EE 369C LECTURE 5

GRIDDING KERNEL DESIGN AND OVERSAMPLING

ASSIGNMENT 2

READ BEATTY PAPER

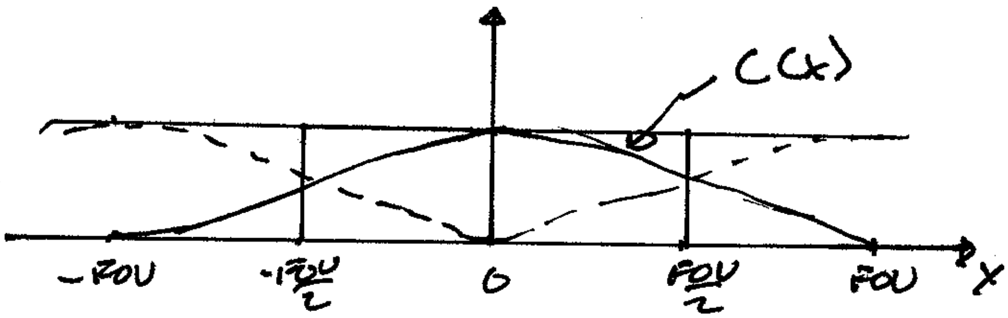
LAST TIME

- GRIDDING
- DENSITY COMPENSATION

THIS TIME

- KERNEL DESIGN
- OVERSAMPLING RATIO
- KERNEL SAMPLING

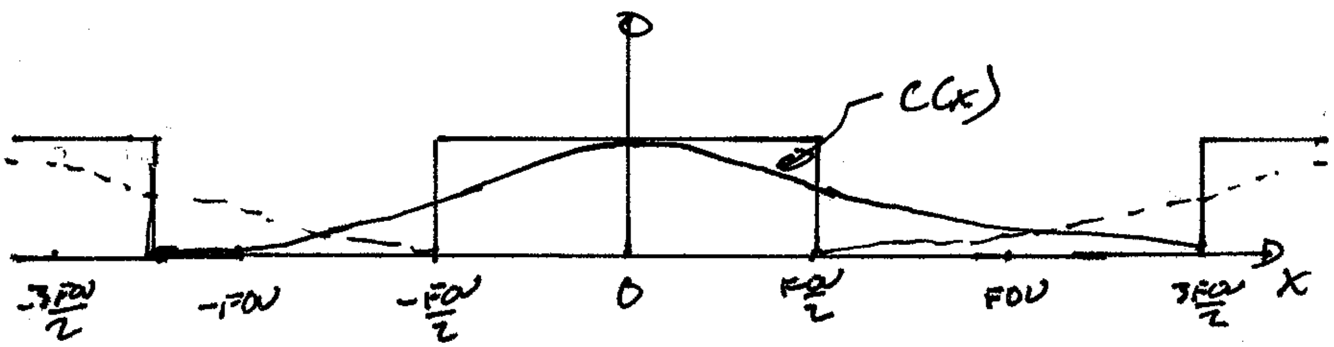
PROBLEM WITH 1X GRID



NO TRANSITION BAND

ALIASED SIGNAL EQUALS APODIZED SIGNAL

2X GRID



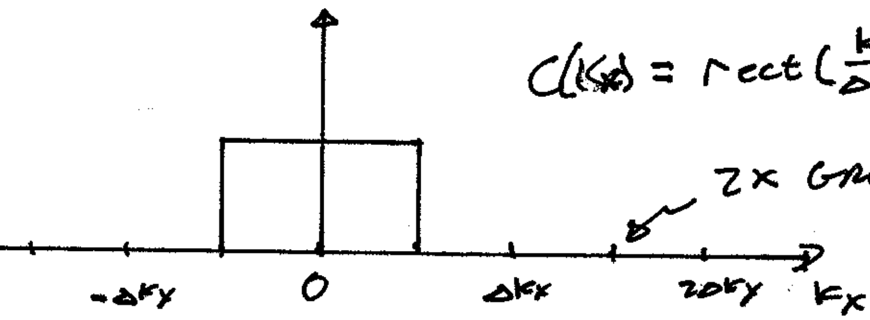
SAMPLE TWICE AS FINELY IN K-SPACE

SPATIAL REPLICAS TWICE AS FAR OUT

MANY KERNELS WORK

WHAT SHOULD WE CHOOSE?

