TODAY

COURSE OVERVIEW

EE469B

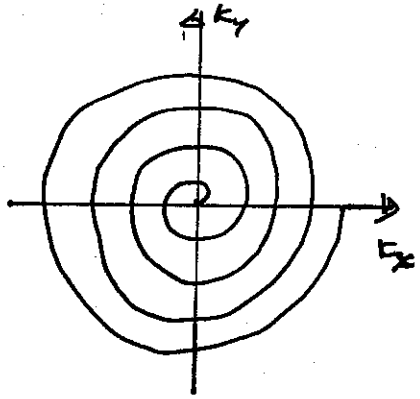
ASSIGNMENT

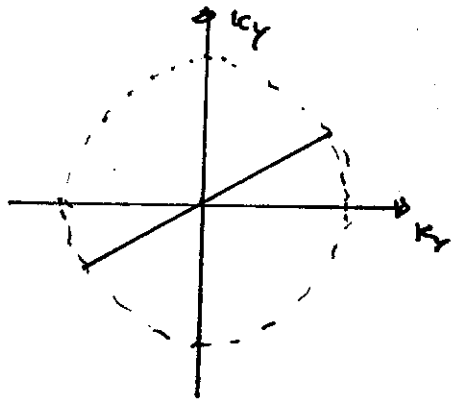
PROJECTS DUE NEXT FRIDAY

MAIN IDEAS FROM THE COURSE

INTERPOLATION

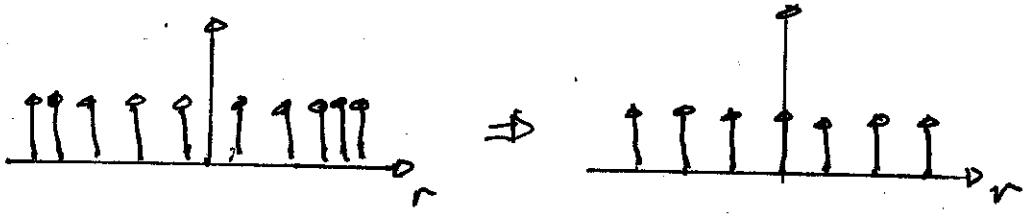
ONE SAMPLE GEOMETRY TO ANOTHER



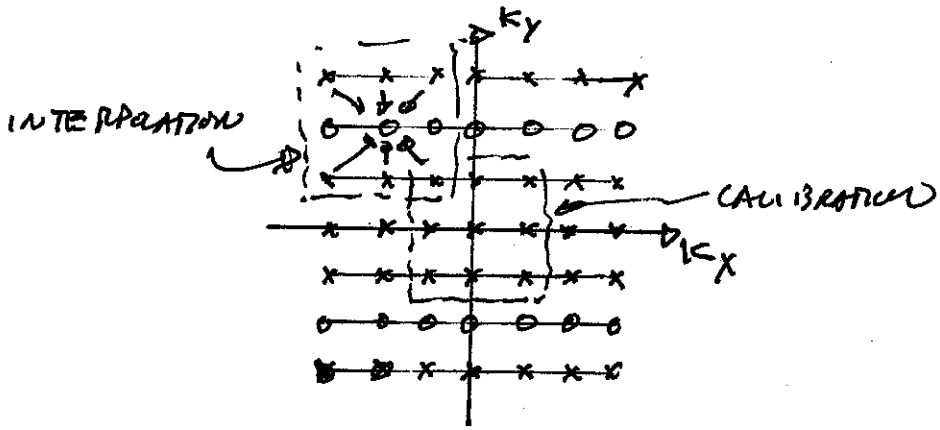


NON-CARTESIAN SAMPLES ⇒ CARTESIAN GRID

NON-UNIFORM SAMPLES ⇒ UNIFORM SAMPLES



FILLING IN MISSING DATA GRAPPA, SPIRiT



PARTICULARLY IMPORTANT PROBLEM

INTERPOLATION FOLLOWED BY A TRANSFORM

NON-UNIFORM DATA IN SPATIAL FREQUENCY

UNIFORM DATA IN IMAGE SPACE

GRIDDING, (DENSITY CORRECTION)

UNIFORM DATA IN IMAGE SPACE

NON-UNIFORM DATA IN SPATIAL FREQUENCY

INVERSE GRIDDING (NO DENSITY CORRECTION)

SYSTEM MODELS

ALL OF THE PROBLEMS WE LOOKED AT COULD BE MODELED AS A MATRIX EQUATION

$$y = Ax$$

- y: ACQUIRED DATA
- A: SYSTEM MATRIX
- x: DESIRED IMAGE

A WAS CALLED THE "PROJECTION OPERATOR" IN PET

ESSENTIALLY A PET SYSTEM SIMULATOR

IN MRI IT WAS CALLED THE ENCODING MATRIX

$$E = \begin{pmatrix} | & & | \\ e^{-j\vec{k}(t_1)\cdot\vec{x}_1} & \cdots & e^{-jk(t_n)\cdot\vec{x}_n} \\ | & & | \end{pmatrix}$$

