# EE369C LECTURE 3

## 2D SAMPLING AND RECONSTRUCTION

ASSIGNMENT 1 DUE OCT 13

NON-CARTESIAN MRI RECONSTRUCTION

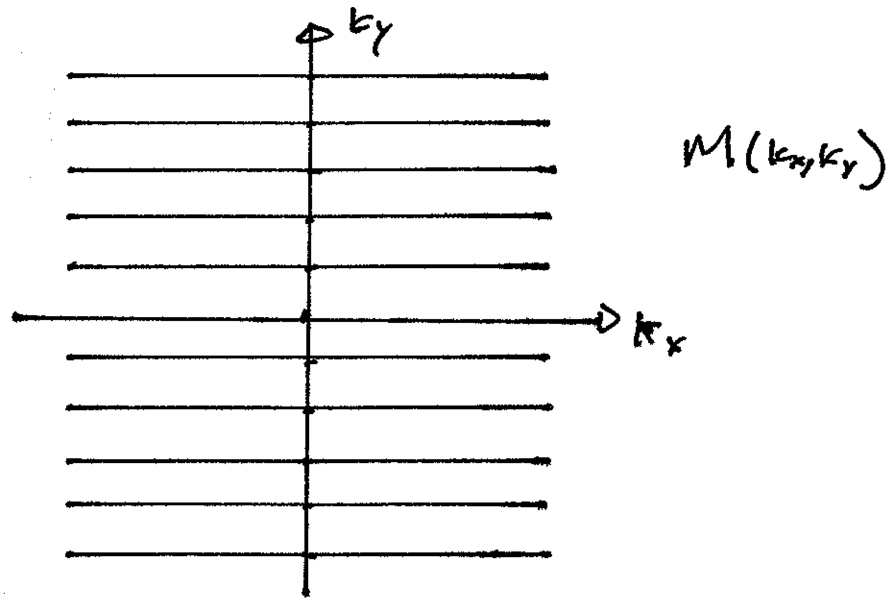
NUFFT PROBLEMS

BASIC GRIDDING IDEA

READ SECTION 13.2 IN BERNSTEIN

# NON-CARTESIAN MRI

TYPICALLY MRI ACQUISITIONS COLLECT DATA ON A CARTESIAN GRID IN SPATIAL FREQUENCY




KNOWN AS 2DFT OR "SPIN WARP", DUE TO KUMAR, WELTI, AND ERNST (YES, THAT ERNST!)

RECONSTRUCTION IS JUST A 2D DFT.