RECT KERNEL



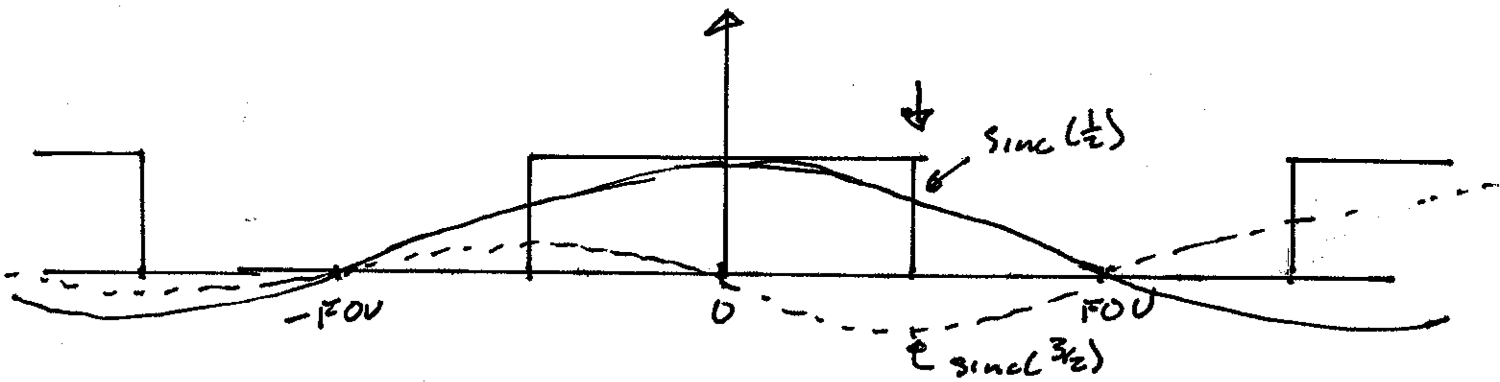
$$C(k_x) = \text{rect}\left(\frac{k_x}{\Delta k_x}\right)$$

EASY TO COMPUTE

NEAREST NEIGHBOR, ONLY HITS ONE GRID POINT

IMAGE SPACE

$$c(x) = \frac{1}{FOV_x}\,\text{sinc}\left(\frac{x}{FOV_x}\right)$$



WE ARE GOING TO DE-APODIZE, SO WHAT WE CARE ABOUT IS RATIO OF MAINLOBE TO SIDELOBE

HIGHEST AT BAND EDGE

MAIN LOBE

$$\text{sinc}\left(\tfrac{1}{2}\right) = \frac{\sin(\pi/2)}{\pi/2} = \frac{2}{\pi} \simeq 0.64$$

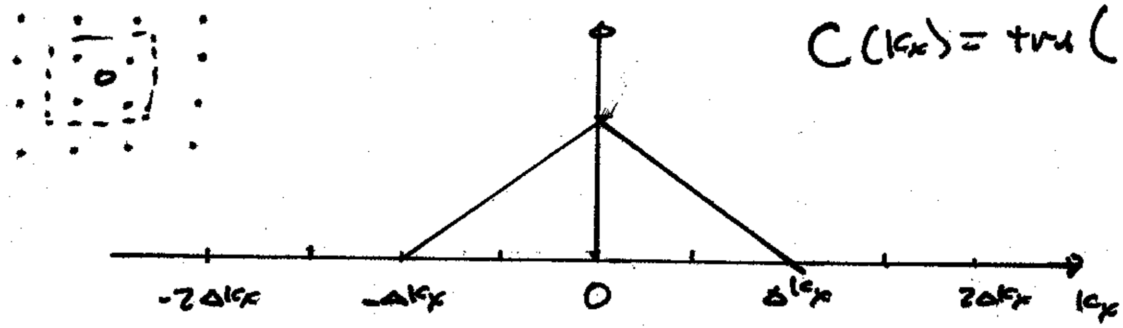
SIDE LOBE

$$\left|\text{sinc}\left(\tfrac{3}{2}\right)\right| = \left|\frac{\sin(3\pi/2)}{3\pi/2}\right| = \frac{2}{3\pi} \simeq 0.21$$

RATIO IS $\underline{\underline{1/3}}$ NOT GOOD

HIGHER OVERSAMPLING CAN BE AS GOOD AS YOU LIKE!