EACH COLUMN IS COMPLEX EXPONENTIAL PRODUCED BY ONE VOXEL

FOR PARALLEL MRI, COIL WEIGHTING ALSO INCLUDED)

FOR MRI, APPLYING A (OR E) IS INVERSE GRIDDING

ANOTHER IMPORTANT OPERATOR

$$\tilde{x} = A^* y$$

$\tilde{x}$: RECONSTRUCTION; y: ACQUIRED DATA

ADJOINT OPERATOR, TAKES DATA AND DOES SOME SORT OF RECONSTRUCTION

IN PET, CT THIS IS "BACKPROJECTION OPERATOR"

IN MRI THIS IS GRIDDING (WITHOUT DENSITY COMPENSATION)

IN PRACTICE, A AND A^* CAN'T BE COMPUTED AND STORED

INSTEAD, IMPLEMENTED AS FUNCTIONS (M-FILES), CONTAIN

- INTERPOLATIONS
- TRANSFORMS
- INTEGRATIONS
- CONVOLUTIONS
- MULTIPLICATIONS

ALL SIMPLE, FAST OPERATIONS

GIVEN

$$y = Ax$$

WHERE y IS KNOWN, AND A (AND A^H) ARE FUNCTIONS

HOW DO WE SOLVE FOR x?

APPLY A^H TO BOTH SIDES

$$(A^H A)x = \underbrace{A^H y}_{\text{SIMPLE RECONSTRUCTION}}$$

AND SOLVE FOR x.

MANY SPECIAL CASES

2DFT MRI

IN THIS CASE A IS FOURIER TRANSFORM MATRIX

AND

$$\underbrace{(A^H A)}_{I} x = \underbrace{A^H y}_{\text{INVERSE FT}}$$

$$x = A^H y$$

NON-CARTESIAN MRI

IN THIS CASE WE CHOOSE TO SOLVE A DIFFERENT PROBLEM

$$W y = W A x$$

W IS A DIAGONAL WEIGHTING, OR PRECONDITIONING MATRIX

THEN

$$A^* W y = (A^* W A) x$$

$$\underbrace{(A^* W A)}_{I} x = A^* W y$$

x: IMAGE; y: K-SPACE

CHOOSE W SO THAT $A^* W A \approx I$

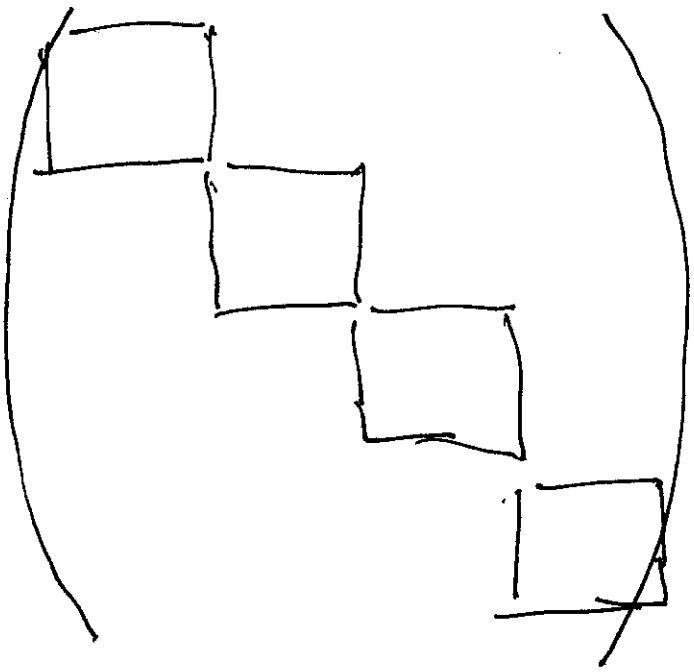
$$x = A^{\#} W y$$

$A^{\#}$: GRIDDING; W: DENSITY COMPENSATION

THIS IS SAME AS WEIGHTED LEAST SQUARES RECONSTRUCTION

CARTESIAN SENSE MRI

HERE A^*A HAS A SPECIAL STRUCTURE (AFTER REORDERING)



A^*A

$$\underset{\text{IMAGE DATA}}{x} = \underset{\text{ALIASED RECONSTRUCTION}}{A^* y}$$

FOR ACCELERATION R, EACH BLOCK IS RXR

SOLVE EACH SUBEQUATION EXPLICITLY

$$x = (A^*A)^{-1} A^* y$$

NON-CARTESIAN PARALLEL MRI

HERE THERE IS NO SPECIAL STRUCTURE TO EXPLOIT

ITERATIVELY SOLVE

$$(A^* A)\, x = A^* y$$

USING CONJUGATE GRADIENT ALGORITHM, OR SOMETHING ELSE

MATLAB LSQR DOES THIS. YOU PROVIDE FUNCTIONS

THAT IMPLEMENT A, A^*

YOU CAN ADD PRECONDITIONING (DENSITY COMPENSATION)

$$(A^* W A)\, x = A^* W y$$

BUT YOU DON'T NEED TO.

SAME APPROACH WORKS FOR SPIRiT, PRUNO,

LOTS OF OTHERS.

