

FAN BEAM PROJECTION RECONSTRUCTION

REBINNING

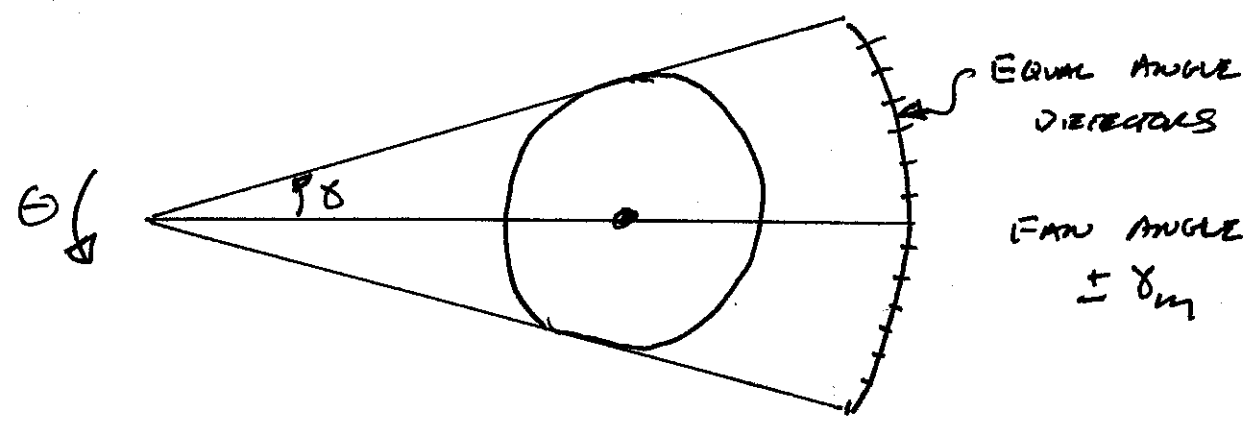
FAN BEAM FILTERED BACKPROJECTION

READING

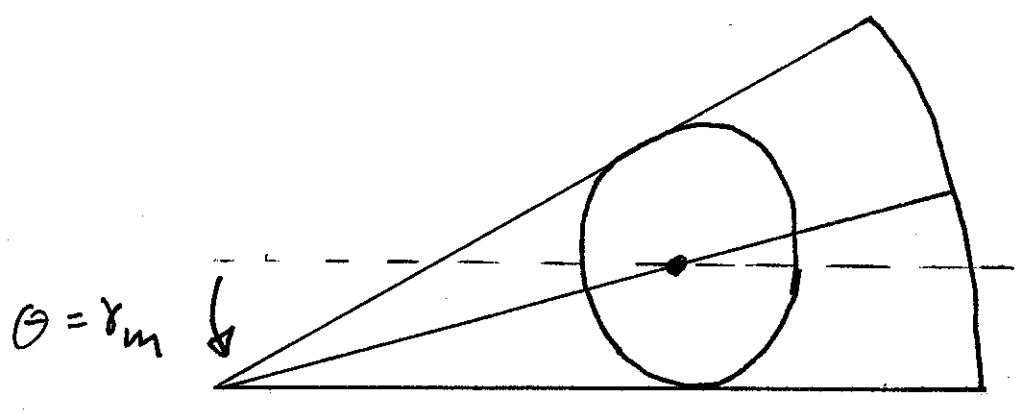
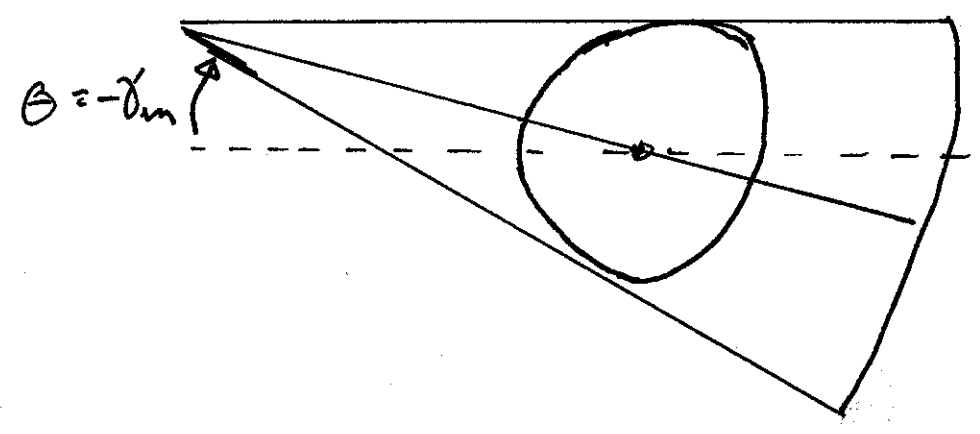
KAK + SLANBY 3.1 - 3.4