TRIANGLE KERNEL



$$C(k_x) = \mathrm{tri}\left(\frac{k_x}{\Delta k_x}\right)$$

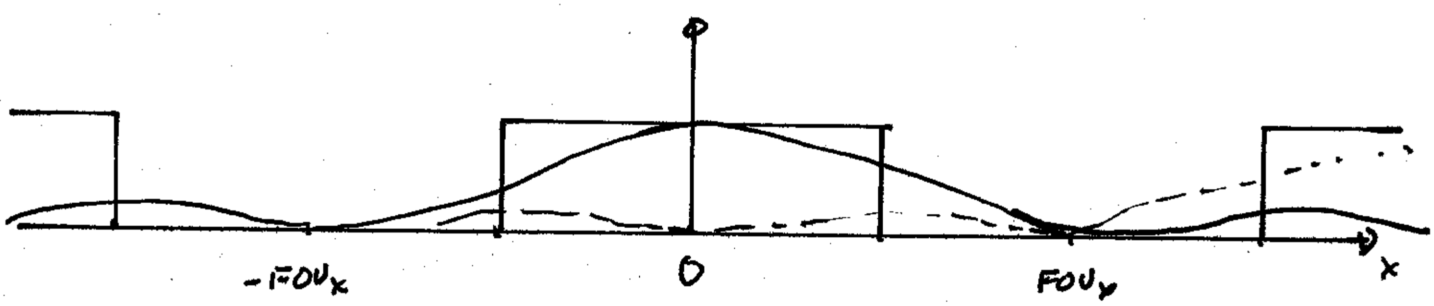
EASY TO COMPUTE (DISTANCE)

LINEAR INTERPOLATION IN K-SPACE

DATA HITS FOUR GRID POINTS, BILINEAR INTERPOLATION

IMAGE SPACE

$$c(x) = \frac{1}{FOV_x} \mathrm{sinc}^2\left(\frac{x}{FOV_x}\right)$$



MAIN LOBE AT BAND EDGE

$$\mathrm{sinc}^2\left(\tfrac{1}{2}\right) = \left(\tfrac{2}{\pi}\right)^2 \simeq 0.41$$

SIDE LOBE AT BAND EDGE

$$\mathrm{sinc}^2\left(\tfrac{3}{2}\right) = \left(\tfrac{2}{3\pi}\right)^2 \simeq 0.045$$

RATIO IS $\underline{\underline{1/9}}$

THIS IS OFTEN GOOD ENOUGH!

WE CAN DO <u>MUCH</u> BETTER

WINDOW FUNCTION KERNELS

MANY SMOOTH LOWPASS FUNCTIONS

WINDOW FUNCTIONS

HERE, KAISER-BESSEL WINDOW

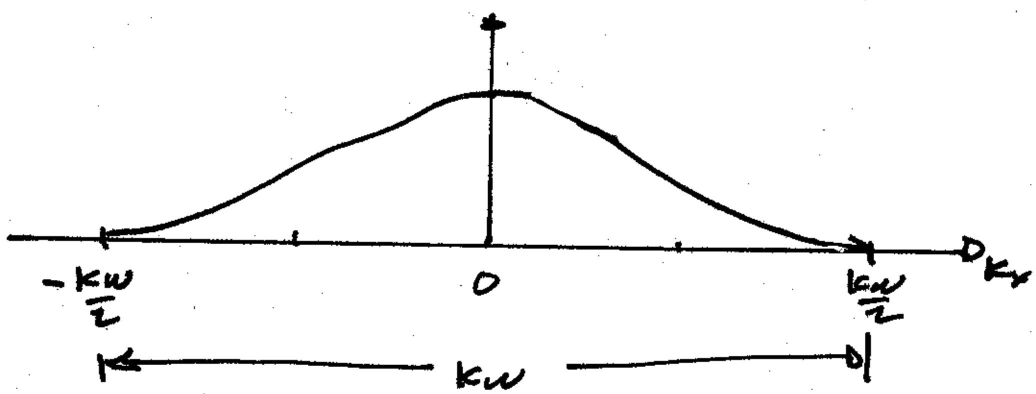
$$C(k_x) = \frac{1}{k_w} I_0\left(\beta \sqrt{1-\left(\frac{k_x}{k_w/2}\right)^2}\right)$$

