LECTURE 13

PROJECTION RECONSTRUCTION

PARALLEL BEAM

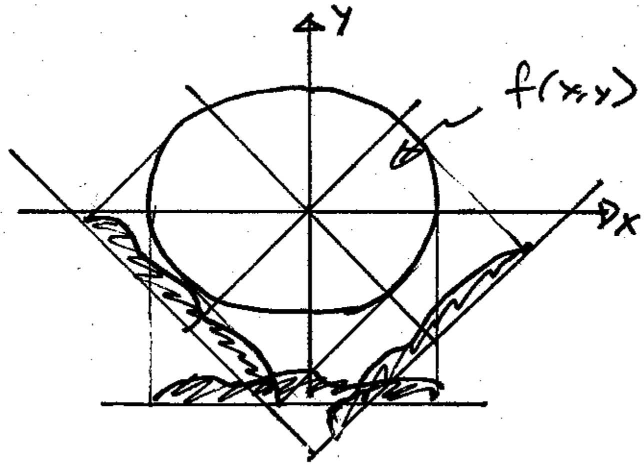
FAN BEAM

BACKPROJECTION

REBINNING

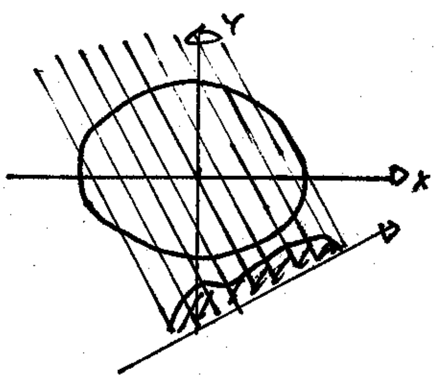
PROJECTION RECONSTRUCTION PROBLEM

GIVEN AN OBJECT f(x,y) WE COLLECT PROJECTIONS

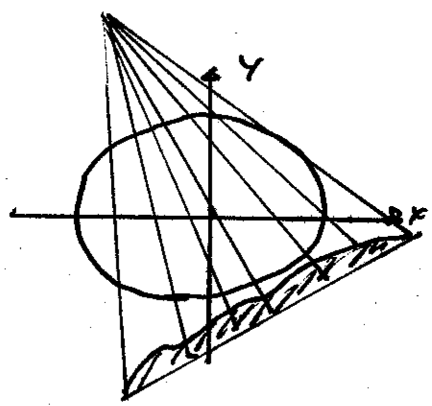


HOW DO WE RECONSTRUCT f(x,y) FROM PROJECTIONS?

