Image Obscuration within Moiré Patterns
Stanford EE 368, Digital Image Processing, Course Report
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Abstract

The pointwise multiplication of banding images - alternating black and white stripes, arranged in linear or sinusoidal patterns - often leads to interesting and unique visual effects. One family of combined banding images, referred to as Moiré fringe patterns, is the result of interference between periodic bands. Moiré images have two components: the base band image and the revealing bands. As the revealing mask is translated across a processed image, different amounts of shift will cause obvious visual effects, particularly beating animation. In this project, we investigate different geometric transforms and image modification techniques to obscure one or multiple images within a single Moiré band pattern, both in black and white and in color.

1. Introduction

Moiré fringe patterns result from the interference between two different periodic banding images[1]. These patterns can be observed in nature, like through window blinds or in between fence posts. Part of their allure is their incorporation of small periodic patterns to reveal macroscopic shapes. Additionally, the translation of one pattern across another can result in a beating animation, adding a perceived motion to an otherwise still scene. Moiré patterns are used commercially in counterfeit protection systems[2], as it's very difficult to replicate an exact fringe pattern.

2. Motivation

We chose to investigate Moiré beating shapes primarily for aesthetic reasons. Just as "Magic Eye" books present a fun, creative, and captivating way to obscure image information in plain sight, we too wanted to explore these ideas. In addition to spatial encoding, the animation that one observes when translating the revealing mask over the base layers is mesmerizing. This project makes extensive use of the Matlab image processing toolbox, with emphasis placed on image capture, morphological image operators, and display methods. Material covered in the EE368 class prepared us well for this creative project.

3. Related Work

Chosson et al. generalizes Moiré fringe line patterns to be described by the following equation, representing the interference lines, indexed by k, between two images [3]: $\frac{\Phi(x,y)}{T_{reveal}} - \frac{\Psi(x,y)}{T_{baseband}} = k$. Amidror [4] defines $G(x, y)$ to be the elevation function that shifts a periodic base banding, later to be obscured with a revealing mask with the same period and orientation. $G(x, y)$ can be discontinuous and complicated, and only needs to be represented as a depth image. $Q(x, y)$ is a geometric transform that maps points in the image space shifted by $G(x, y)$ back to the original periodic gratings. By substituting $\Phi(x, y) = Q(x, y)$ and $\Psi(x, y) = Q(x, y) - G(x, y)$ we can recover the Moiré representation of the depth profile.

$$\frac{Q(x,y)}{T} - \frac{Q(x,y) - G(x,y)}{T} = k \Rightarrow \frac{G(x,y)}{T} = K$$

Furthermore, Chossen proves that all geometric transforms applied to the base band and the revealing mask will simplify to meet the conditions laid out for $\frac{G(x,y)}{T} = k$, allowing us to see our original depth image, no matter the transformation. transformation[3].

4. Band Generation

Band generation begins with tuning parameters for the desired revealing mask. We further explore manipulating these parameters in Section 6. Once band period, size, and orientation have been determined, we generate an alternating black and white banding of equal thickness for the revealing mask. We then generate a second copy of this banding pattern. This secondary copy will be offset in further steps to create our underlying image. The first copy of the banding image is our revealing mask. Alternatively, rather than generating a black and white banding with equal spacing, we can also generate single pixel bands of colors, repeating with the same period as the black and white banding.

5. Image Embedding Methods

Grayscale images representing a depth map are embedded in the base banding patterns, to be revealed later with the revealing mask. To embed the depth image within the base banding, we shift position of each banding pixel along the vertical direction by the scaled depth value. The brightest depth map pixels are shifted the greatest distance. We make sure to constrain this shift amount to the maximum period of the banding in the original band image. This ensures that each pixel intensity will have a unique translation amount. The amount each pixel in the base banding is shifted by is described by the equation[4]:

$$y_{new} = y + round(pixelIntensity \times bandPeriod)$$

Figure 1 shows a grayscale image and it's embedding within a horizontal, black and white base band layer.