WHERE

β - SHAPE PARAMETER

k_w - WIDTH IN SPATIAL FREQUENCY

$I_0(\cdot)$ - ZERO ORDER MODIFIED BESSEL FUNCTION OF THE FIRST KIND (BUILT IN TO MATLAB)



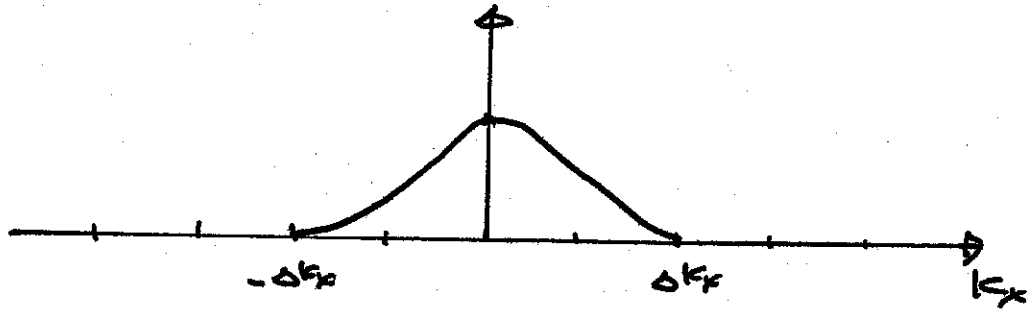
HOW DO WE CHOOSE β, k_w?

FOR 2X CASE, ANALYZED BY JACKSON, ET AL

GIVEN k_w, PROVIDES β TO MINIMIZE ALIASING

EXAMPLE

4 SAMPLE KB KERNEL



FROM JACKSON, $\beta = 9$

SAME COMPUTATION AS tri(·)

MAIN LOBE AT BAND EDGE

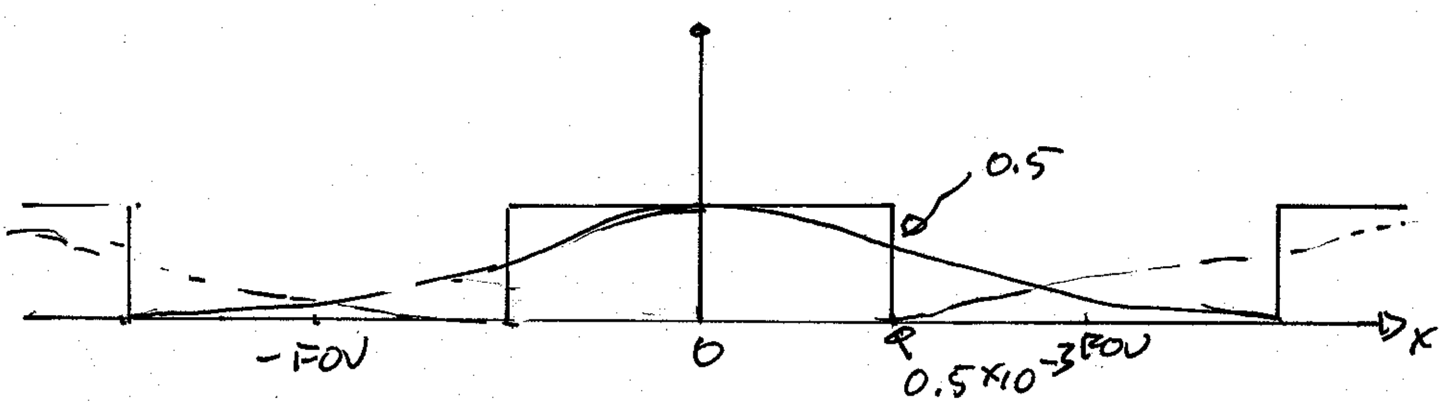
~ 0.5

1 ST SIDE LOBE AT BAND EDGE

$\sim 0.5 \times 10^{-3}$

RATIO IS $\underline{\underline{1/10^3}}$!

