EE369C: Assignment 5: Solutions

Due Nov 8

The problems this week will be concerned with a simple 2DFT SENSE reconstruction. The SENSE data is a simulated resolution phantom, so that the sensitivies are known exactly. The data is in

`http://ee369c.stanford.edu/mr_data/sense_2dft.mat`

This simulates a 24 cm FOV acquisition, with 24 cm circular diameter coils directly above and below the object. The acquisitions is an axial slice, so the coils are aligned with the y axis.

1. g-Factor The most important characteristic of a SENSE acquisition is the geometry factor g. This tells you how well conditioned the reconstruction problem will be. The two sensitivity maps are stored in `S1` and `S1`. Assume that the noise covariance matrix is $\sigma^2 I$, meaning that the noise is uncorrelated between coils, and is of equal intensity.

a) Make an image of the g-factor if the the phase encode is along the y axis. Show the image with the range of g-factors limited to 1 (the minimum possible) to 4. Plot a cross section along the y axis. Where is the largest g-factor? What is it?

Solution:
The g-factor map map, and a cross section, are plotted in Fig. 1. This is very well conditioned, and causes little SNR loss (10 to 15%) in addition to the unavoidable scan time loss.

b) Repeat the calculation for phase encoding along the x axis. Show the image, and plot a cross section along the x axis. Again, limit the g-factor image to a range of 1 to 4.

Solution:
The g-factor map map, and a cross section, are plotted in Fig. 2. This is very poorly conditioned, and causes tremendous SNR loss (a factor of 5 or more) in addition to the unavoidable scan time loss.

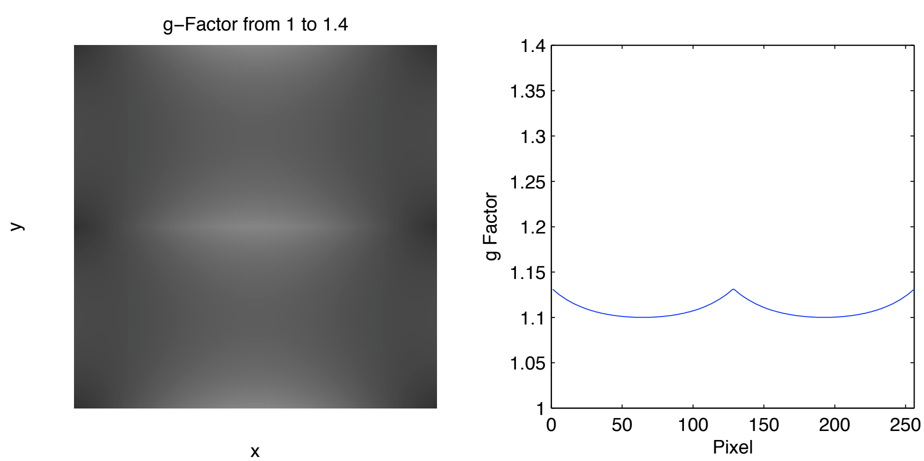

Figure 1: g-Factor for acceleration by a factor of 2 in y. This is very well conditioned. The range has been narrowed to 1 to 1.4 to make the variations visible.

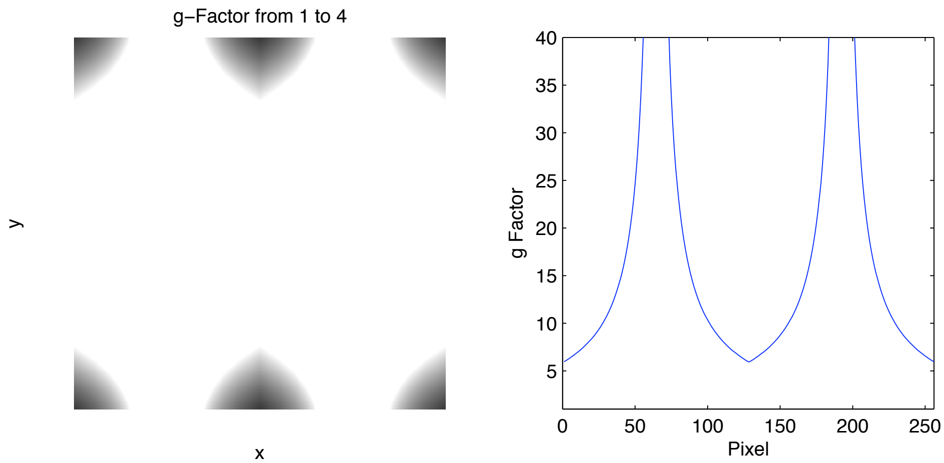

Figure 2: g-Factor for acceleration by a factor of 2 in y. This is very poorly conditioned. The g factor is greater than 4 almost everywhere, which is a huge SNR loss.