FAN BEAM GEOMETRY

EQUAL ANGLE CASE



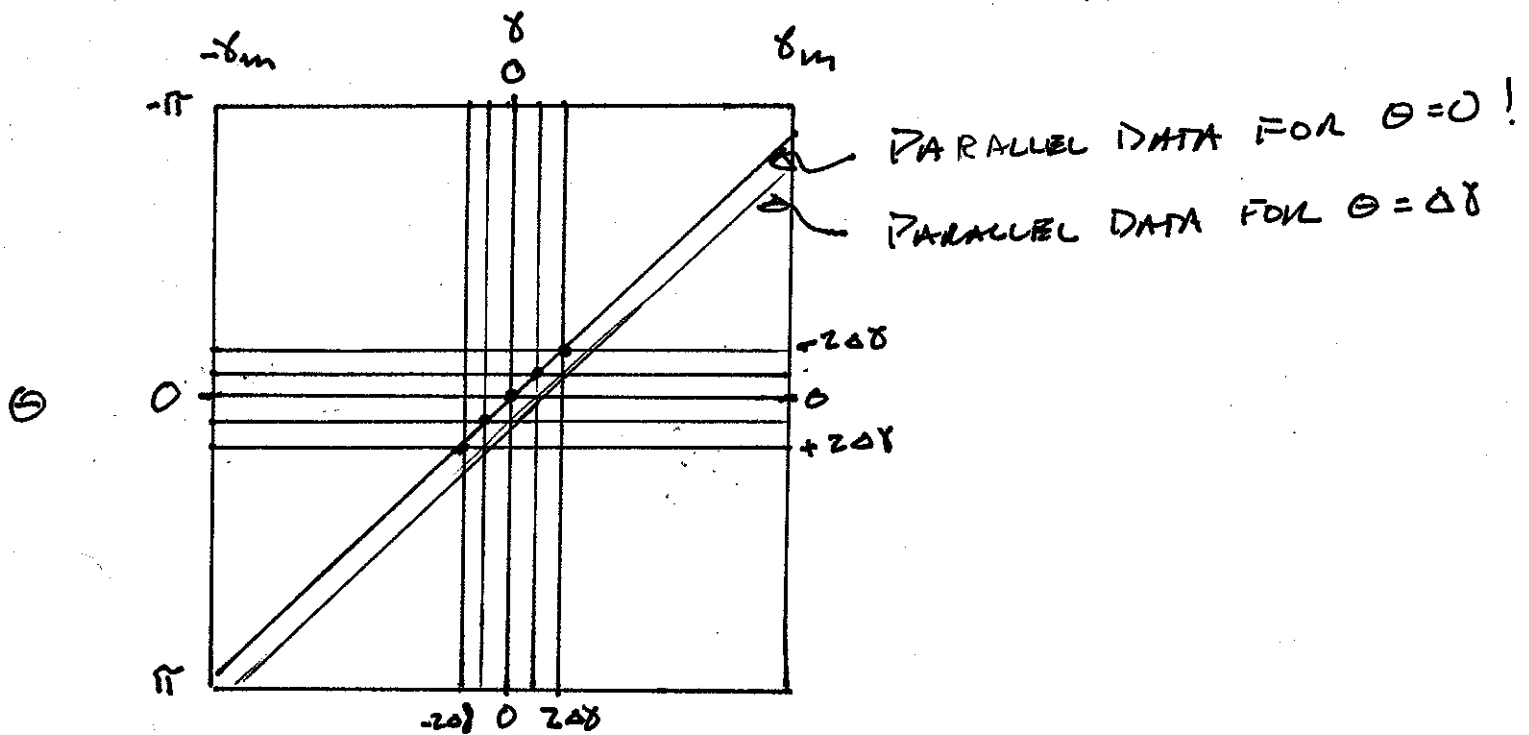
REBINNING



FOR EVERY $\Delta\theta$ FROM $-\delta_m$ TO δ_m , ANOTHER RAY BECOMES PARALLEL TO THE $\theta=0, \gamma=0$ RAY

