EE369C LECTURE 4

GRIDDING RECONSTRUCTION

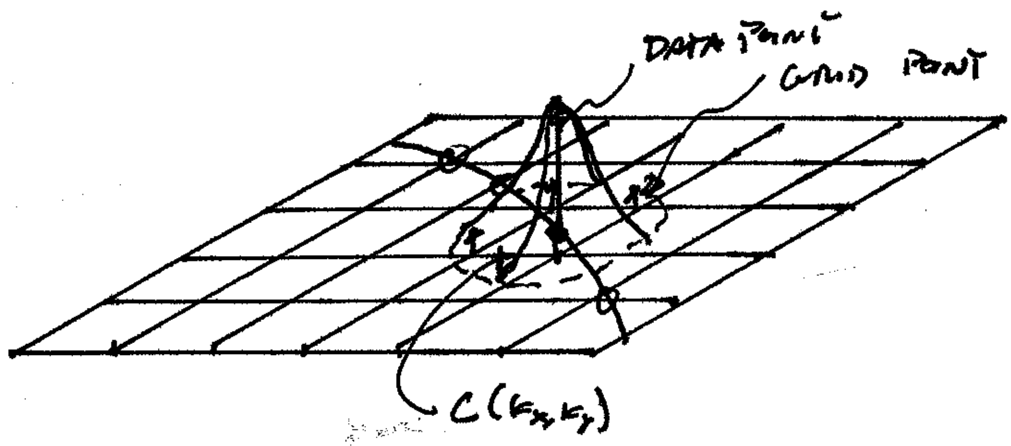
OFFICE HOURS ON WEB SITE

READ 13.2 IN BERNSTEIN

MORE COMPLETE GRIDDING ALGORITHM

DENSITY COMPENSATION

FROM LAST TIME: SIMPLE GRIDDING IDEA



CONVOLVE DATA WITH KERNEL

RESAMPLE ON GRID

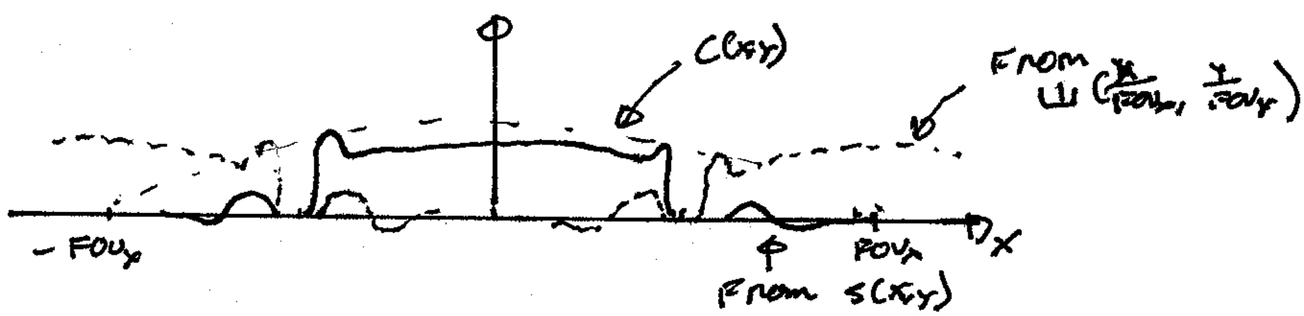
IN K-SPACE

$$\hat{M}(k_x,k_y) = \left[\left(M(k_x,k_y)\cdot S(k_x,k_y)\right) * C(k_x,k_y)\right] Ш\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)$$

IN IMAGE SPACE

$$\hat{m}(x,y) = \left[\left(m(x,y) * s(x,y)\right)\cdot c(x,y)\right] * Ш\left(\frac{x}{FOV_x}, \frac{y}{FOV_y}\right)$$

IN 1D:

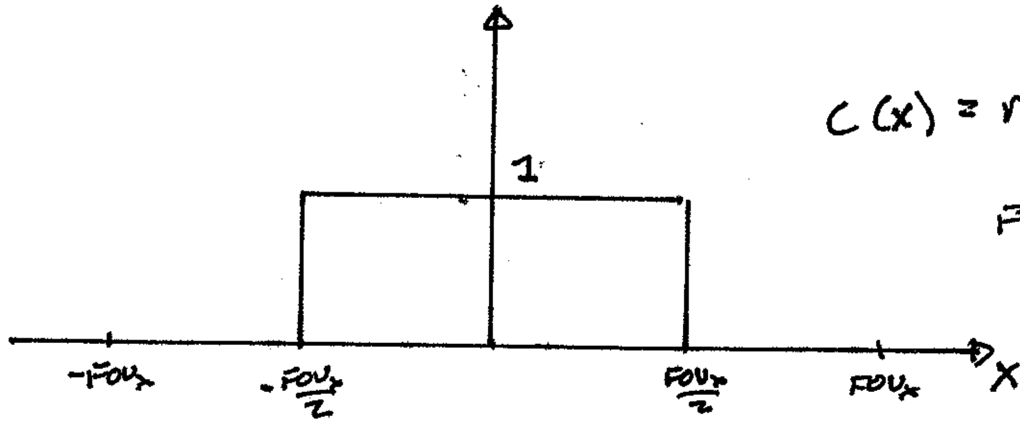


APODIZATION AND $C(k_x, k_y)$

IDEAL APODIZATION

$$c(x) = \mathrm{rect}\left(\frac{x}{FOV_x}\right)$$

$$FOV_x = \frac{1}{\Delta k_x}$$



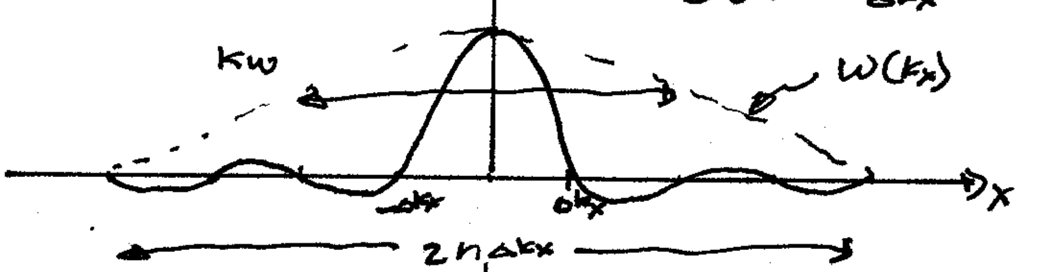
IDEAL KERNEL

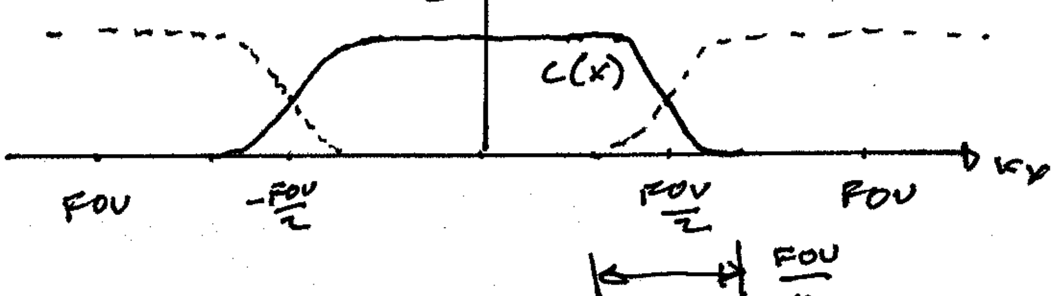
$$C(k_x) = \mathcal{F}\{c(x)\}$$
$$= FOV_x \,\mathrm{sinc}(FOV_x\, k_x)$$
$$= \frac{1}{\Delta k_x} \mathrm{sinc}\left(\frac{k_x}{\Delta k_x}\right)$$

DECAYS SLOWLY, NEED LONG SEGMENT FOR GOOD APPROXIMATION

WINDOWED SINC

$$C(k_x) = \frac{1}{\Delta k_x} \mathrm{sinc}\left(\frac{k_x}{\Delta k_x}\right) W(k_x)$$





