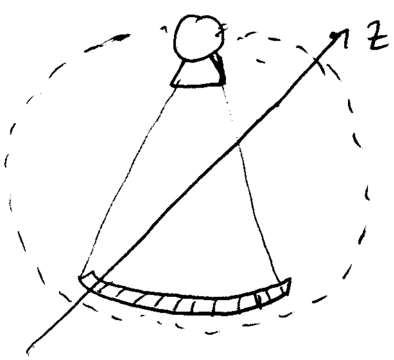
3D CT

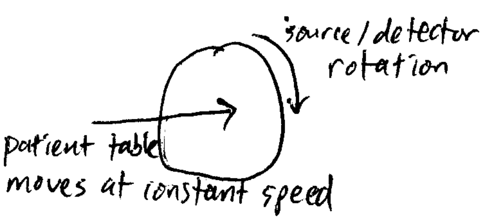
Helical CT
Cone Beam CT
3D Radon Space



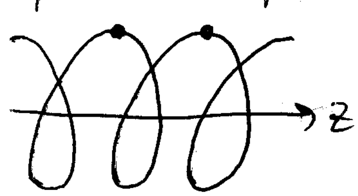
Single row, one rotation = one axial slice

Step + shoot: one slice at a time
translate patient (in z) between slices
need to unwind cables

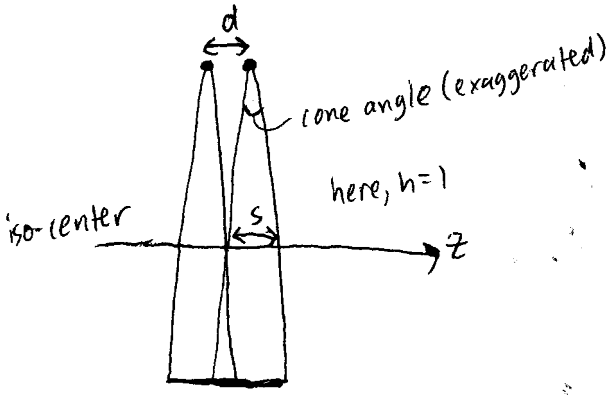
Slip ring: continuous rotation
transfers data + high voltage power
enables continuous data collection
as patient table translates in z



effective source path relative to patient is a spiral



Spiral / Helical CT (~1990)



projection q_j, q_{j+R}, $R = \#$ proj/rot

pitch $h = d/s = vt/s$

table velocity

rotation time

coverage at iso-center

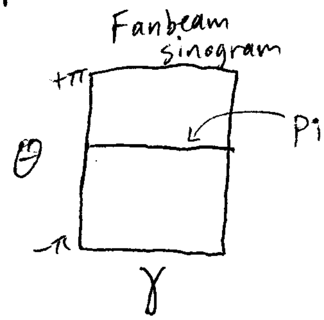
typical h is 1–1.5

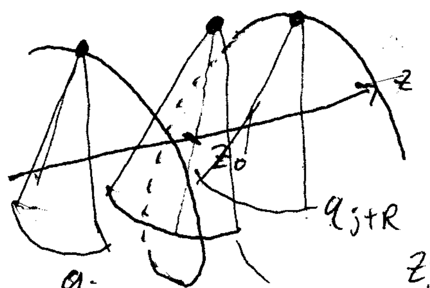
higher pitch = faster translation, less coverage/vol

pitch determines artifact levels in helical CT

Suppose we want an axial image at $z = z_0$.

Simple approach: linear interpolation of sinogram (fan-beam) at $z = z_0$ (360° LI)





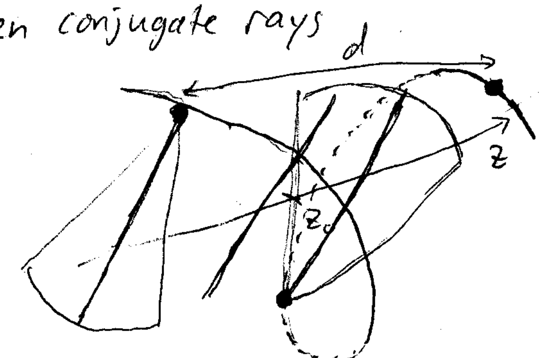
$$p_i \approx \frac{z_{j+R} - z_0}{d} q_j + \frac{z_0 - z_j}{d} q_{j+R}$$

Works pretty well, lose resolution in z

Biggest source of error? Inconsistencies at beginning/end of sinogram. Due to motion, object variation in z

Soln: Underscan — downweight beginning/end since only 180° + fan angle is needed

180° LI: Use linear interpolation between conjugate rays



Multi-slice CT

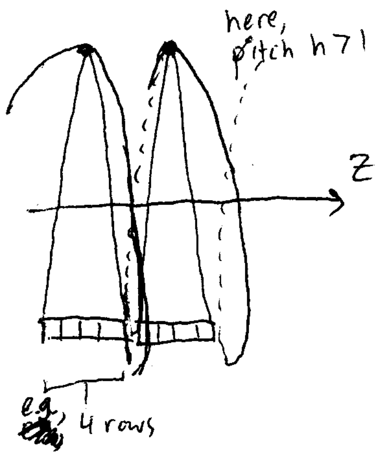
Additional rows of detectors (slices) allow:

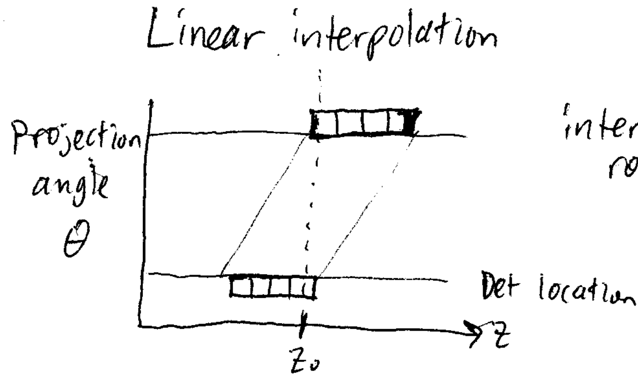
- More coverage (in z) per rotation
- Faster scans (reduces patient motion, better monitor contrast uptake)
- Better tube utilization.

History: 2, 4, 8, ..., 128, 320 rows
↖ 20 cm coverage, entire heart in < 0.5 s rot

Cons: Increased scatter, cone beam artifacts

multislice CT vs CBCT
2, 4, 16, 64, ... flat panel, large area detector

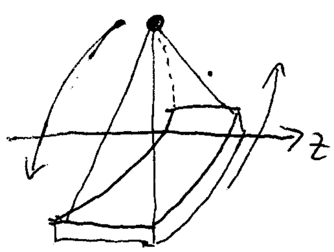




interpolate between rows

What happens as cone angle gets larger?

Cone Beam CT (CBCT)



For now, assume circular path (don't translate table)

Can we get volumetric (3D) images?

320 slices can image entire heart/brain (20 cm in z) in < 0.5 s rot.

Flat panel detector used in fluoroscopy (real-time x-ray images for interventional/image guided procedures)
- Can do C-arm CT

Most common recon is FDK (Feldkamp-Davis-Kress), approximate recon

FDK is natural extension of fanbeam FBP:

1) Density correct by $\underset{\text{fan angle}}{\cos\gamma} \cdot \underset{\text{cone beam angle}}{\cos\tau} = \underset{\text{overall angle}}{\cos\xi}$

2) Rho filter each row separately

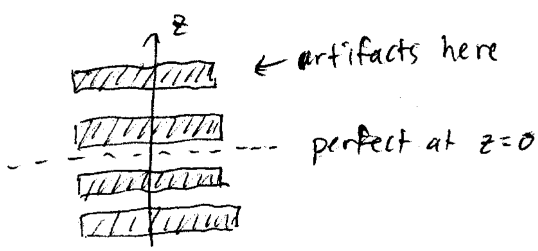
3) Backproject along cone (3D), weighting by $(D/d)^2$ with depth

4) Repeat for each projection

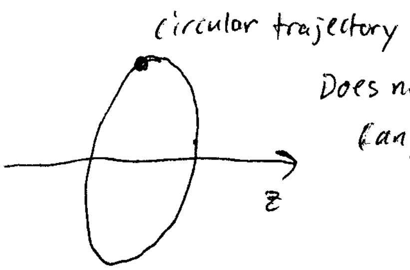
Works quite well, simple to implement, exact for $z=0$ axial plane

Suffers from "cone-beam artifacts" at large z/cone angles, especially if there are rapid variations in z

Example:

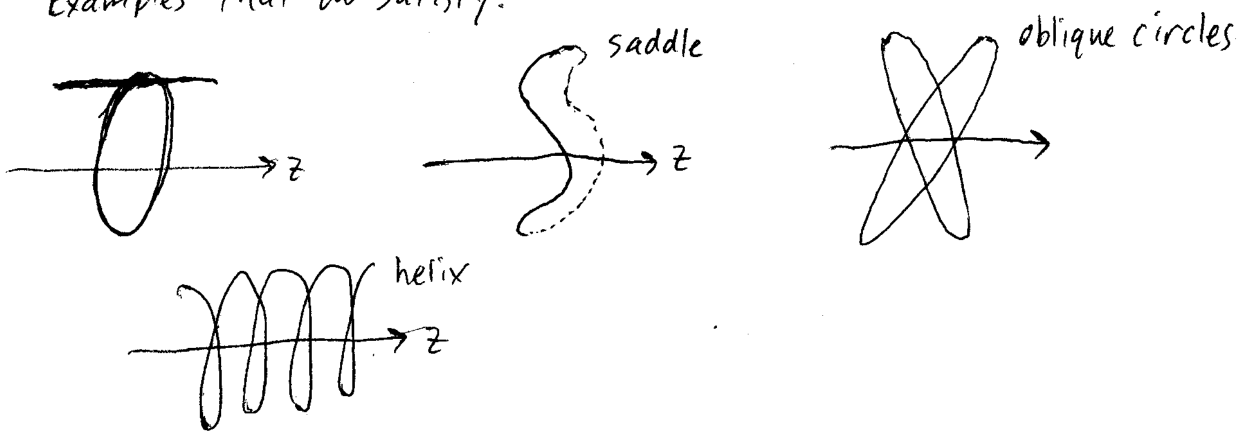


Why? Tuy's condition for exact recon: every plane through the object must intersect the source trajectory at least once