WE CAN PLOT THIS ON A SINOGRAM

ASSUME $\Delta\theta = \Delta\delta$



WE CAN CREATE PARALLEL PROJECTIONS BY SAMPLING THE SINOGRAM ALONG DIAGONAL LINES

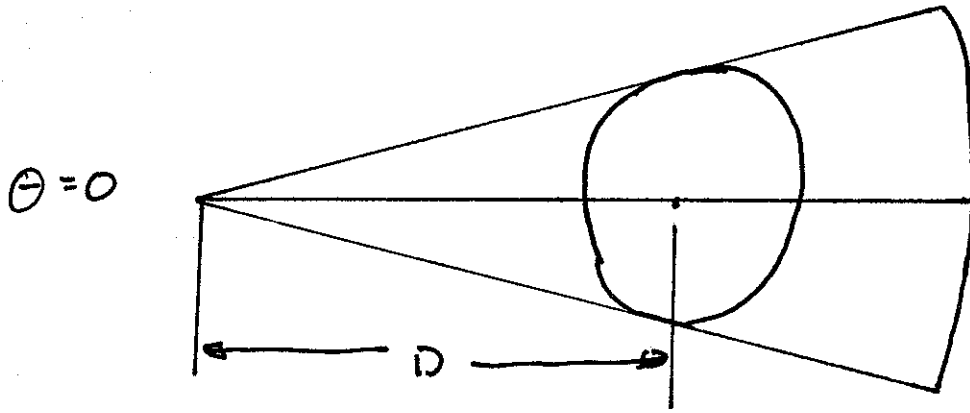
EASY IF $\Delta\theta = \Delta\delta$, OR MULTIPLES

REQUIRES 2D INTERPOLATION IN GENERAL.

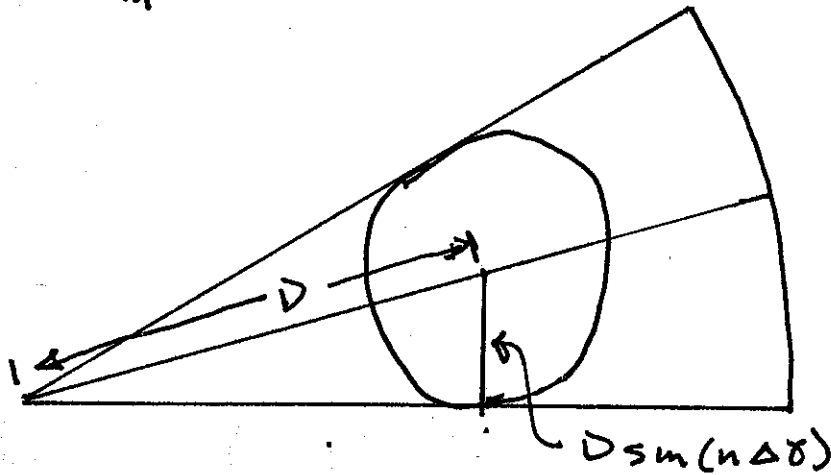
PROBLEM

(4)

ALTHOUGH THE REBINNED DATA IS PARALLEL,
IT IS NOT UNIFORMLY SPACED!