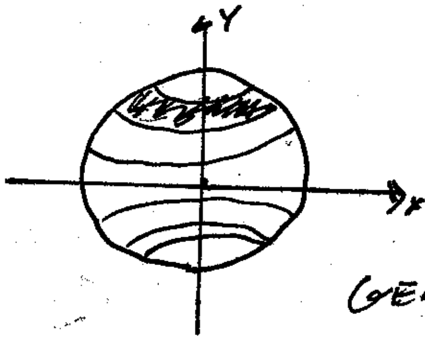
MANY SPECIAL CASES



PARALLEL BEAM



FAN BEAM



GENERAL

MANY MODALITIES

X-RAY

PET

SPECT

MRI

OPTICAL

ULTRASOUND

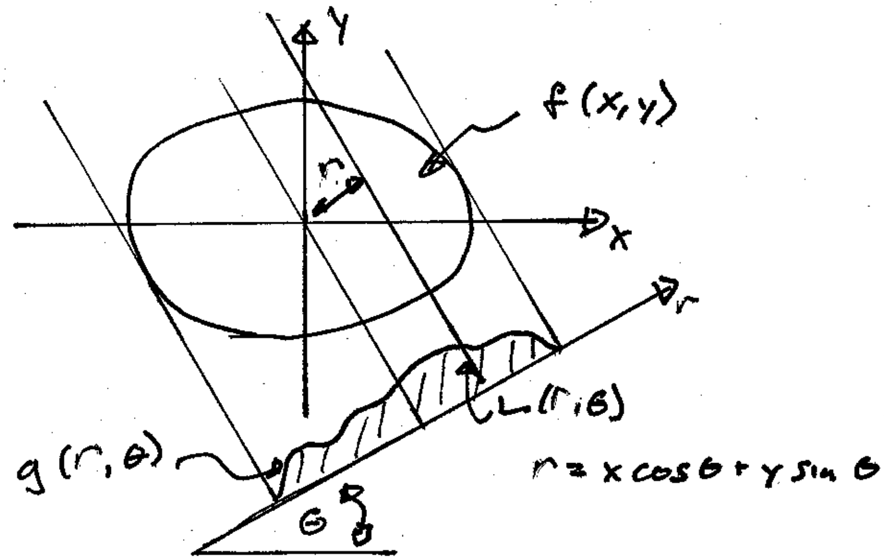
RECONSTRUCTION APPROACHES

PARALLEL BEAM - CENTRAL SECTION THEOREM, BACKPROJECTION

CONVERT TO PARALLEL BEAM

BACKPROJECT ACCORDING TO GEOMETRY

PARALLEL PROJECTION DATA



THE PROJECTION DATA AT AN ANGLE θ IS

$$g(r,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - r)\,dx\,dy$$

FOR X-RAY CT

$f(x,y) = \mu(x,y;\bar{E})$ ATTENUATION AT EFFECTIVE ENERGY $\bar{E}$

$g(r,\theta) = -\log\left(\frac{I_d}{I_0}\right)$ NEGATIVE LOG OF DETECTED INTENSITY OVER SOURCE INTENSITY

HOW DO WE FIND $f(x,y)$ FROM $g(r,\theta)$?

TAKE 1D FOURIER TRANSFORM OF PROJECTION

$$\tilde{\mathcal{F}}_{1D}\{g(r,\theta)\} = \int_{-\infty}^{\infty} g(r,\theta)\, e^{-i2\pi k_r r}\, dr$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - r)\,dx\,dy\right] e^{-i2\pi k_r r}\, dr$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \underbrace{\left[\int_{-\infty}^{\infty}\delta(x\cos\theta + y\sin\theta - r)\, e^{-i2\pi k_r r}\, dr\right]}_{e^{-i2\pi k_r(x\cos\theta + y\sin\theta)}} dx\,dy$$

$$= \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-i2\pi(x k_r\cos\theta + y k_r\sin\theta)}\, dx\,dy}_{F(k_x,k_y)\big|_{\substack{k_x = k_r\cos\theta \\ k_y = k_r\sin\theta}}}$$

$$= F(k_r\cos\theta,\, k_r\sin\theta)$$

PROJECTION AT AN ANGLE θ CORRESPONDS TO A DIAMETER IN SPATIAL FREQUENCY

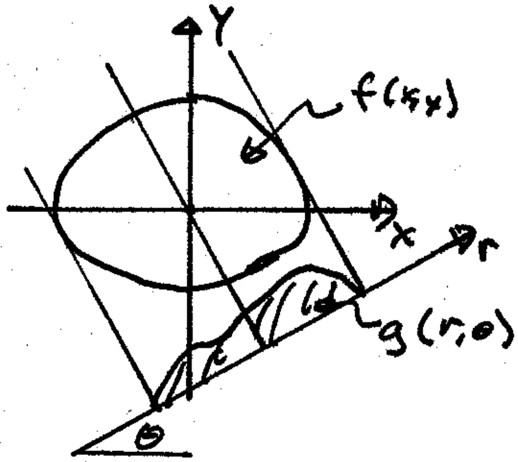
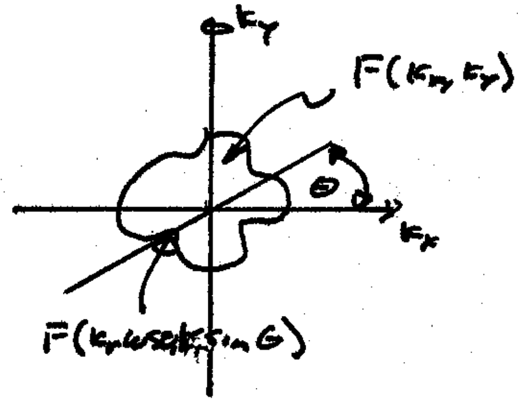
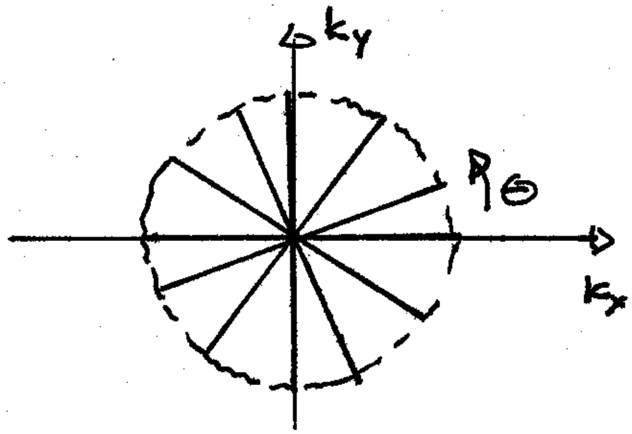


