) = \frac{\omega(\underline{x})}{2\pi}\ \mathrm{Hz}$$

## DEMODULATION

SIGNAL EQN

$$s(t) = \int_{\underline{x}} m_{xy}(\underline{x})\, \overbrace{e^{-i\phi_c(\underline{x})}}^{\text{UNKNOWN}}\, \overbrace{e^{-i\omega_0(\underline{x})t}}^{\text{MEASURED}}\, \underbrace{e^{-i2\pi \underline{k}(t)\cdot\underline{x}}}_{\text{APPLIED}}\, d\underline{x}$$

$\phi_c(\underline{x})$ CONSTANT, MANY CONTRIBUTORS

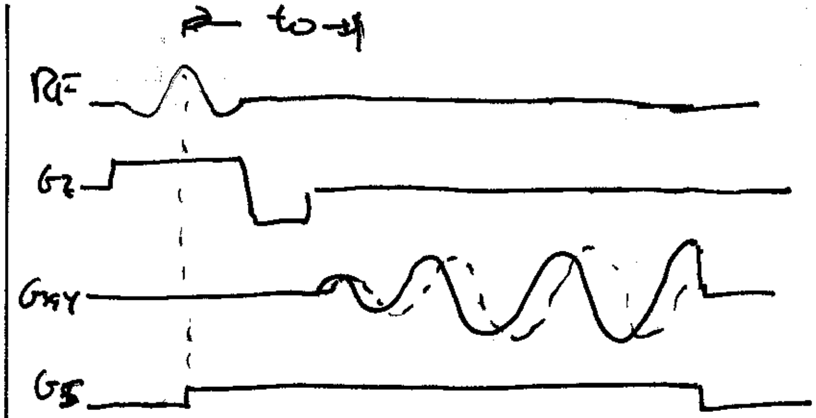
$\omega_0(\underline{x})$ LOCAL FREQ AT $\underline{x}$

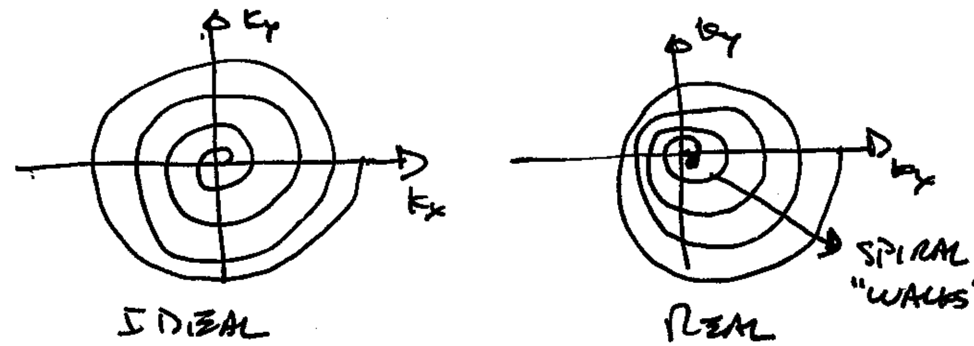
$t$ STARTS AT
- RF FOR GRADIENT ECHO
- SPIN ECHO

# LINEAR OFF-RESONANCE CORRECTION

$$\omega(\underline{x}) = \alpha x + \beta y$$
$$= \gamma G_{x,s}\, x + \gamma G_{y,s}\, y = \gamma \underline{G}_s \cdot \underline{x}$$

$$s(t) = \int_{\underline{x}} m_{xy}(\underline{x})\, e^{-i\omega(\underline{x})t}\, e^{-i2\pi \underline{k}(t)\cdot \underline{x}}\, d\underline{x}$$
$$= \int_{\underline{x}} m_{xy}(\underline{x})\, e^{-i2\pi\left(\frac{\gamma}{2\pi}\underline{G}_s \cdot \underline{x}\right)t}\, e^{-i2\pi \underline{k}(t)\cdot \underline{x}}\, d\underline{x}$$
$$= \int_{\underline{x}} m_{xy}(\underline{x})\, e^{-i2\pi\left[\underline{k}(t) + \underbrace{\frac{\gamma}{2\pi}\underline{G}_s t}_{\text{EXTRA GRADIENT}}\right]\cdot \underline{x}}\, d\underline{x}$$