TRANSITION WIDTH

$$\sim \frac{1}{K_W} = \frac{1}{n\Delta K} = \frac{1}{n} FOV_x$$

WINDOW WIDTH

$\sim 2n$

COMPUTATION

$(2n)^2$ PER DATA POINT

N^2 DATA POINTS

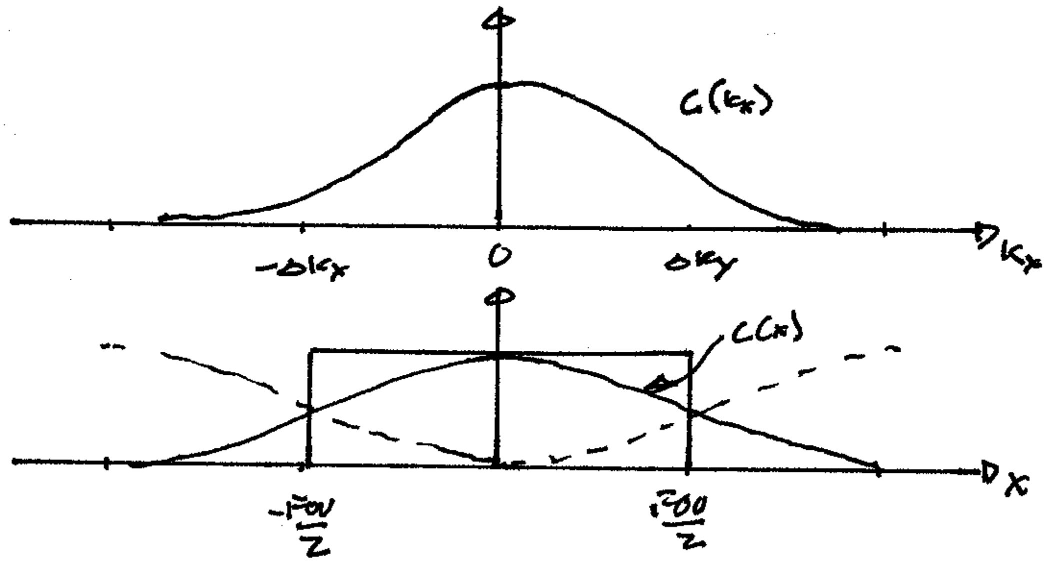
WANT

n SMALL FOR COMPUTATION

n LARGE FOR ALIASING LIMITATION

SMALL KERNELS

MINIMIZE COMPUTATION WITH A WINDOW FUNCTION



OVERSAMPLING

WE CHOOSE RECONSTRUCTION GRID

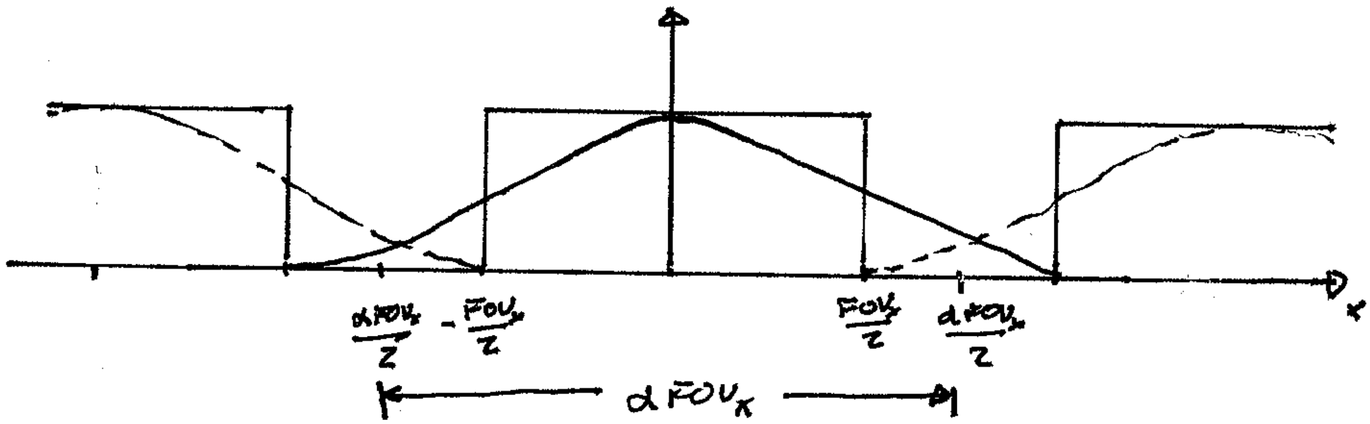
IF WE RECONSTRUCT ON A FINER GRID

$$\left(\frac{\Delta k_x}{\alpha}, \frac{\Delta k_y}{\alpha}\right) \qquad \alpha > 1$$