IMAGE SPACE



K SPACE

REPEAT AT MANY ANGLES



COVERS DISK IN SPATIAL FREQUENCY

FOURIER RECONSTRUCTION

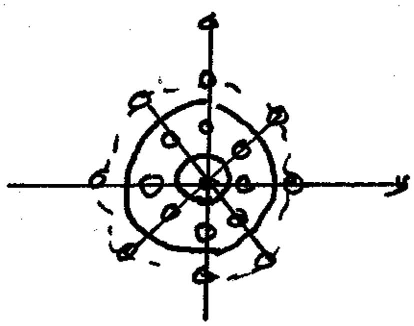
TRANSFORM PROJECTIONS

GRIDDING INTERPOLATION

INVERSE TRANSFORM

DENSITY CORRECTION

PREVIOUSLY, WE SHOWED THAT WE CAN ANALYTICALLY COMPUTE THE DENSITY



N PROJECTIONS, Δk_r SAMPLING

CENTRAL DISK

N SAMPLES

$\left(\frac{\Delta k_r}{2}\right)^2 \pi$ AREA

RING n

$2N$ SAMPLES

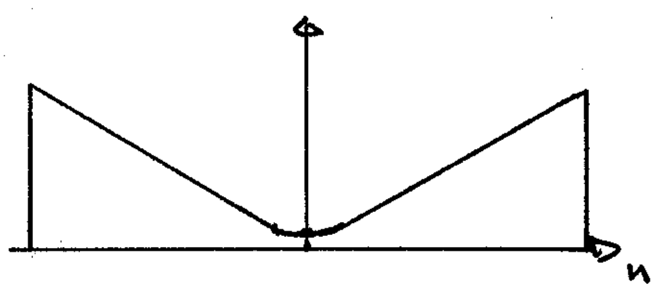
$(\Delta k_r (n+1/2))^2 \pi - (\Delta k_r (n-1/2))^2 \pi$

AREA PER SAMPLE IS a_n

$$a_0 = \frac{(\Delta k_r)^2 \pi}{N} \frac{1}{4} \qquad H(k_r) = (a_0, a_1, \ldots a_n \ldots) = \frac{(\Delta k_r)^2 \pi}{N} \left(\frac{1}{4}, 1, 2, \ldots n \ldots\right)$$