* DC SHIFTS ($\underline{G}_s$ DURING $t_0$)
* LOSS OF FOV, SAMPLING DENSITY
* RECONSTRUCT WITH

$$\underline{k}(t) + \frac{\gamma}{2\pi}\underline{G}_s \cdot t$$

WHERE t STARTS AT RF

WORKS WELL

* ESTIMATE $\underline{G}_s$ FROM MAP

<u>SIMPLE</u> + <u>ROBUST</u>

## MULTI-FREQUENCY RECONSTRUCTION

ESTIMATE FREQUENCIES FROM MAP

RECONSTRUCT AT L FREQUENCIES

FOR EACH PIXEL, PICK CLOSEST FREQUENCY RECONSTRUCTION

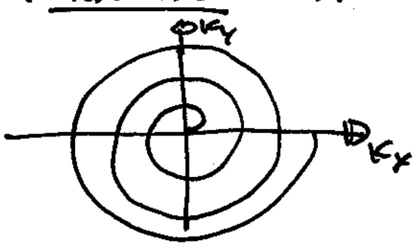
ISSUES

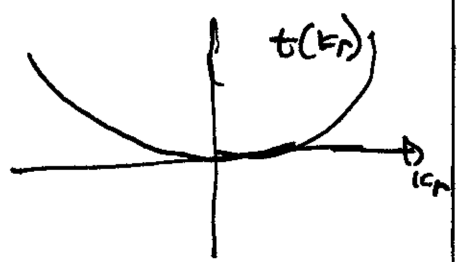
- HOW MANY FREQUENCIES
- HOW TO COMPUTE IMAGES QUICKLY

## TIME MAPS

OFTEN WE CAN DEFINE A UNIQUE TIME FOR EACH K-SPACE POSITION

EXAMPLE: SPIRALS




TIME DEPENDS ON $k_r$

## DISCRETE FREQUENCY ALGORITHM

RECONSTRUCT

$$s(t)\, e^{i(n\Delta\omega)t}$$

FOR $n = -L/2$ TO $L/2$

CHOOSE $\Delta\omega$ SO THAT

$$\Delta\omega T = \pi/2$$

WHERE $T$ IS READOUT DURATION

THIS IS 1/4 CYCLE OVER READOUT

DEFINE PHASE FUNCTION

$$P(k_x,k_y) = e^{i\Delta\omega \cdot t(k_r)}$$

PHASE DUE TO ONE STEP IN $\Delta\omega$

IF $\hat{m}_0(k_x,k_y)$ IS GRIDDING OF $s(t)$ THEN

$$\hat{m}_1(k_x,k_y) = \hat{m}_0(k_x,k_y)\, P(k_x,k_y)$$

$$\hat{m}_{-1}(k_x,k_y) = \hat{m}_0(k_x,k_y)\, P^*(k_x,k_y)$$

## DISCRETE FREQUENCY ALGORITHM

GRID $s(t) \Rightarrow \hat{M}_0(k_x,k_y)$

COMPUTE $t(k_x,k_y)$

COMPUTE $P(k_x,k_y) = e^{i\Delta\omega \cdot t(k_x,k_y)}$

RECURSIVELY COMPUTE

$$\hat{m}_\ell(k_x,k_y) = \hat{m}_{\ell-1}(k_x,k_y)\, P(k_x,k_y)$$

$$\hat{m}_{-\ell}(k_x,k_y) = \hat{m}_{-(\ell-1)}(k_x,k_y)\, P^*(k_x,k_y)$$

COMPUTE $m_\ell(x,y)$ BY L 2D DFT'S

## TOTAL

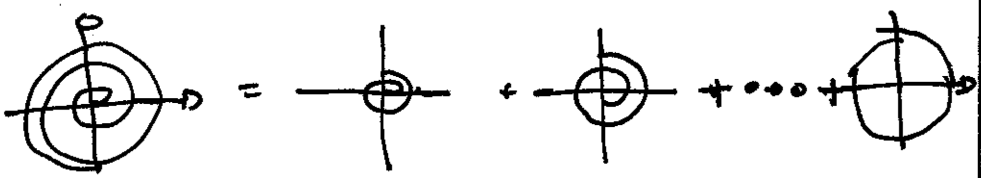
1 GRIDDING OPERATION

L DIRECT MATRIX PRODUCTS

L 2D DFT'S

## OTHER OPTIONS

### DISCRETE TIME ALGORITHM



EACH SEGMENT AT ONE TIME

RECONSTRUCT $m_\ell(x,y)$ FOR EACH SEGMENT

RECONSTRUCT AT $\omega$

$$m_\omega(x,y) = \sum_\ell m_\ell(x,y)\, e^{i\omega(\ell\Delta t)}$$

TAKES MORE SEGMENTS THAN DISCRETE FREQUENCY ALGORITHM

## BETTER INTERPOLATION

BOTH CASES $\Rightarrow$ NEAREST NEIGHBOR!