THEN

$$\hat{M}(k_x,k_y) = \left[\left(M(k_x,k_y)\cdot S(k_x,k_y)\right) * C(k_x,k_y)\right]\cdot \text{Ш}\left(\frac{k_x}{\Delta k_x/\alpha}, \frac{k_y}{\Delta k_y/\alpha}\right)$$

$$\hat{m}(x,y) = \left[\left(m(x,y) * s(x,y)\right)\cdot c(x,y)\right] * \text{Ш}\left(\frac{x}{\alpha FOV_x}, \frac{y}{\alpha FOV_y}\right)$$



ALLOWS TRANSITION BANDS

LIMITS ALIASING

REDUCES APODIZATION

DIVIDE OUT REMAINING APODIZATION

IMPROVED GRIDDING RECONSTRUCTION

$$\hat{m}(x,y) = \frac{\left[(m(x,y) * s(x,y)) \cdot c(x,y)\right] * Ш\left(\frac{x}{\alpha FOV_x}, \frac{y}{\alpha FOV_y}\right)}{c(x,y)}$$

HOW DO WE CHOOSE α, $C(k_x,k_y)$?

WE'LL COME BACK TO THIS.

DENSITY CORRECTION

SAMPLING PATTERN IS

$$S(k_x,k_y) = \sum_i \delta(k_x - k_{xi}, k_y - k_{yi})$$

MOST ACQUISITIONS HAVE NON-UNIFORM SAMPLING DENSITIES

THIS CREATES RECONSTRUCTION ARTIFACTS

THE INVERSE FT OF $S(k_x,k_y)$ IS $s(x,y)$

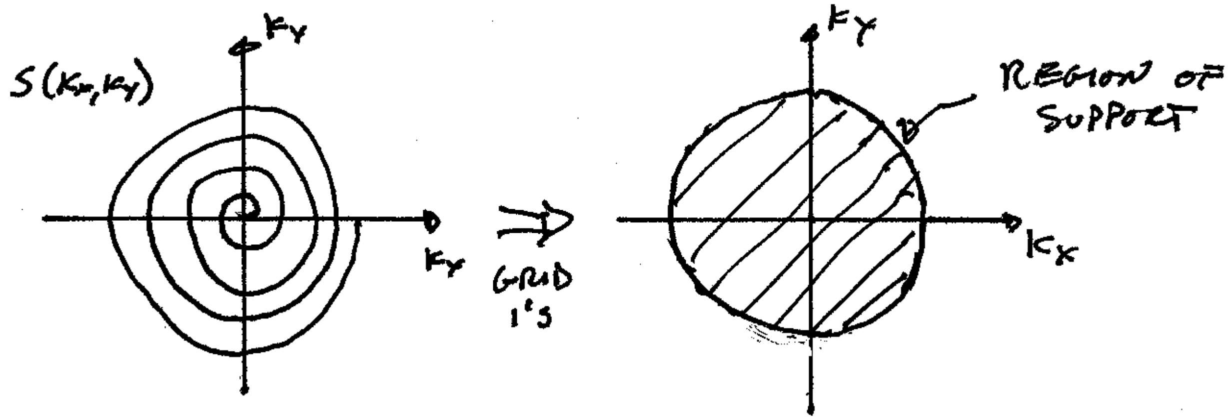
RECONSTRUCTION OF SAMPLING PATTERN

IMPULSE RESPONSE OR POINT SPREAD FUNCTION OF THE ACQUISITION

WE WANT $s(x,y)$ TO APPROXIMATE AN IMPULSE

BASIC IDEA

IF WE HAVE UNIFORM DATA (GRID 1's) WE WANT UNIFORM MAGNITUDE k-SPACE DATA



THE IMPULSE RESPONSE IS FT^{-1} OF THE ROS

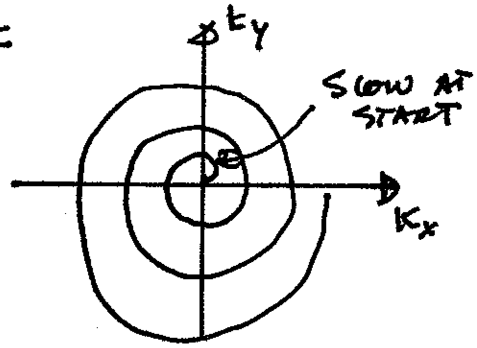
BEST WE CAN HOPE FOR!