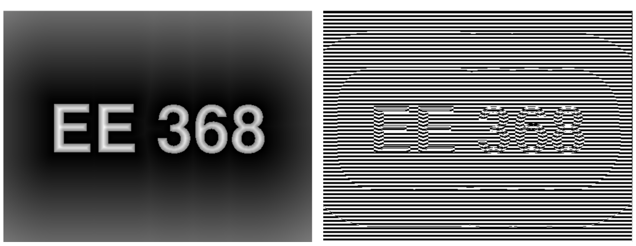

Figure 1. Embedding an image into the base bands

By using a grayscale depth map, rather than a black and white image, we achieve multiple perceived intensities with the superposition of the revealing mask. Additionally, the shapes at different depths will beat through different colors or intensities as the revealing mask is translated across the image. For simplicity, we've decided that all revealing masks will be translated only in the vertical direction.

5.1. Black and White morphological image modification

By turning a simple black and white image into a grayscale image, we achieve more visually stimulating effects as the revealing mask is translated over the the base bandings. In effect, we are simulating depth images without actually having depth data. We use morphological image operators such as dilation, erosion, and skeletonization to separate the outlines, background and foreground of the image. We then take the Euclidean distance transform between the skeleton of the foreground and the shape outline, as well as between the skeleton of the background and the shape outline[3].

To present a sharp contrast between the foreground and the background, we remap distance transform values to lie between 1 to 0.5 for the foreground and 0.0 to 0.5 for the background. Edges between the original foreground and background shapes are presented in these depth maps as a step between 0.5 and 0.0, a large step when the revealing mask added. The original image and the resulting image from our morphological image processing are show in Figure 2.

Figure 2. Before and after mophological image processing

5.2. Depth Image embedding

In addition to accepting simulated depth images, our banding offset algorithm obviously works with true spatial depth information as well. We've enabled our project to capture depth maps with a Kinect camera and embed this image into the base bands. The depth resolution is limited to the period size of the revealing mask to prevent aliasing in the final superimposed image. Figure 3 shows the original depth map and the recovered image with a revealing mask.

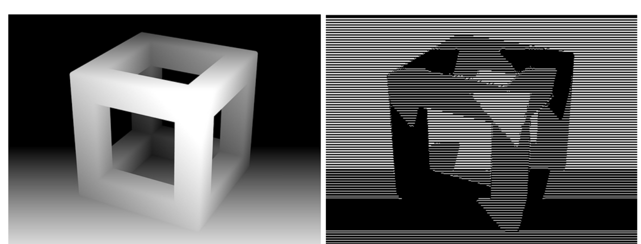

Figure 3. Hidden and revealed image

5.3. Multi-image embedding

Our image processing pipeline is not limited to the obscuration of single images. The same process also allows for multiple depth maps to be obscured within the same image, each of which can be revealed with a corresponding revealing mask. To do so, we generate two embedded image patterns and their revealing masks independently of each other [5]. Then, we combine the two embedded images with a pointwise OR operation for black and white images, or a pointwise multiplication for color images.

Figure 4. Multiple images within the same base banding

The results shown in Figure 4 show a combined baseband image and the two revealed images when using different revealing masks. More than two images could be combined with this same method, but artifacts between images begin to show up if parameters in the revealing masks are too similar.

5.4. Beating Shapes

Shifting the revealing layer in the vertical direction by one pixel at a time shifts all unoccluded pixels in the base banding by one grayscale level. This creates a beating effect. The number of steps in the beating effect is determined by the period of the base banding. Figure 1 has a period of $T = 4$ and shows four frames of the successive image sequence.

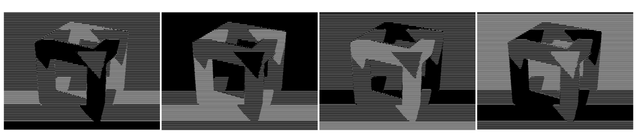

Figure 5. Beating Moiré fringe patterns

6. Band Manipulation Methods

With these aforementioned banded image generation processes, we explore modifying our baseline parameters and the resulting images. We look at changes to band spacing, band angle, the introduction of color banding, and larger geometric transforms that can be applied to the entire image.