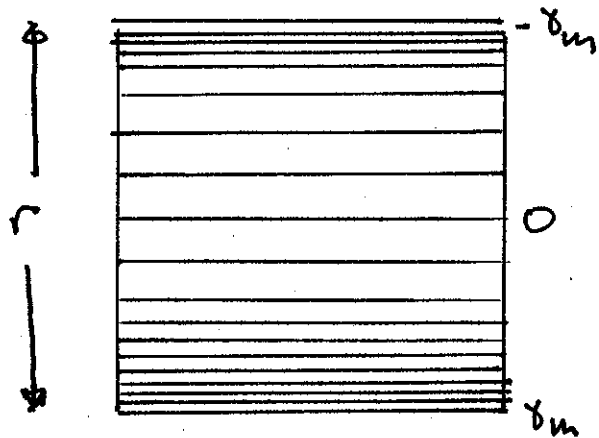


$\Theta = n \Delta \delta = \delta_m$



$\sin(n \Delta \delta) < n \Delta \delta$

PROJECTIONS GET CLOSER AS ANGLE INCREASES



RAY SPACING FOR $\theta = 0$

$$r_n = D \sin(\theta_n) = D \sin(n \Delta \theta)$$

RESAMPLE TO GET UNIFORM SPACING

ALSO A DENSITY CORRECTION ISSUE, IMPORTANT FOR FAN BEAM FILTERED BACKPROJECTION

SUMMARY OF REBINNING

- 1) RESAMPLE SINOGRAM ALONG DIAGONAL LINES TO GENERATE PARALLEL BEAM DATA
- 2) RESAMPLE PARALLEL PROJECTIONS TO GET UNIFORMLY SAMPLED DATA
- 3) RHO FILTER
- 4) BACKPROJECT

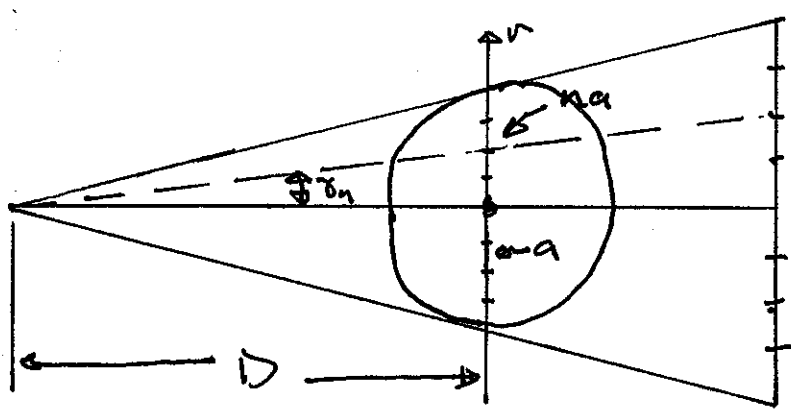
FAN BEAM FILTERED BACKPROJECTION

FILTERED BACKPROJECTION IS SIMPLE FOR THE PARALLEL BEAM CASE

CAN WE FIX IT UP FOR FAN BEAM DATA?

CONSTANT DETECTOR SPACING CASE

(CONSTANT ANGULAR SPACING SIMILAR)



DETECTOR SPACING AT CENTER OF FOU IS 'a'