SIGNIFICANT IMPROVEMENT WITH BETTER INTERPOLATION

FACTOR OF 2 FASTER FOR LARGE L (>10)

# AUTOFOCUS RECONSTRUCTION

MAP-BASED RECON PROBLEMS:

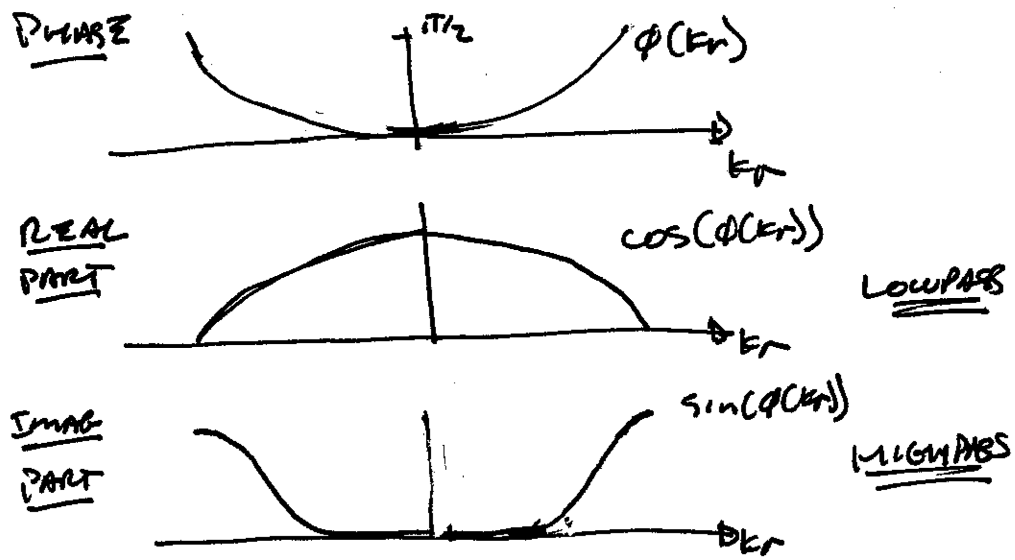
- MAPS TAKE TIME
- LIMITED RESOLUTION
- OFF-RESONANCE DISTORTION
- FAIL AT BOUNDARIES, DISCONTINUITIES

AUTOFOCUS

- USE IMAGE ITSELF AS FOCUS METRIC

# SPIRAL, RADIAL PR AUTOFOCUS

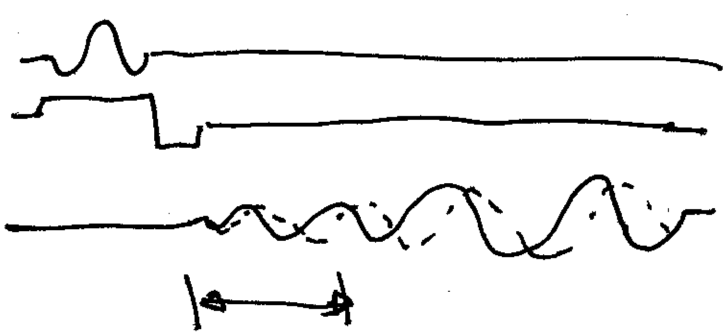





AS $\phi(k_r)$ INCREASES (MORE OFF-RESONANCE) IMAGINARY COMPONENT INCREASES

# SPIRAL AUTOFOCUS

1) RECONSTRUCT LOW FREQUENCY PHASE REFERENCE $m_R(x,y)$, $p(x,y) = e^{i\angle m_R(x,y)}$