MUCH BETTER THAN REQUIRED FOR MR.



REDUCED OVERSAMPLING RATIO

WITH 2X GRID, A 4 SAMPLE KERNEL GIVES AN ALMOST PERFECT RECON

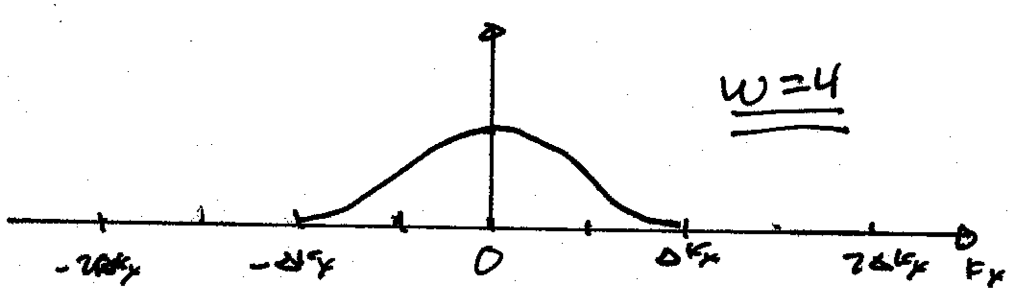
HOW MUCH CAN I REDUCE THE OVERSAMPLING, AND STILL HIT A SPECIFIC ALIASING LEVEL?