$$a_n = \frac{(\Delta k_r)^2 \pi}{N} n$$

"RHO" FILTER

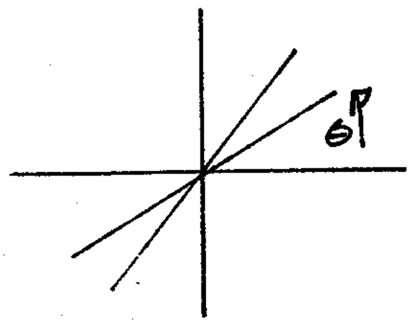


IMPORTANT THAT D.C. NOT BE ZERO

OTHER DERIVATIONS GIVE SLIGHTLY DIFFERENT ANSWERS

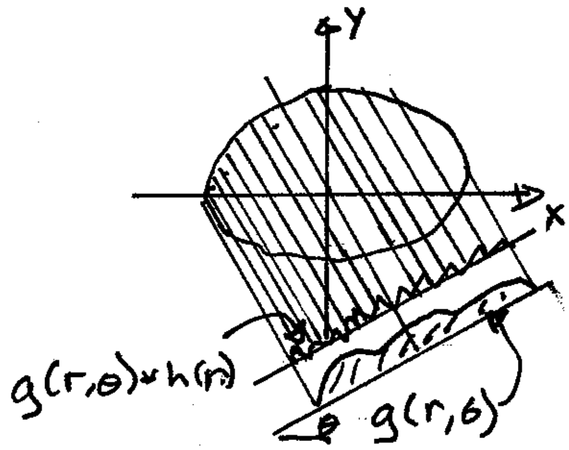
MORE COMMON APPROACH   IMAGE DOMAIN

SUMMARY OF FOURIER DOMAIN APPROACH



1) TRANSFORM PROJECTIONS
2) DENSITY CORRECT (MULTIPLY BY $H(k_r)$)
3) ADD TO GRID
4) INVERSE TRANSFORM

IN THE IMAGE DOMAIN THIS CORRESPONDS TO



1) NOTHING
2) CONVOLVE WITH $h(r) = \mathcal{F}_{1D}^{-1}\{H(k_r)\}$
3) SPREAD FILTERED PROJECTION ACROSS IMAGE SPACE "BACKPROJECTION"
4) NOTHING