SHORT IN TIME, LITTLE OFF-RESONANCE

2) MULTIFREQUENCY RECON $m_\ell(x,y)$

3) COMPUTE FOCUS METRIC

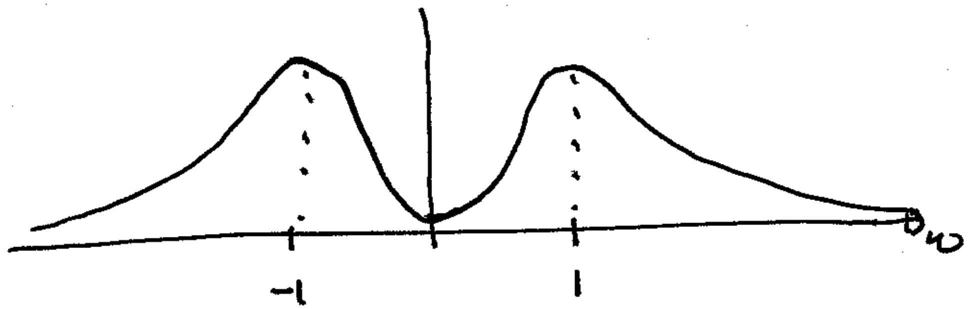
$$f_\ell(x,y) = \sum_{W} \left| \operatorname{Imag}\{ m_\ell(x,y)\, p^*(x,y) \} \right|^{\alpha}$$

W IS CENTERED NEIGHBORHOOD (3×3, 4×4, 5×5)

4) FOR EACH PIXEL, CHOOSE RECON WITH SMALLEST $f_\ell(x,y)$

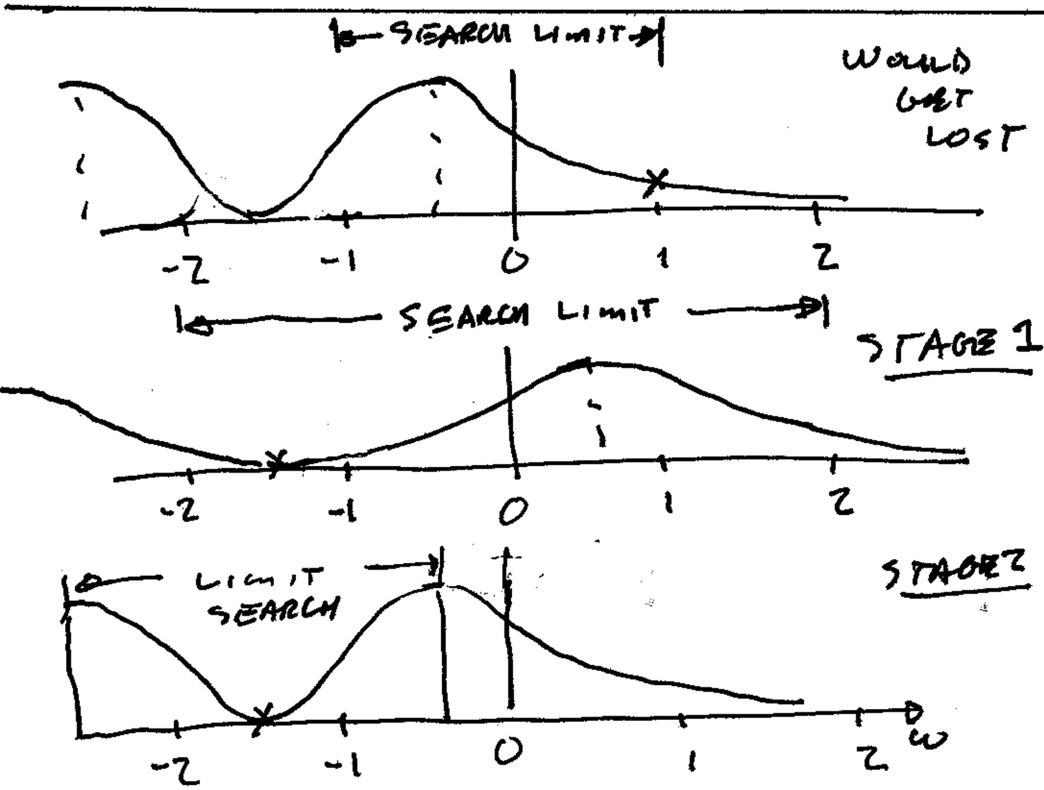
5) $\alpha$ IS SOMETHING LIKE 1/2 TO 1

MINIMUM IS LOCAL




WILL CONVERGE IF WITHIN ±1 CYCLE

DIVERGES OUTSIDE THIS RANGE




FOR LARGER RANGE

1) AUTOFOCUS WITH FIRST HALF OF
DATA ⇒ TWICE FREQUENCY RANGE

2) AUTOFOCUS OVER FULL DATA
LIMIT TO ±1 CYCLE FROM 1ST STAGE

ADD MORE STAGES AS NEEDED

SUMMARY

CONSTANT, LINEAR TERMS EASY
PART OF GRIDDING

HIGHER ORDER TERMS NEED
MULTI-FREQUENCY RECON

SEVERAL ALGORITHMS
MAP BASED
AUTOFOCUS

EACH HAS ADVANTAGES