A LOT

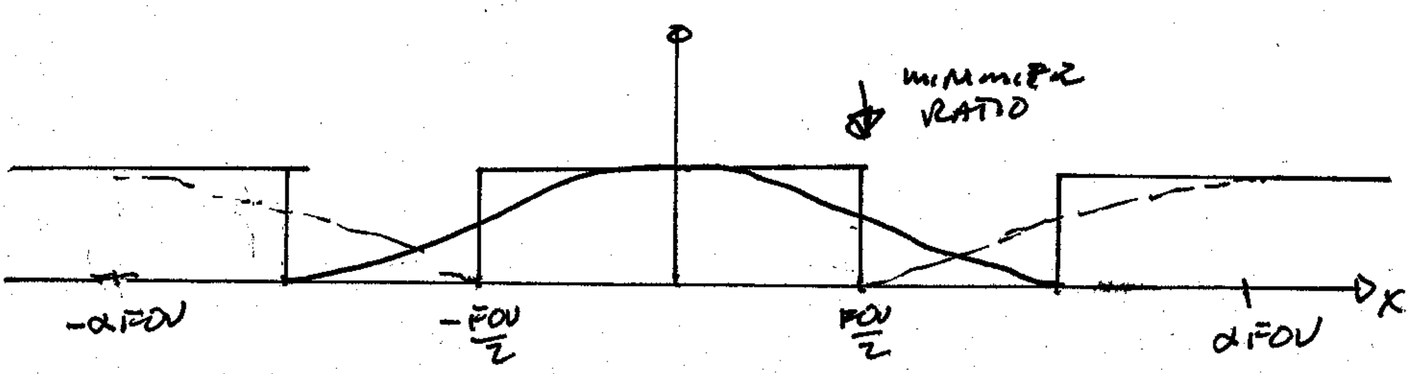
NEEDS FFTW TO MAKE SENSE

OPTIMUM KB KERNEL

GIVEN A KERNEL WIDTH W IN OVERSAMPLED GRID UNITS

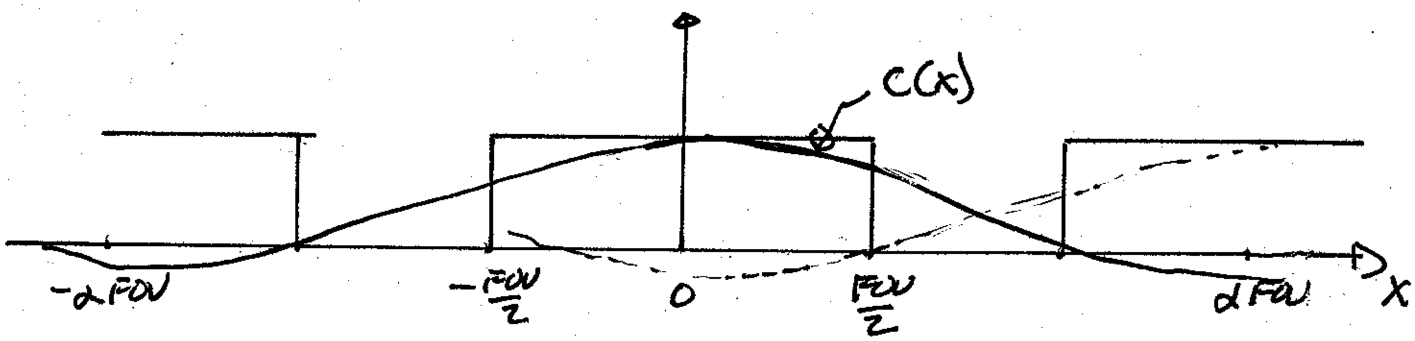


AND AN OVERSAMPLING FACTOR α, FIND β TO MINIMIZE ALIASING



LS

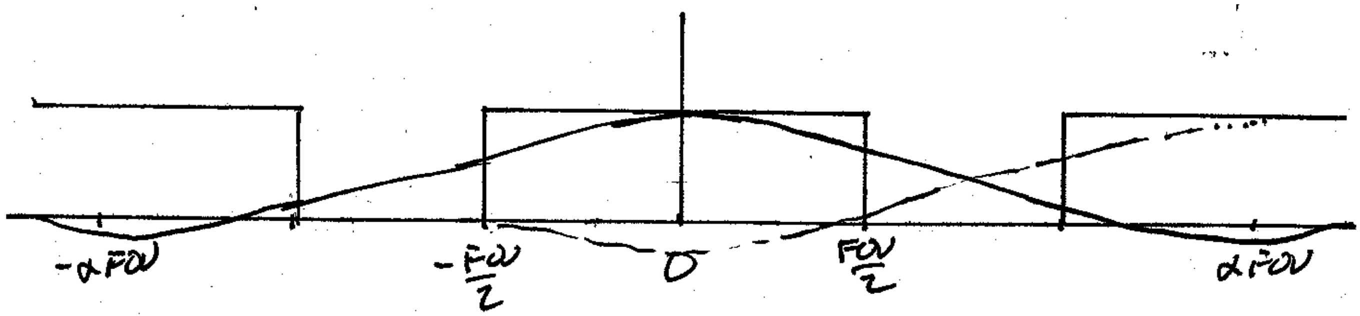
SOLUTION 1 (WAJER, ISMRM 1999)



PUT ZERO OF C(x) AT BAND EDGE

$$\beta = \pi\sqrt{\left[\frac{W}{\alpha}\left(\alpha-\frac{1}{2}\right)\right]^2 - 1}$$

SOLUTION 2 (BEATTY)



PUT ZERO INSIDE FOV

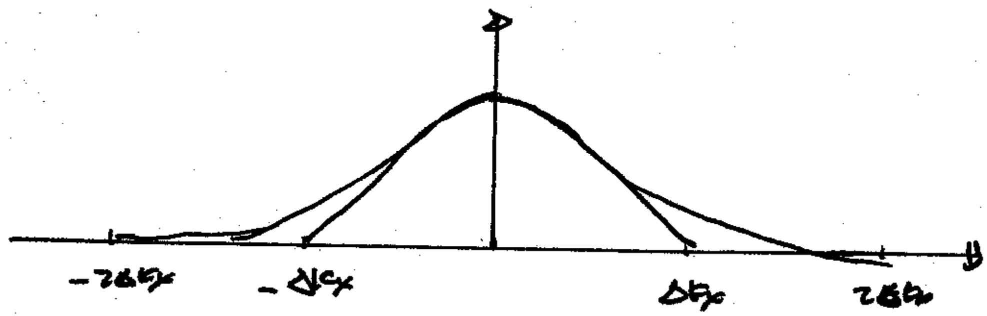
$$\boxed{\beta = \pi\sqrt{\left[\frac{W}{\alpha}\left(\alpha-\frac{1}{2}\right)\right]^2 - 0.8}}$$