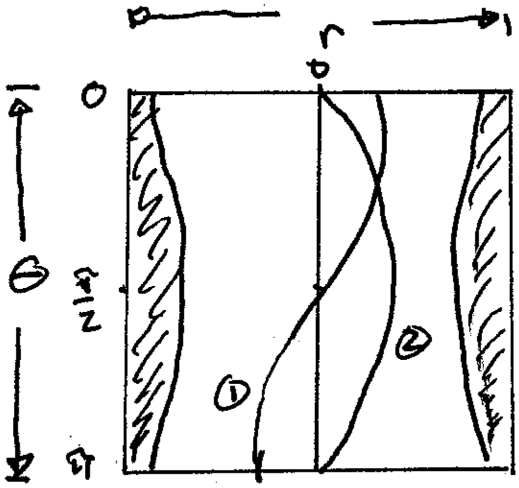
THIS IS CALLED

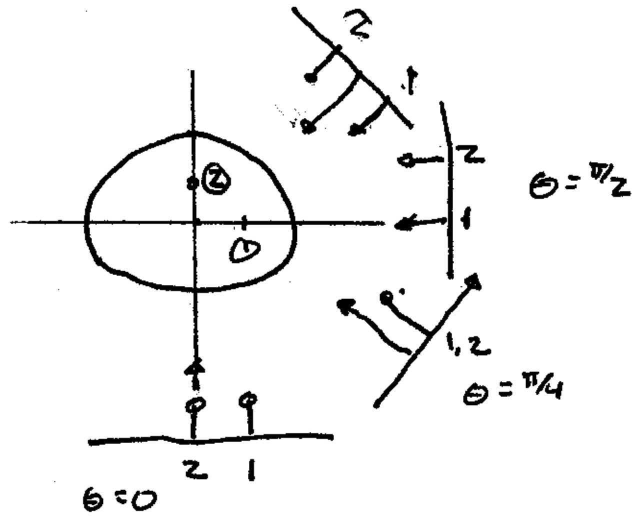
FILTERED BACKPROJECTION
CONVOLUTION BACKPROJECTION

VERY COMMON

IMPLEMENTATION

INITIAL DATA IS IN RADON SPACE $g(r, \theta)$



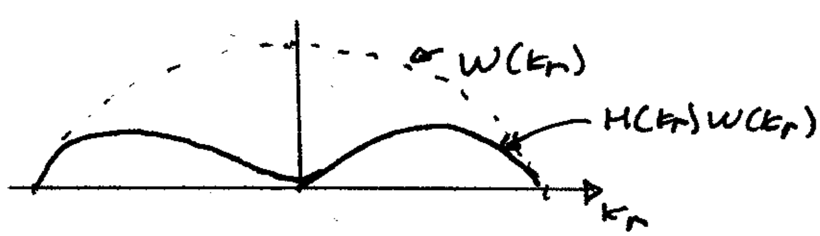


SINOGRAM — EACH POINT TRACES OUT A SINUSOID

CONVOLVE EACH PROJECTION WITH

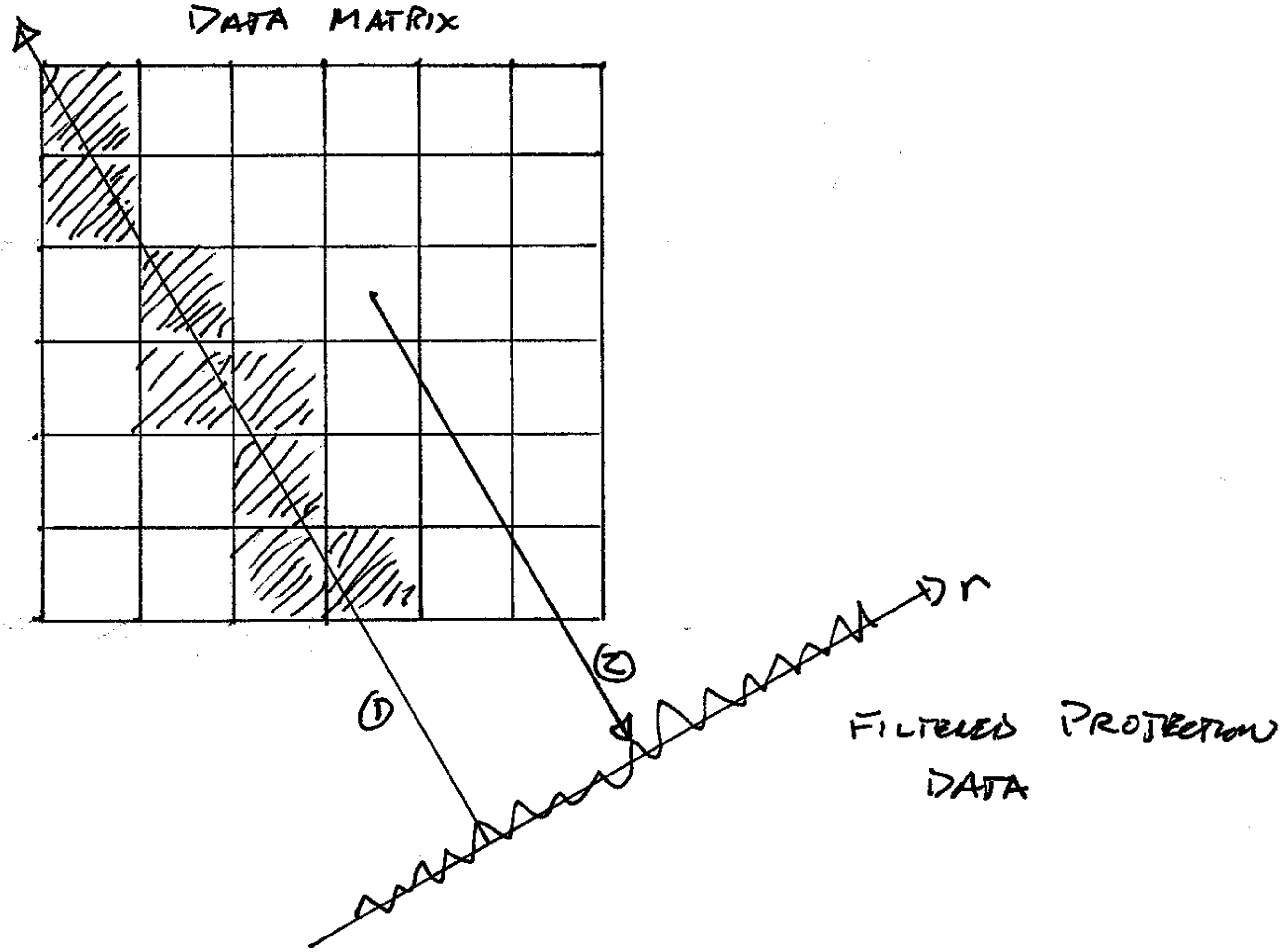
$$h(r) = \mathcal{F}^{-1}_{1D}\{H(k_r)\}$$