WE WANT TO BE ABLE TO BACKPROJECT ALONG THE FAN

TWO ISSUES

- 1) DENSITY CORRECTION
- 2) RHO FILTER

DENSITY CORRECTION

JUST AS IN REBINNING, WE HAVE TOO MANY RAYS AT THE EDGE OF FOV

THE OFFSET r_n OF THE n^{th} RAY IN THE PROJECTION IS

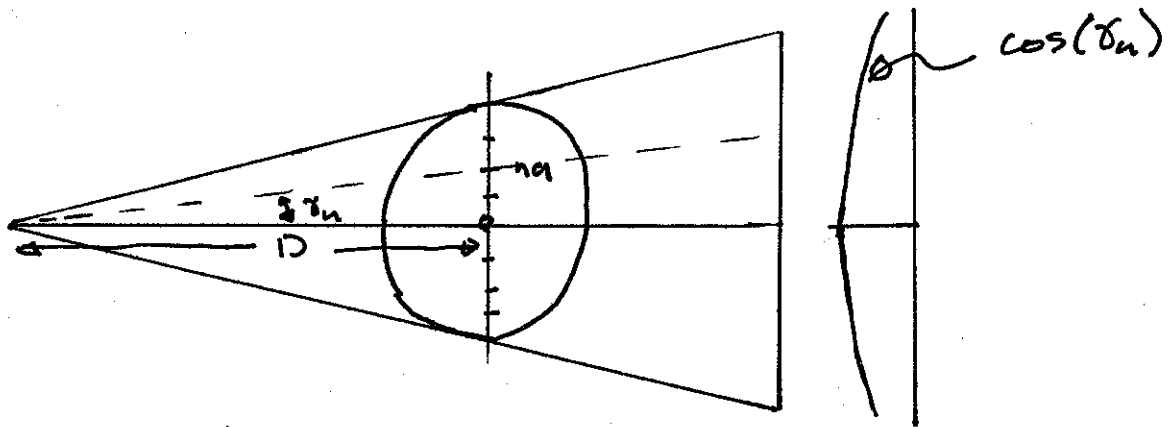
$$r_n = D \sin(\delta_n) \quad (\text{NAD FOR EQUAL ANGLES})$$

THE SPACING (AREA) IS THE DERIVATIVE

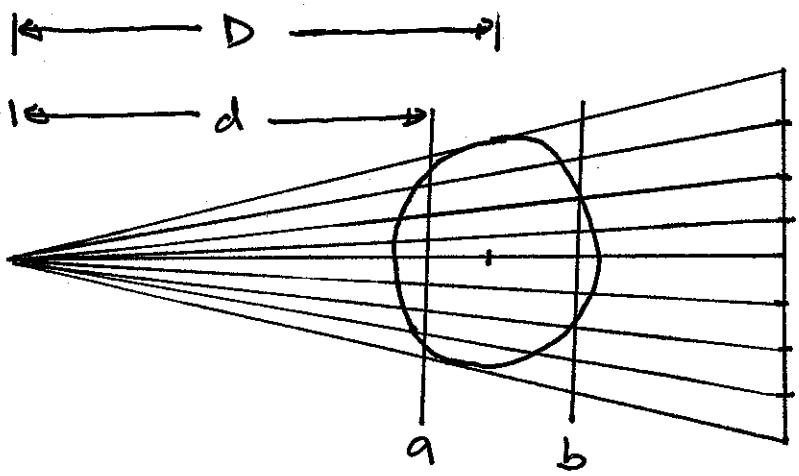
$$r'_n = D \cos(\delta_n)$$

THIS IS THE DENSITY COMPENSATION FUNCTION FOR EQUAL DETECTOR SPACING

$$\cos(\delta_n) = \frac{D}{\sqrt{D^2 + (n\Delta)^2}}$$



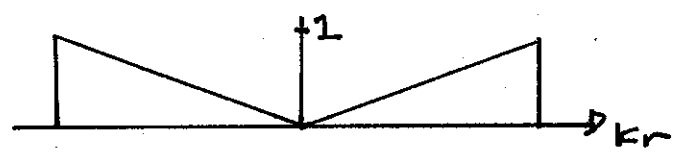
RHO FILTER



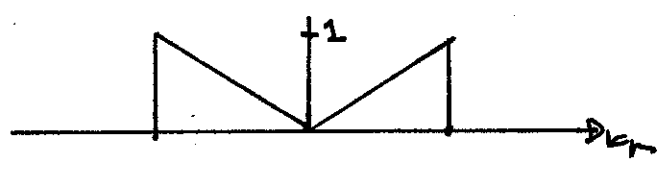
WE WANT THE SAME RHO FILTER AT "a" AND "b"
IN CYCLES/CM

SAMPLING DENSITY IS HIGHER AT "a"

ASSUME WE RHO FILTERED THIS PROJECTION, WHAT
 WOULD IT LOOK LIKE AT a AND b?



AT "a"



AT "b"

WE CAN MAKE THESE LOOK THE SAME OVER THE
 REGION THAT b COVERS BY SCALING THE AMPLITUDE
 OF a.