WHY IS THIS A PROBLEM?

SAMPLING IS SELDOM UNIFORM

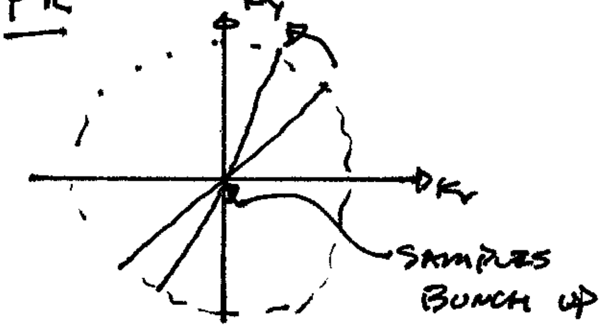
- RATE ALONG TRAJECTORY
- PATTERN DENSITY

SPIRAL



K-SPACE VELOCITY

PR



PATTERN DENSITY

CORRECTION OPTIONS

IDEAL: PRECOMPENSATION

$$\hat{M}(k_x,k_y) = \left[\left(M(k_x,k_y)\cdot\frac{S(k_x,k_y)}{\rho(k_x,k_y)}\right) * C(k_x,k_y)\right] \text{Ш}\left(\frac{k_x}{\Delta k_x/\alpha}, \frac{k_y}{\Delta k_y/\alpha}\right)$$

DENSITY $\rho(k_x,k_y)$ IS COMPENSATED ON A DATA SAMPLE BASIS BEFORE CONVOLUTION

WE NEED TO KNOW $\rho(k_x,k_y)$ FIRST.

APPROXIMATE: POST COMPENSATION

$$\hat{M}(k_x,k_y) = \left[\frac{\left(M(k_x,k_y)\cdot S(k_x,k_y)\right) * C(k_x,k_y)}{\rho(k_x,k_y)} \right] \text{Ш}\left(\frac{k_x}{\Delta k_x/\alpha}, \frac{k_y}{\Delta k_y/\alpha}\right)$$

DENSITY IS CORRECTED ON A <u>GRID POINT</u> BASIS

<u>AFTER</u> CONVOLUTION

WE CAN ESTIMATE DENSITY DURING GRIDDING

BY GRIDDING 1'S (EASIER)

ESTIMATING $\rho(k_x,k_y)$

<u>GEOMETRY</u>

SPIRAL, PR, LISSAJOU

<u>IN GENERAL</u>

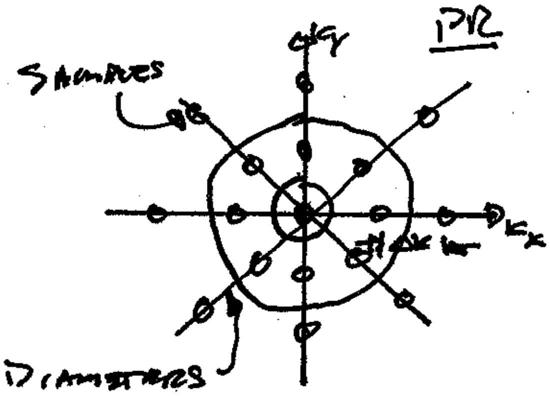
GRIDDING

NUMERICAL CALCULATION

GEOMETRY

HOW MUCH AREA IS ASSOCIATED WITH EACH SAMPLE

THIS GIVES $1/\rho$, COMPENSATION FILTER



DC DISK

$$\frac{1}{N}\pi\left(\frac{\Delta k}{2}\right)^2$$

1ST SAMPLE

$$\frac{\pi}{2N}\left(\left(\frac{3\Delta k}{2}\right)^2-\left(\frac{\Delta k}{2}\right)^2\right)$$

n^{th} SAMPLE

$$\frac{\pi}{N}(\Delta k)^2 n$$

DISCRETE RHO FILTER

NOT ZERO AT ORIGIN!