OFTEN $H(k_r)$ IS WINDOWED TO REDUCE RINGING



RESULT IS A FILTERED SINOGRAM.

BACK PROJECTION



TWO APPROACHES

1) PUSH RAY THROUGH MATRIX, ADDING TO PIXELS IT CROSSES

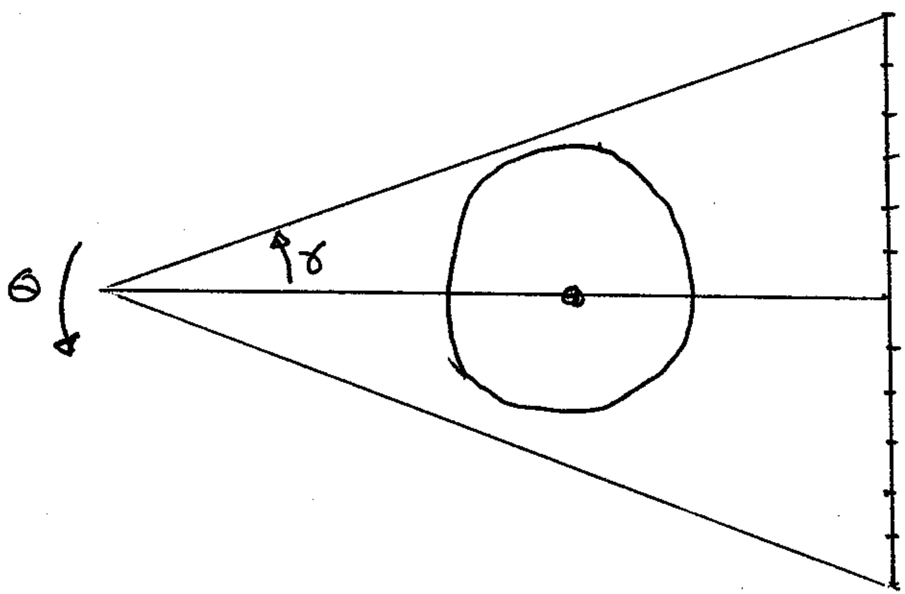
2) TAKE THE LOCATION OF EACH PIXEL, AND INTERPOLATE INTO PROJECTION DATA

FAN BEAM RECONSTRUCTION

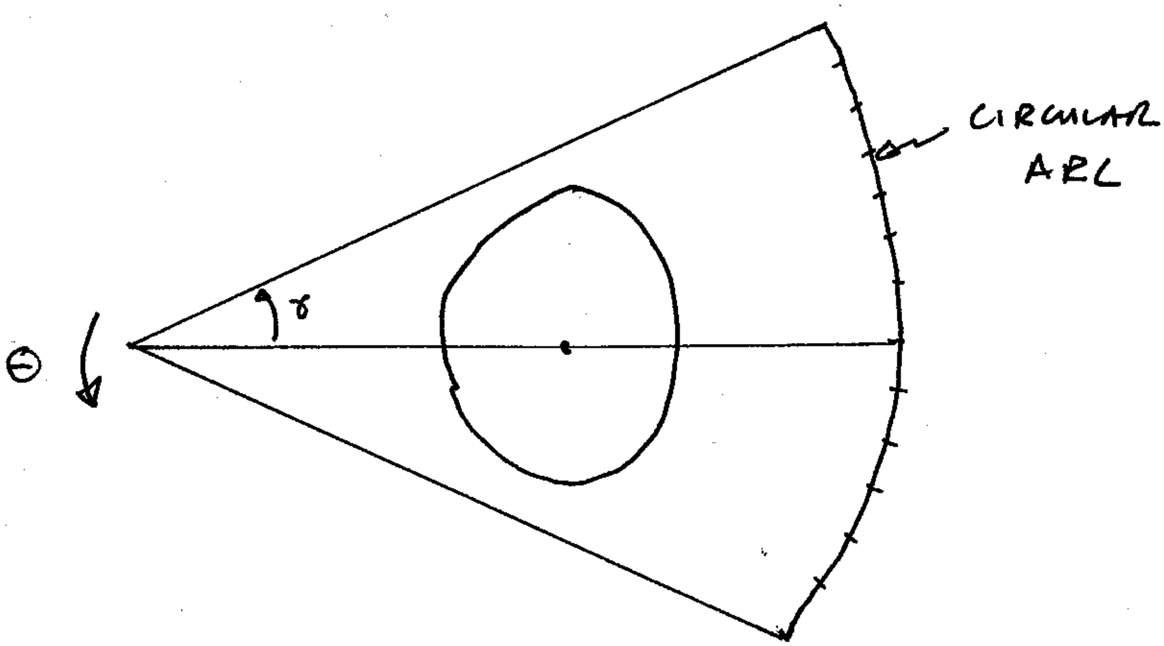
MOST CT SYSTEMS USE A FAN BEAM GEOMETRY