MUCH BETTER AS $\alpha \to 1$

W vs β

β IS A STRONG FUNCTION OF W

ALL KERNELS SIMILAR WITH RESPECT TO ΔK_x

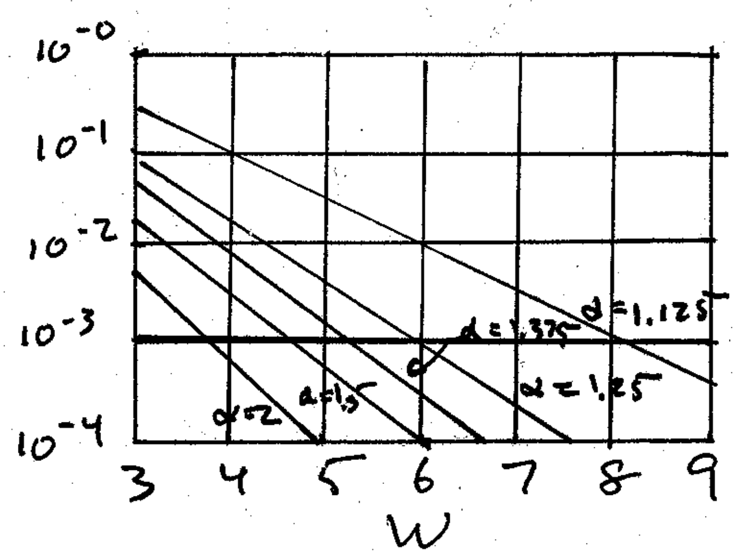


AS W INCREASES, $C(K_x)$ GETS SMOOTHER

W AND α VS ALIASING

FOR A GIVEN ALIASING AMPLITUDE, WHAT W AND α SHOULD I CHOOSE?

FIG 3 IN BEATTY PAPER



0.1% ALIASING

$\alpha = 1.125 \quad W=8$

$\alpha = 1.25 \quad W=6$

$\alpha = 2 \quad W=4$

KERNEL SAMPLING

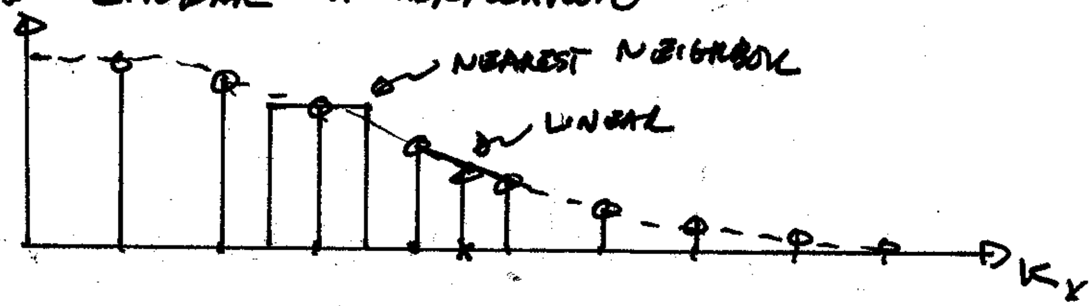
KERNEL IS RECOMPUTED MANY TIMES

DOMINATES COMPUTATION

HOW CAN I PRECOMPUTE KERNEL

COMPUTE AT S SAMPLES / GRID POINT

- NEAREST NEIGHBOR LOOK UP
- LINEAR INTERPOLATION



SURPRISINGLY HUGE DIFFERENCE

NEAREST NEIGHBOR

$$\max(\varepsilon_1) = \frac{0.91}{\alpha S}$$

LINEAR INTERPOLATION

$$\max(\varepsilon_1) = \frac{0.37}{(\alpha S)^2}$$

α - OVERSAMPLING RATIO

S - KERNEL OVERSAMPLING

ε_1 - ALIASING ERROR OVER FOV

EXAMPLE

SET $\varepsilon_1 = 10^{-4}$, ASSUME $\alpha = 1.25$, $W = 6$ (ALIASING AMPLITUDE OF 10^{-3})

NEAREST NEIGHBOR

$S = 7280$ $\quad$ $WS = (7280)(6) = 43{,}680$ SAMPLES

LINEAR

$S = 49$ $\quad$ $WS = 6(49) = 294$ SAMPLES

IMPORTANT BECAUSE MEMORY MORE IMPORTANT THAN COMPUTATION OFTEN. $\quad$ LS