SIMILAR ARGUMENTS FOR SPIRALS, OTHERS

GRIDDING DENSITY

ESTIMATE $\rho(k_x,k_y)$ BY GRIDDING 1'S

$$\hat{\rho}(k_x,k_y) = \left[S(k_x,k_y) * C(k_x,k_y)\right]\cdot Ш\left(\frac{k_x}{\Delta k_x/\alpha},\frac{k_y}{\Delta k_y/\alpha}\right)$$

THIS IS ON GRID POINTS

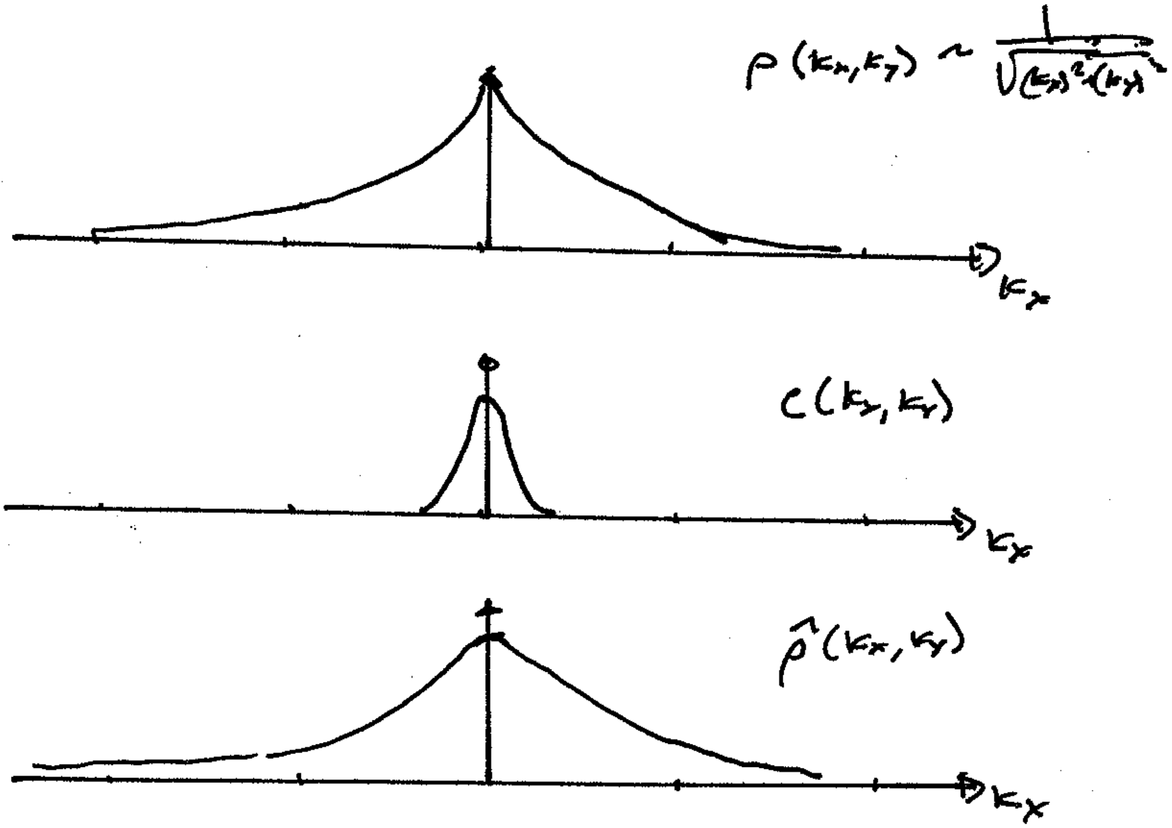
POSTCOMPENSATION

OK IF $\rho(k_x,k_y)$ VARIES SLOWLY

FAILS WHEN $\rho(k_x,k_y)$ CHANGES RAPIDLY

COMMON IN OLDER PAPERS

EXAMPLE



LOW FREQUENCIES BLURRED OUT

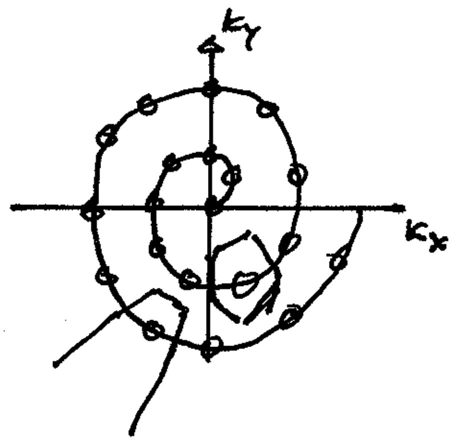
PRODUCES BASELINE, AND LOW FREQUENCY ARTIFACTS

ODD! THIS IS WHERE WE HAVE THE MOST DATA

NUMERICAL CALCULATION

VORONOI DIAGRAM

PARTITION PLANE INTO POLYGONS CLOSEST TO EACH SAMPLE



DENSITY IS $1/\text{AREA}$

FAST, PART OF MATLAB

EDGE SAMPLES UNBOUNDED

ANOTHER NUMERICAL PERSPECTIVE

GRIDDING $1/\text{DENSITY}$ SHOULD PRODUCE UNIFORM k-SPACE DATA

GRIDDING CONVOLUTION CAN BE WRITTEN AS A MATRIX EQUATION

IN 1D