6.1. Band Spacing

When generating the bandings for the base layer and the revealing mask, we choose a period in pixels for how large we desire our bands to be. Larger periods give rise to wider spacing, but also present the opportunity to achieve more discrete levels of perceived grayscale. This is a result from the way we form our shifted baseband images, moving each pixel in the y direction by the value of the depth map, proportional to the period of the banding: $y_{new} = y + round(pixelIntensity \times bandPeriod)$.

While this constraint ensures that our maximum shift is no larger than the period of the banding to prevent aliasing, the larger band period also allows for more discrete perceived grayscale levels when interfered with the revealing mask. For example, a banding with a period of $T = 2$ would only allow for the revealing mask to either be fully transmissive, or fully occluding at each translation in the y direction, resulting in only two different perceived grayscale levels. A banding with a period of $T = 4$ Allows for full occlusion, three-quarters occlusion, half transmission, and quarter transmission, resulting in four perceived grayscale levels. The results show in Figure 6 show the difference between band periods of $T = 2$ and $T = 4$ pixels.

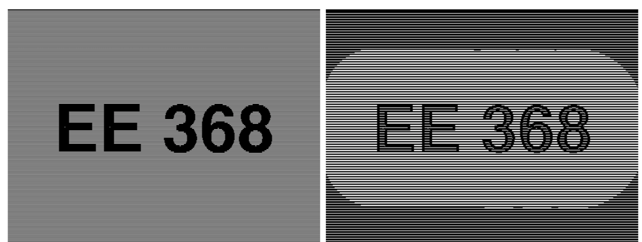

Figure 6. Banding with Period of $T = 2$ and $T = 4$

6.2. Band Angle

Keeping the band period the same, we can rotate the angle of both the base banding and the revealing mask by the same angle, θ, and achieve another family of moire patterns, still fulfilling the $\frac{G(x,y)}{T} = k$ requirements [4]. While we are still able to recover the embedded image from the occlusion at different angles, the perceived grayscale value may appear different, due to the interpolation use when rotating the base banding. Different rotations have different interpolations, resulting in different overlap at the same position on the image. Figure 7 shows the resulting interference after a $20°$ and $40°$ rotation to the bandings, with period kept the same.

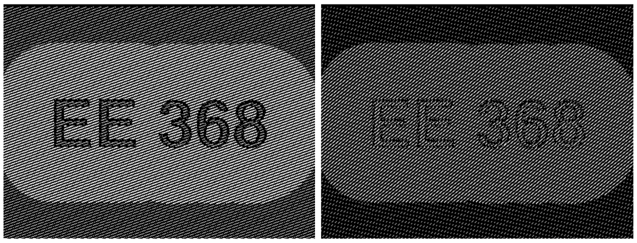

Figure 7. Banding with rotation of 20° and 40°

Additionally, band rotation obscures the desired embedded image in the original base bands better than in traditional horizontal bandings. This is advantageous for hiding visually sensitive images or making bandings more difficult to copy. Figure 8 shows the a horizontal and rotated base banding without a revealing mask, both with the same image and same band period. It is more difficult to spot the original image in the rotated bands.

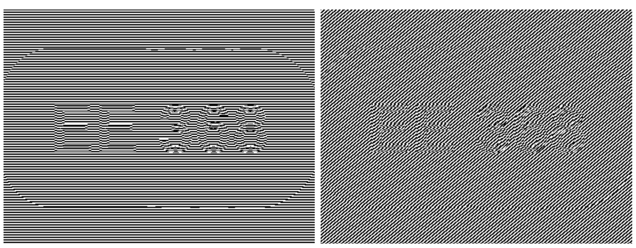

Figure 8. Unmasked banding with rotation of 0° and 40°

6.3. Band Color

As previously mentioned, Moiré fringe patterns can be produced in color if the color banding matches the same period, angle, and transformation as the revealing mask. Similarly to how changing the band period increases the number of perceived grayscale levels, Larger periods result in a greater number of colors. This is demonstrated in Figure 9 with four and six different band colors.