TO FIND THE SCALING FACTOR, SOLVE FOR RHO FILTER

$$h(r) = \int_{-\infty}^{\infty} |k_r| e^{j2\pi k_r (r \frac{d}{D})} dk_r$$

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 SCALED  
 PROJECTION  
 SIZE at d

DEFINE

$$\hat{k}_r = \frac{d}{D} k_r$$

THEN

$$h(r) = \int_{-\infty}^{\infty} \left| \frac{\hat{k}_r}{d/D} \right| e^{j2\pi \hat{k}_r r} \frac{d\hat{k}_r}{(d/D)}$$

$$= \frac{1}{(d/D)^2} \int_{-\infty}^{\infty} |\hat{k}_r| e^{j2\pi \hat{k}_r r} d\hat{k}_r$$

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 SCALING RHO FILTER
 DEPENDS NO "d" DEPENDENCE
 ON "d"

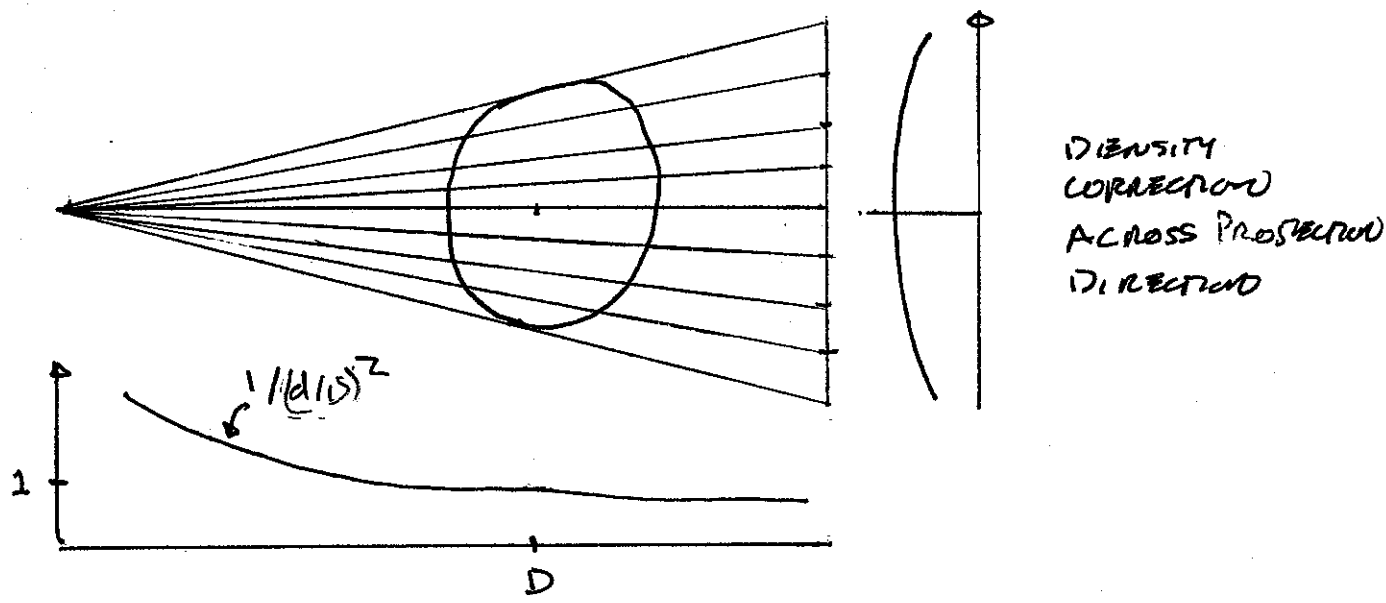
EACH DEPTH HAS ITS OWN RHO FILTER

IMPLEMENT BY

- 1) RHO FILTER AT d=D, SCALING IS 1
- 2) SCALE BY $\frac{1}{(d/D)^2}$ FOR OTHER DEPTHS

THIS IS A WEIGHTING ALONG THE BEAM.

BOTH DENSITY CORRECTION AND RHO FILTER CORRECTION ARE PERFORMED BY WEIGHTING



RHO FILTER CORRECTION ALONG PROJECTION DIRECTION

FAN BEAM FILTERED BACKPROJECTION

- 1) DENSITY CORRECT
- 2) RHO FILTER
- 3) BACKPROJECT ALONG FAN, WEIGHTING BY $1/(d/d_0)^2$ WITH DEPTH
- 4) REPEAT FOR EACH PROJECTION