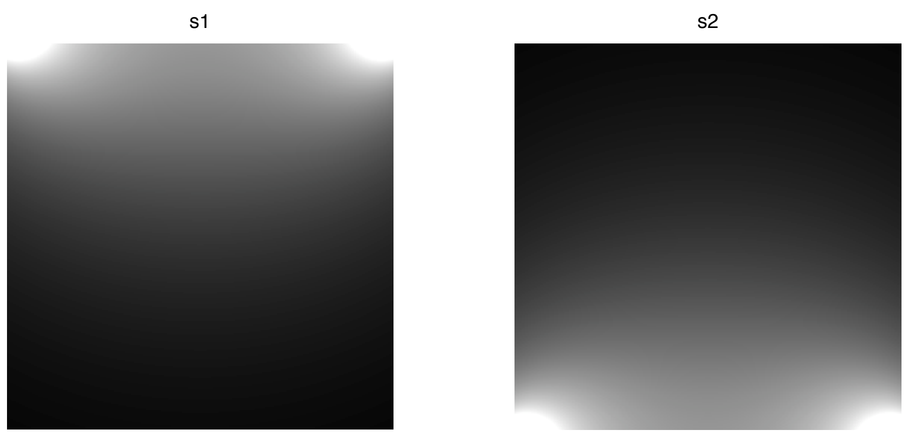

Figure 3: Sensitivities of the two coils.

c) For the second case, the g-factor is large in specific places. What is it about the coil sensitivities there that cause this to happen?

Solution:
The sensitivities are shown in Fig. 3. They vary most rapidly in y, which makes them best suited for undersampling in y. In x, the sensitivies are very flat. Two aliased pixels in x then have almost the same encoding due to the sensitivities, and this makes the g-factor, and the sense decomposition very poorly conditioned. This causes large SNR losses. At the two points where the g-factor blows up, the sensitivies are the same for the two aliased pixels, and we have no ability to sort out where the signals came from.

2. 2DFT SENSE Reconstruction Two acceleration factor 2 aliased images are in the matlab variables `im1a` and `im2a`. Write an mfile that performs the SENSE decomposition. Show the resulting image.

Solution:
Here is the matlab code to do the decomposition:

```
function im = sense2x(im1a, im2a, s1, s2)

%
%function im = sense2x(im1, im2, s1, s2)
%
%  im1, im2 -- aliased images
%  s1, s2 -- sensitivities
%
%  Simple R=2 Sense reconstruction
%  Assumes the sensitivities cover the full FOV
%

[n m] = size(im1a);

% to simplify indexing below, extend aliased image to full FOV

im1ap = [im1a(n/2+1:n,:); im1a; im1a(1:n/2,:)];
im2ap = [im2a(n/2+1:n,:); im2a; im2a(1:n/2,:)];

im = zeros(2*n,m);

for ii=1:n,
    for jj = 1:m,
        S = [s1(ii,jj) s1(ii+n,jj);
             s2(ii,jj) s2(ii+n,jj)];
        p = inv(S'*S)*S'*[im1ap(ii,jj); im2ap(ii,jj)];
        im(ii,jj) = p(1);
        im(ii+n,jj) = p(2);
    end;
end
```

Note that this is very simple, it assumes the sensitvities are known thoughout the FOV, which is generally not true. In this case, you have to figure out on a pixel-by-pixel basis how many pixels alias. This is described in the Bernstein book.

The resulting image is shown in Fig. 4.

3. SENSE and Averaging One of the tradeoffs with SENSE is that it reduces SNR in exchange for speed. Occasionally, the SNR will be too low, and the operator will decide to average multiple SENSE acquisitions to get the SNR back.

a) Assume that the acquisitions has been accelerated by a factor R, and then averaged R times. In this amount of time, we could have simply acquired a fully encoded data set. Find an expression for the ratio of the SNR's of the two alternatives.

   Solution:
   The SNR loss for a single SENSE accleration R scan compared to a full scan is $\frac{1}{g\sqrt{R}}$. If we

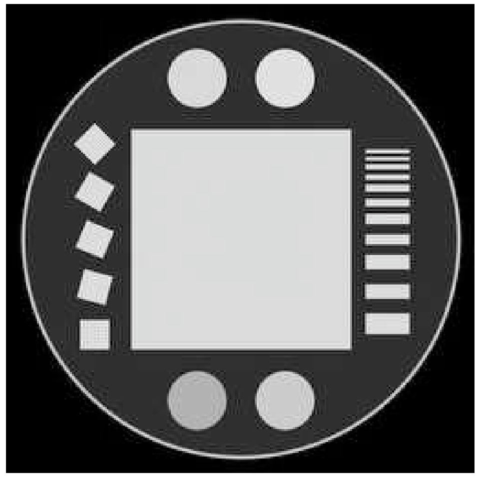

Figure 4: R=2 SENSE reconstruction.

average R times, we get back a factor or $\sqrt(R)$. The result is a net loss of

$$\left(\frac{1}{g\sqrt{R}}\right)\sqrt{R} = \frac{1}{g}.$$

Effectively we recover the scan time loss, $\sqrt{R}$, but the g-factor loss remains.

b) When would acceleration and averaging be a reasonable thing to do?

Solution:
The single SENSE scans are shorter by a factor of R, and will have reduced motion artifacts. Averaging R scans will result in a motion blurred image, with reduced the aliasing artifacts usually associated with motion. Hence, this makes sense when you have surplus SNR, and are willing to trade it for reduced motion artifacts.

For stationary objects, it doesn't pay off to do this.