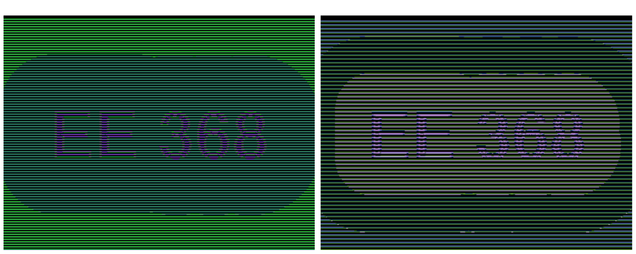

Figure 9. Four and six color banding

6.4. Band Translation

To make banding patterns more complex, we introduce a geometric shift at each pixel in both the banding and the revealing mask. Like the banding rotation, this fulfills all mathematical requirements for Moiré patterns. For the purposes of our demonstration, we defined a cosine shift, moving each pixel along the horizontal direction by a specific amplitude. The amplitude of the shift changes depending on the cosine of the position. Both the period of the wave and the amplitude of the offset are adjustable. This works with both black and white and color banding images. Two different cosine waves are demonstrated in Figure 10.

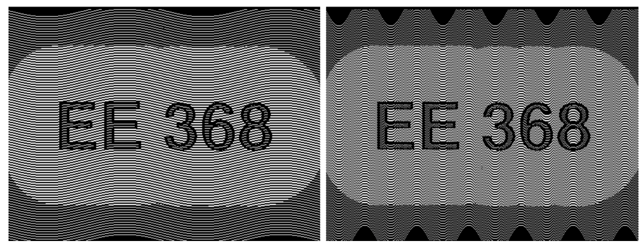

Figure 10. Banding images with cosine wave transformations

Wave translations can be thought of as position dependent angular rotations. Like the increased visual complexity of the obscured image when band angles are rotated, the obscured image banding becomes almost completely impossible to understand without the revealing mask. This is show in Figure 11

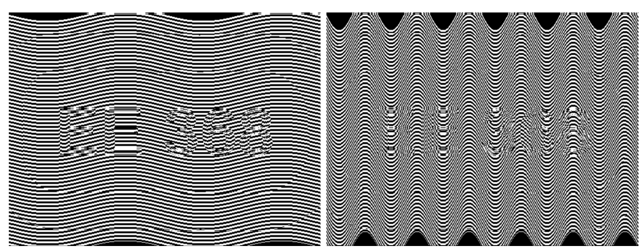

Figure 11. Banding images with cosine wave transformations

7. Future Work

We'd like to investigate other transformations that could be applied that satisfy the equation for $\frac{G(x,y)}{T} = k$, like rotational Moiré pattern revelation. We believe that different revealing masks with horizontal, vertical, and rotational preference could all be introduced into the same baseband image, each revealing a different depth map when translated with the correct revealing mask with the same orientation preference. One could potentially create 3D patterns that give the appearance of beating fringe patterns depending on the angle of observation, like the projection of a fence down onto the ground when viewed at an angle. Additionally, we suspect there to be an audio analogy to these image transformation processes, with the revealing pattern representing the waveform amplitude and the orientation representing the phase. These ideas could also be combined with phone cameras, where the shifted band image is passed between two individuals, to be revealed later using an app with knowledge of the encoding.

Another avenue of research would be to quantify how well an image can be obscured - how much of the original image remains to the naked eye. This is a difficult thing to automate as it involves input from human observers. Each obscured image would need a collection of guesses on what the original image might contain. This type of aggregate research would be fun to run as a web game - perhaps, similar to the Magic Eye game mentioned earlier in this paper, people would enjoy attempting to determine what the obscured image truly represented.

8. Acknowledgments

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