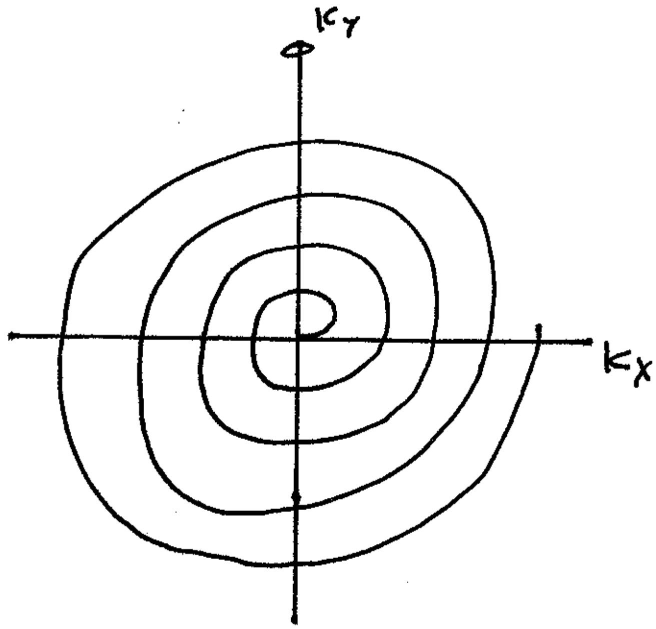
MUCH OF THE SUCCESS OF MRI IS DUE TO SPIN-WARP

- FAST RECONSTRUCTION
- IMMUNE TO SYSTEM IMPERFECTIONS
- VERY ROBUST

THERE ARE MANY OTHER POSSIBILITIES, AND THESE CAN HAVE ADVANTAGES

# EXAMPLES

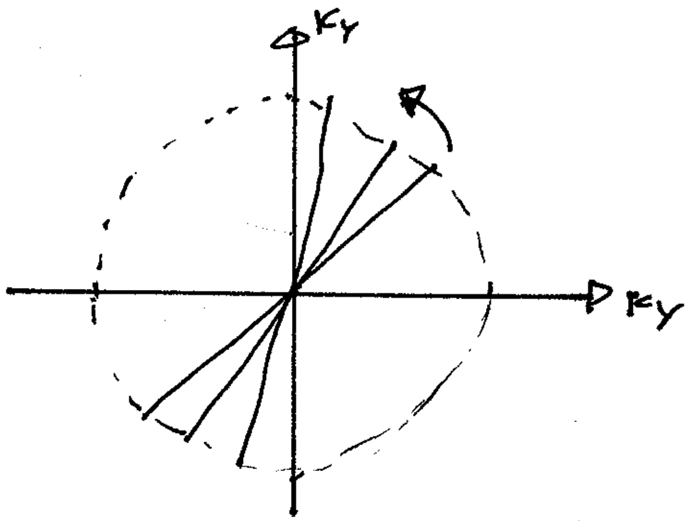
## SPIRAL




USEFUL WHEN MOTION IS AN ISSUE

- CARDIAC
- fMRI

## PROJECTION




USEFUL FOR UNDERSAMPLING

- REAL-TIME IMAGING
- CONTRAST ENHANCED IMAGING
- SHORT $T_2$
- CT, PET

## PROBLEM

DATA DOES NOT FALL ON 2D CARTESIAN GRID.

HOW CAN WE RECONSTRUCT THE DATA?

## GENERAL PROBLEM: NON-UNIFORM FFT

NuFFT

FOUR GENERAL PROBLEMS, DEPENDING ON WHETHER THE SOURCE OR DESTINATION IS NON-UNIFORM, (OR BOTH, OR NEITHER)

| SOURCE DOMAIN | DESTINATION DOMAIN | |
|---|---|---|
| UNIFORM | UNIFORM | DFT |
| UNIFORM | NON-UNIFORM | "PROBLEM 2" NuFFT INVERSE GRIDDING |
| NON-UNIFORM | UNIFORM | "PROBLEM 1" GRIDDING |
| NON-UNIFORM | NON-UNIFORM | "PROBLEM 3" |

PROBLEM CLASSIFICATION DUE TO DUTT + ROKHLIN SIAM J. SCI. COMPUT. (1993) pp 1368-1393

PROBLEM 2 TURNS UP IN ITERATIVE RECONSTRUCTION OF PET + CT DATA

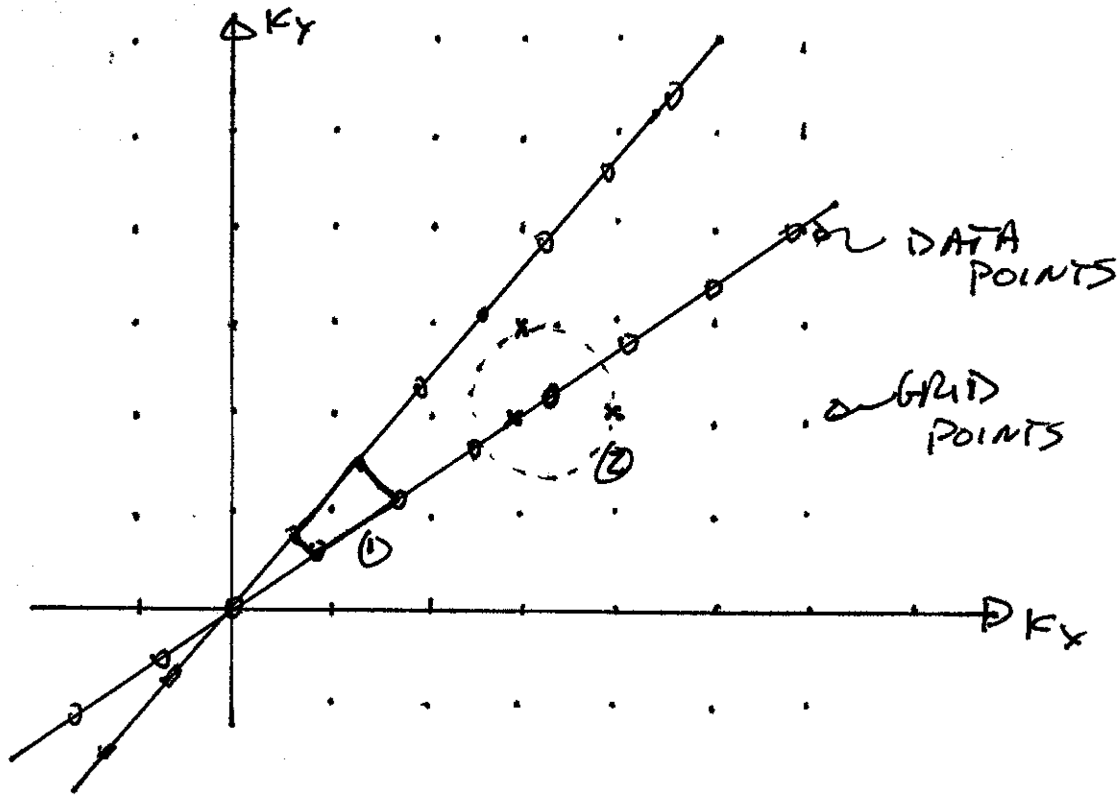
PROBLEM 1 IS NON-CARTESIAN MRI RECONSTRUCTION GRIDDING

# APPROACHES TO PROBLEM 1

CONSIDER THE PROJECTION CASE.