Does not satisfy Tuy's condition (any axial plane, $z \neq 0$)

Examples that do satisfy:

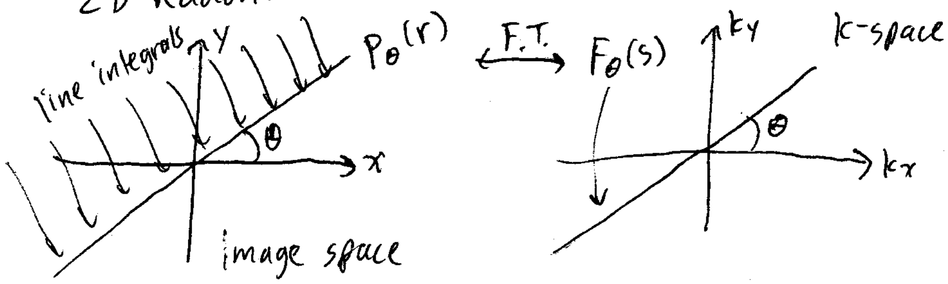


3D Radon Space

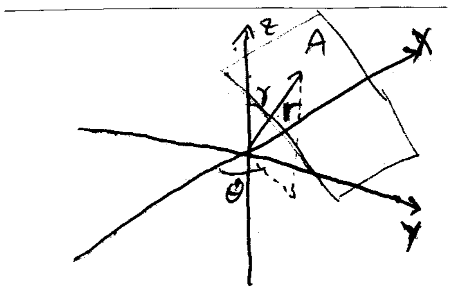
More advanced / exact reconstruction algorithms often convert measured data to 3D Radon space, then do an inverse Radon transform to recover image

Measured helical/cone beam $\rightarrow$ 3D Radon $\rightarrow$ 3D image
(or some variant)

2D Radon:



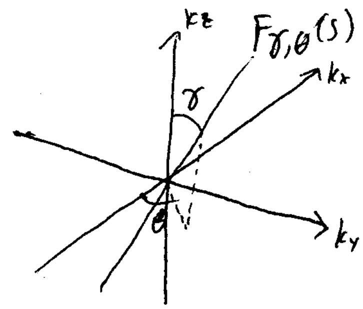
3D Radon:



Consider surface A (plane)
normal vector $n_{\gamma,\theta}$
distance r to origin

Let $p_{\gamma,\theta}(r)$ be plane integral

Then $p_{\gamma,\theta}(r) \xleftrightarrow{F.T.} F_{\gamma,\theta}(s)$



Given all such plane integrals $p_{\gamma,\theta}(r)$, we can recover image via k-space

Also, expression for inverse 3D recon:

$$f(x,y,z) = -\frac{1}{8\pi^2} \iint_S \frac{\partial^2 p_{\gamma,\theta}(r)}{\partial r^2}\, dS$$

integral over unit sphere S

In practice, neither approach is used — $p_{\gamma,\theta}(r)$ not measured directly
But it tells us that only all plane integrals are needed, a weaker condition than needing all line integrals

Instead, consider method by Grangeat (1991):

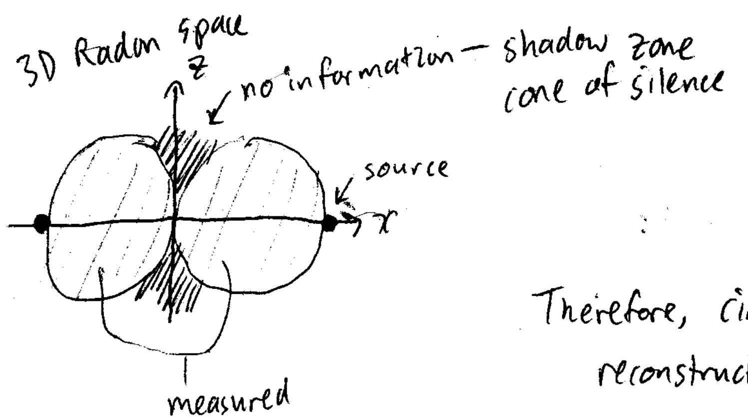
1) Cone beam data $\rightarrow \frac{\partial}{\partial r} p_{\gamma,\theta}(r)$ (deriv of Radon):
Involves partial derivatives, line integrals, and weighting on detector plane

2) $\frac{\partial}{\partial r} p_{\gamma,\theta}(r) \rightarrow$ reconstructed 3D object ; Filtered backprojection operations

Cone Beam Data Incompleteness:

Circular trajectory doesn't acquire all data in 3D Radon space

Points that are measured lie within a torus (x-z plane shown below)



Therefore, circular cone beam cannot be reconstructed exactly, e.g. cone-beam artifacts