$$\underbrace{\begin{bmatrix} \vdots \\ d_{g,i} \\ \vdots \end{bmatrix}}_{\text{GRID DATA}} = \underbrace{\begin{bmatrix} \ddots & & 0 \\ & C(k_{s,j}-k_{g,i}) & \\ 0 & & \ddots \end{bmatrix}}_{\text{Kernel}} \underbrace{\begin{bmatrix} \vdots \\ d_{s,j} \\ \vdots \end{bmatrix}}_{\text{DATA}}$$

OR

$$\underline{d}_g = G\,\underline{d}_s$$

WE WANT TO FIND WEIGHTING VECTOR

$$\underline{w} = 1/\rho$$

SUCH THAT

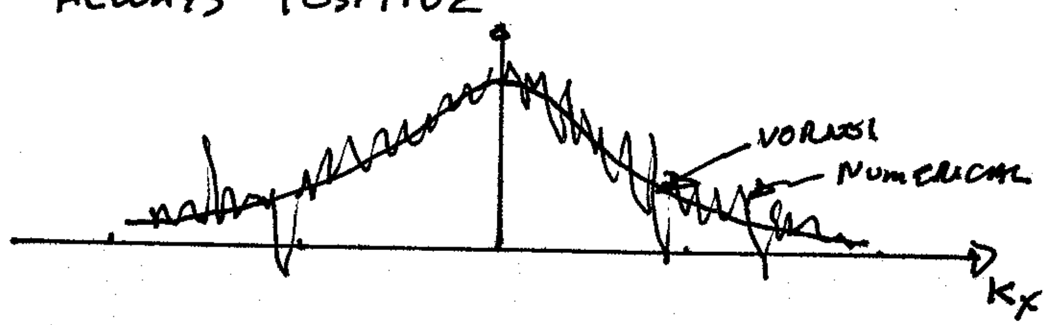
$$1 = G\,\underline{w}$$

IF WE GRID $\underline{w}$, WE GET 1'S ON THE GRID POINTS

LARGE PROBLEM, BUT G IS SPARSE
EASILY SOLVED IN MATLAB WITH LSQR( )

UNUSUAL CHARACTERISTICS

WEIGHTING NO LONGER ASSOCIATED WITH AREA OR DENSITY
DEPENDS SOMEWHAT ON $C(k_x, k_y)$
NOT ALWAYS POSITIVE



DENSITY COMPENSATION

DEPENDS ON YOUR PROBLEM

POST COMPENSATION + GRIDDING MAY BE ENOUGH
AND IS EASY

PRE COMPENSATION IS BETTER
MORE DIFFICULT TO COMPUTE

CAN COMBINE THE TWO
GOOD INITIAL ESTIMATE WITH PRE COMPENSATION
CORRECT FOR ACTUAL DATA WITH POSTCOMPENSATION

NEXT TIME

KERNEL DESIGN
OVERSAMPLING RATIO
KERNEL SAMPLING