TWO BASIC TYPES

EQUAL SPACED DETECTORS



EQUAL ANGLE DETECTORS



WE WILL FOCUS ON THE EQUAL ANGLE CASE FOR NOW

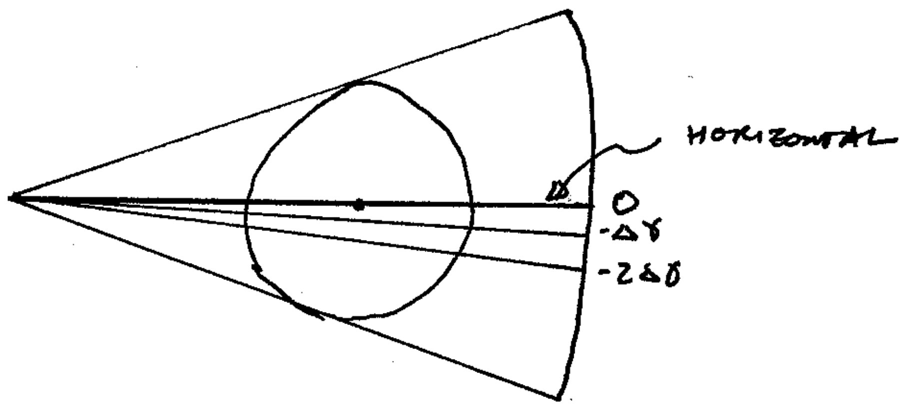
FIRST APPROACH

CONVERT THE PROBLEM TO A PARALLEL BEAM PROBLEM REBINNING

BASIC REBINNING IDEA

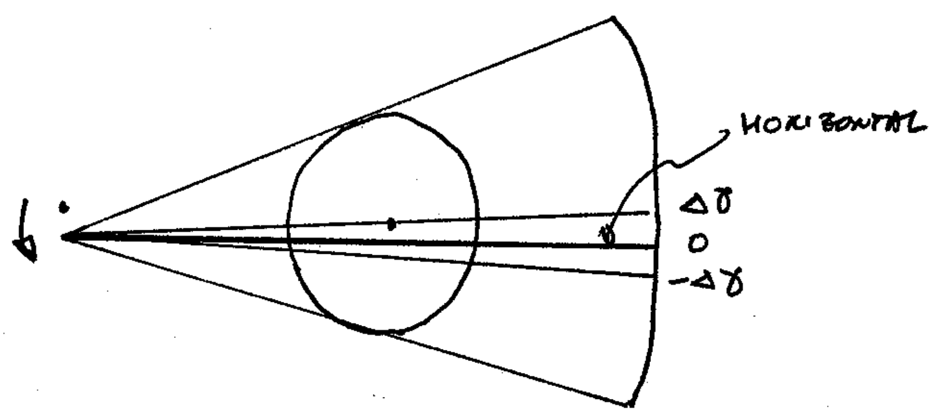
ASSUME BEAMS SPACED BY $\Delta\gamma$, AND THAT WE ROTATE BY $\Delta\gamma$ STEPS