PET

HERE

$$\underset{\text{EVENTS}}{y} = A \underset{\text{SOURCE DISTRIBUTION}}{x}$$

A IS A PET SIMULATION, A^* IS BACKPROJECTION

STATISTICS OF y ARE POISSON

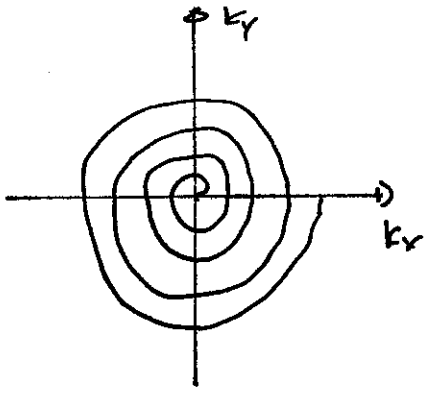
ITERATIVE SOLUTION THAT MAXIMIZES LIKELYHOOD (ML-EM)

PROJECTION AND BACKPROJECTION OF RATIOS

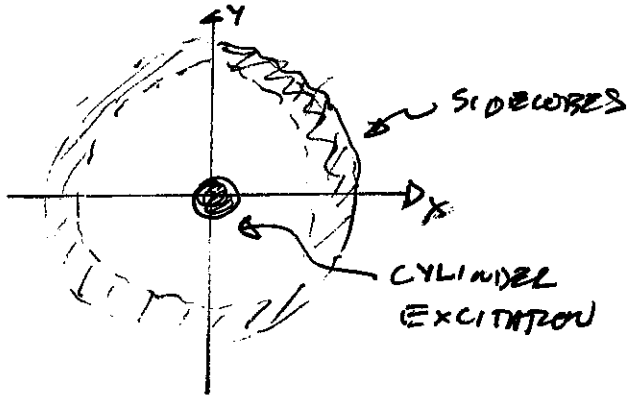
EE4693 RF PULSE DESIGN FOR MRI

ALL THE SAME IDEAS, NEW APPLICATIONS

NON-CARTESIAN MRI ⇒ MULTIDIMENSIONAL RF PULSES



SPIRAL k-SPACE TRAJECTORY + RF



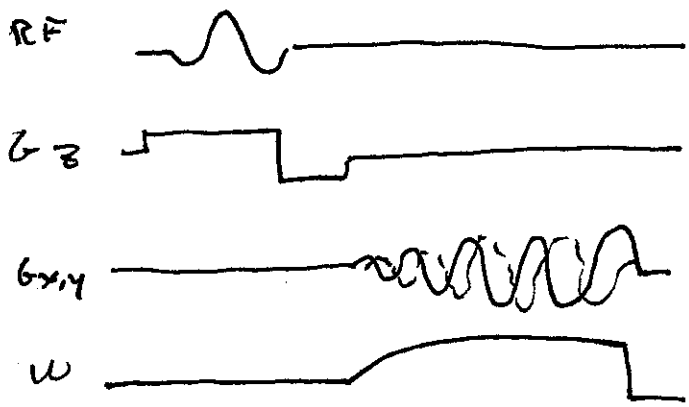
PARALLEL MRI ⇒ PARALLEL TRANSMIT

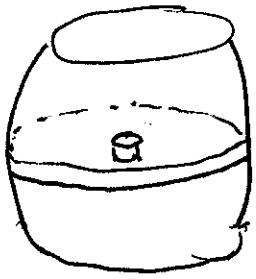
TAKE ANY PULSE SEQUENCE AND REVERSE IT

USE DENSITY COMPENSATION AS RF

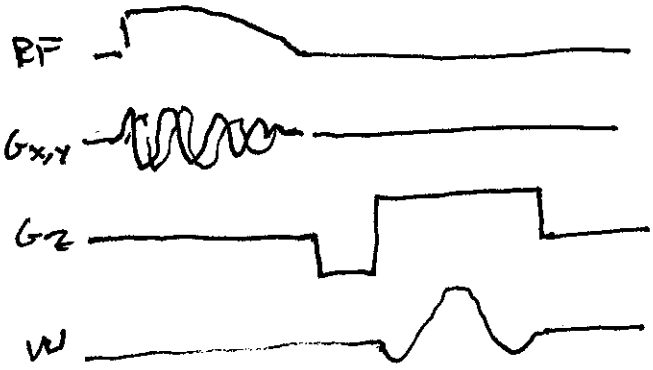
USE RF AS DENSITY COMPENSATION

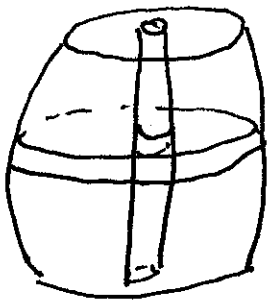
THIS WILL DO SOMETHING USEFULL





EXCITES SLICE, IMAGES WITH SPIRAL





EXCITES CYLINDER READS OUT ALONG CYLINDER

MR M-MODE

ADDED BONUS

MANY OF THE PROCESSES ARE NON-LINEAR (ROTATIONS) BUT HAVE EXPLICIT SOLUTIONS!