WE HAVE DATA ALONG DIAMETERS

WE WANT DATA ON 2D GRID




## TWO APPROACHES

1) FOR EVERY GRID POINT INTERPOLATE BETWEEN NEAREST DATA POINTS

2) FOR EVERY DATA POINT, ADD CONTRIBUTION TO NEARBY GRID POINTS

APPROACH (1) WORKS WELL
DOESN'T USE ALL THE DATA

APPROACH (2) USES ALL THE DATA
REQUIRES COMPENSATION FOR DENSITY

# BASIC GRIDDING IDEA

DATA FALLS ON NON-CARTESIAN POINTS

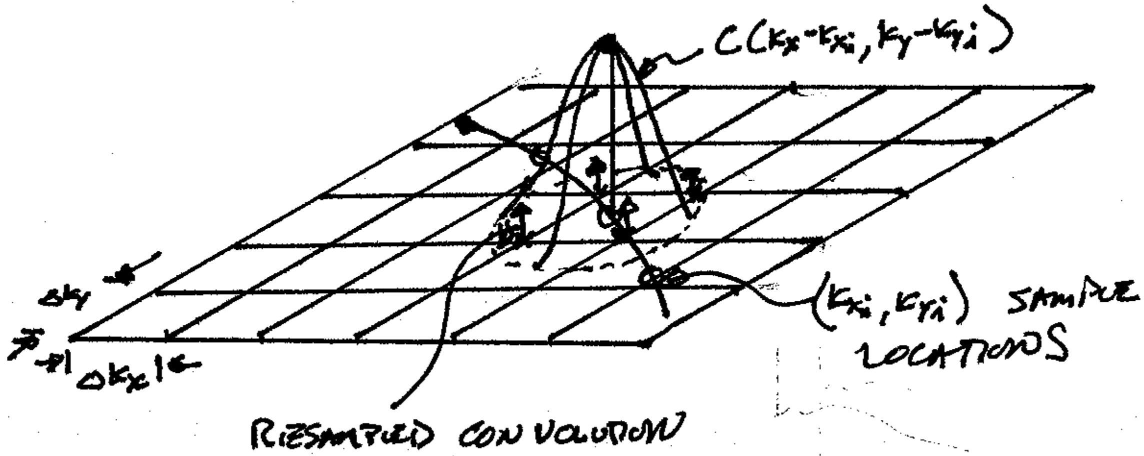
$(k_{x_i}, k_{y_i})$

CONVOLVE EACH POINT WITH A KERNEL

$C(k_x, k_y)$

RESAMPLE CONVOLUTION ON GRID POINTS

DFT TO RECONSTRUCT IMAGE



# SIMPLE VERSION

SAMPLING FUNCTION AT $(k_{x_i}, k_{y_i})$

$$S(k_x, k_y) = \sum_i^2 \delta(k_x - k_{x_i}, k_y - k_{y_i})$$

SAMPLED DATA, $m(k_x, k_y)$ CONTINUOUS FREQUENCY DOMAIN DATA

$$m(k_x, k_y)\, S(k_x, k_y)$$

CONVOLVE WITH KERNEL

$$[m(k_x, k_y) S(k_x, k_y)] * C(k_x, k_y)$$

SAMPLE ON 2D GRID

$$\hat{m}(k_x, k_y) = \left( [m(k_x, k_y) S(k_x, k_y)] * C(k_x, k_y) \right) Ш\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)$$

INVERSE TRANSFORM

$$\hat{m}(x, y) = \left[ (m(x,y) * s(x,y)) \cdot c(x,y) \right] * Ш\left(\frac{x}{FOV_x}, \frac{y}{FOV_y}\right)$$