$\theta = 0$

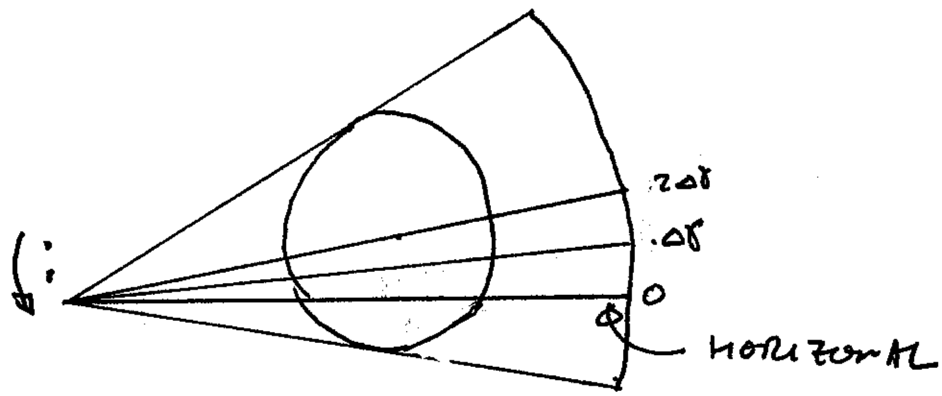


AFTER INCREASING θ BY $\Delta\gamma$

$\theta = \Delta\gamma$

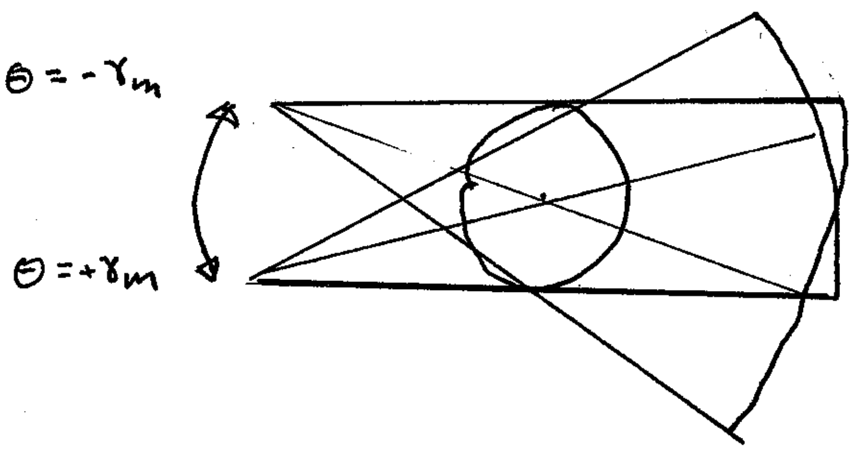


$\theta = 2\Delta\gamma$



WE CAN COLLECT ALL THESE PARALLEL BEAMS INTO ONE PARALLEL PROJECTION

THE EDGE RAYS ARE;



ONE PROJECTION REQUIRES $2\gamma_m$, WHERE THE FAN BEAM GOES FROM $\pm\gamma_m$

TOTAL ANGLE REQUIRED FOR RECONSTRUCTION IS

$$\pi + 2\gamma_m$$

TOTAL NUMBER OF PROJECTIONS

$$N = \frac{\pi + 2\gamma_m}{\Delta\theta}$$

PRACTICAL ISSUES

GENERALLY $\Delta\gamma$ AND $\Delta\theta$ ARE NOT THE SAME

ALTHOUGH THE RAYS ARE PARALLEL AFTER REBINNING THEY ARE NOT EVENLY SPACED.

THESE ARE BOTH INTERPOLATION PROBLEMS WE WILL TALK ABOUT NEXT TIME