WHERE

$m(x,y)$ IS IMAGE DATA

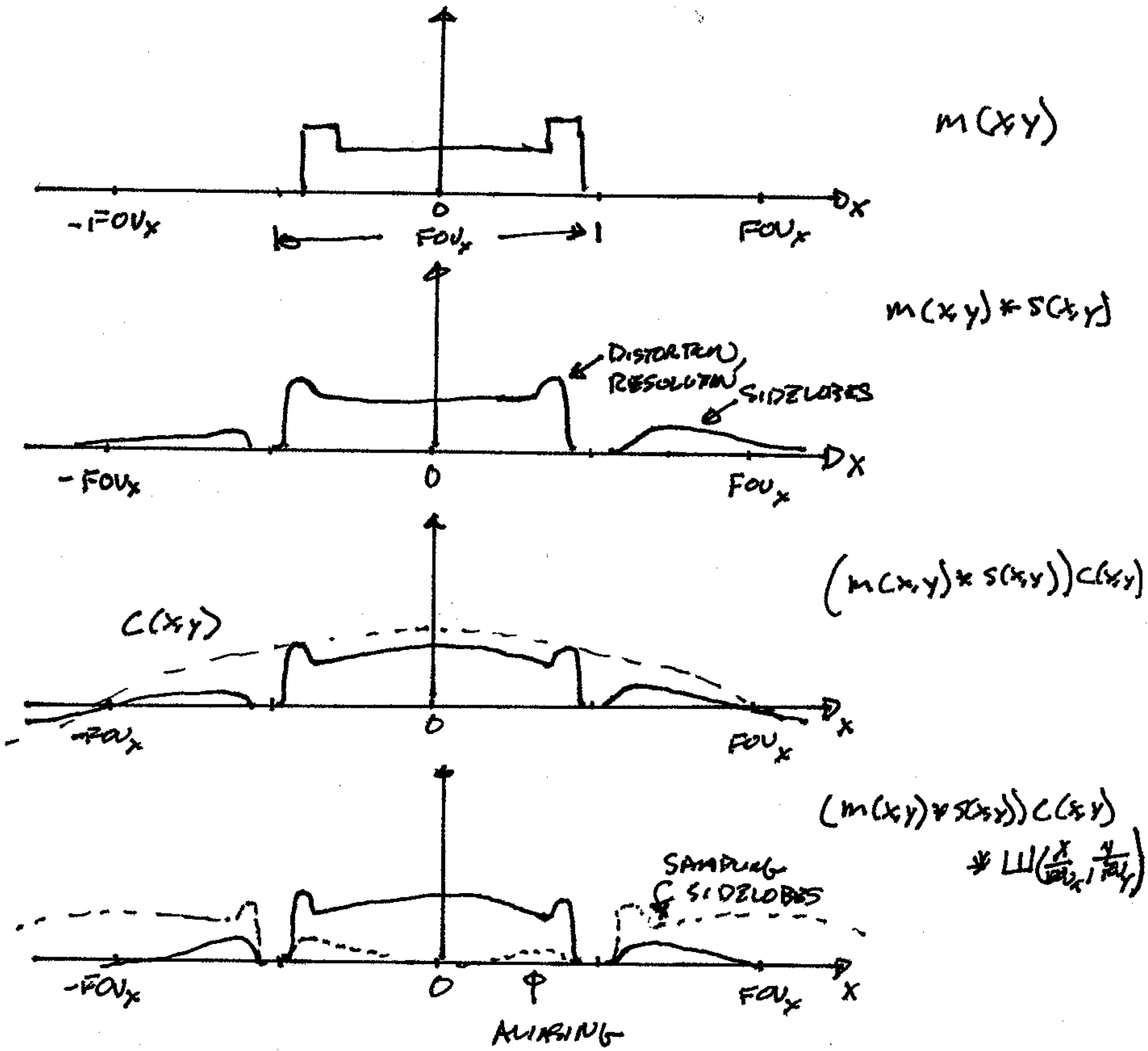
$s(x,y)$ IMPULSE RESPONSE OF SAMPLING PATTERN

$c(x,y)$ APODIZATION

$FOV_x, FOV_y$ ARE $\frac{1}{\Delta k_x}, \frac{1}{\Delta k_y}$

# GRAPHICAL INTERPRETATION

## 1D, IMAGE DOMAIN



## RESULT

THE RECONSTRUCTION $\hat{m}(x,y)$ IS

- BLURRED, DISTORTED
- ALIASED (DATA SAMPLING IN SPATIAL FREQUENCY)
- APODIZED
- ALIASED (GRID SAMPLING)

HOW CAN WE FIX THIS? IDEALLY

- NO ALIASING
- NO APODIZATION
- FAST

## DESIGN ISSUES

- CONVOLUTION KERNEL $C(k_x, k_y)$
  - APODIZATION
  - ALIASING
- GRID DENSITY $\Delta k_x, \Delta k_y$
  - ALIASING
  - APODIZATION
- SAMPLING PATTERN
  - IMPULSE RESPONSE
  - SIDELOBES, ALIASING
  - DENSITY