Subsampling and reconstruction of bandlimited images with universal sampling sets

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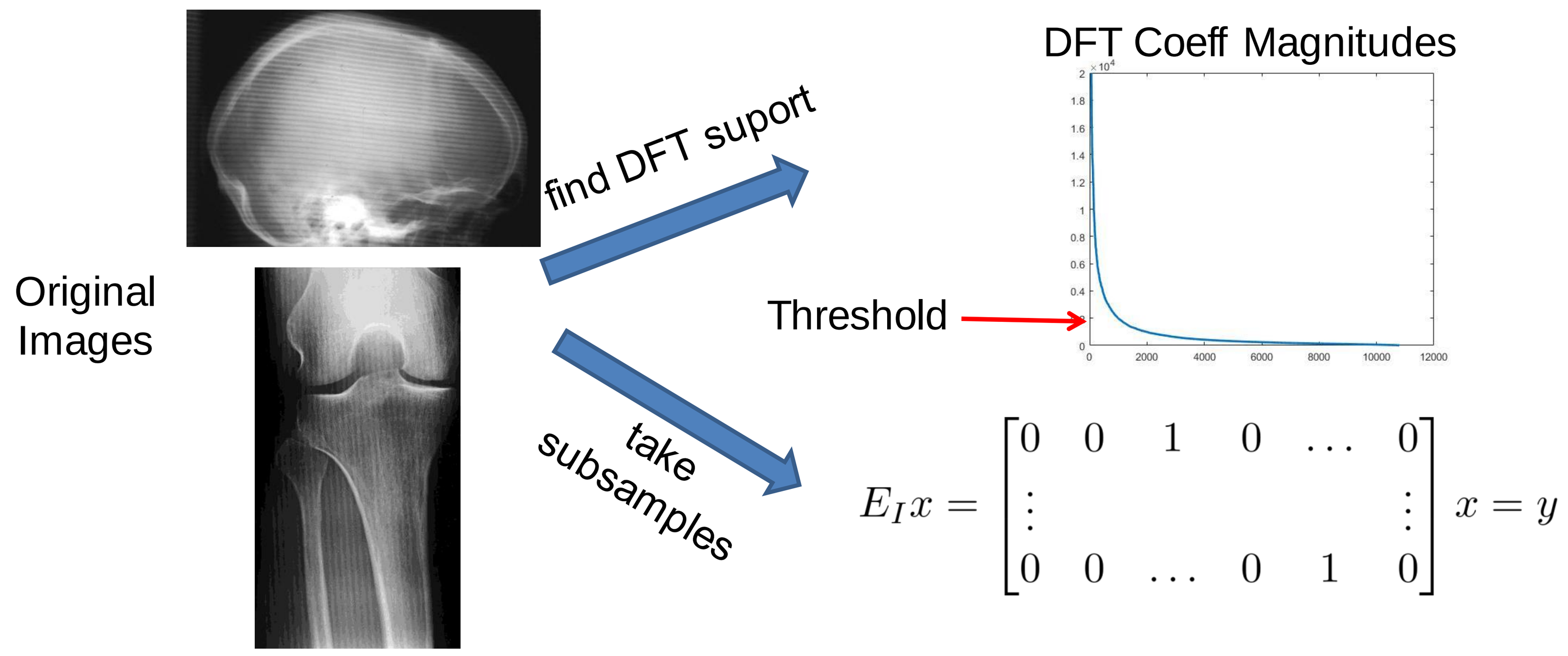
Introduction

Reconstruction of signals from a limited set of subsamples is a fundamental problem in signal processing. It is often done in image processing for compression, computation, and sensing purposes.

We investigate the subsampling and subsequent reconstruction of bandlimited images using the framework of universal sampling sets [1]. We then compare these results to the popular sparse reconstruction algorithms iterative hard thresholding (IHT) and compressive sensing matching pursuit (CoSaMP) from compressed sensing [2]. Our comparison focuses on what information is required to perform the reconstructions, for what rates the reconstructions can succeed, and the visual quality of the reconstructions.

Methods:Subsampling

Let **x[n]** be a real-valued, length **N**, discrete-time signal such as a vectorized image. In theory, any signal **x[n]** which is bandlimited to **k** distinct frequency components can be reconstructed from subsamples taken at the indices **I** of a universal sampling set (USS). Let **M** be the number of subsamples.



- Can take subsamples of the original image (no anti-aliasing) or of the sparse approximation (AA).
- For IHT/CoSaMP, the samples are taken using a random sensing matrix with entries **~N(0,1/M)**.

Methods:Reconstruction

By writing out **x[n]** as a sum of its frequency components, we can see the method for reconstruction. Let **J** be the set of **k** indices which form the support of the DFT of **x[n]**.

$$x[n] = \frac{1}{N}\sum_{i\in J} a_i e^{j\frac{2\pi}{N}ni} = F^{-1}E_J^T a$$

where **F** is the DFT matrix.

$$y = E_I x = E_I F^{-1} E_J^T a$$

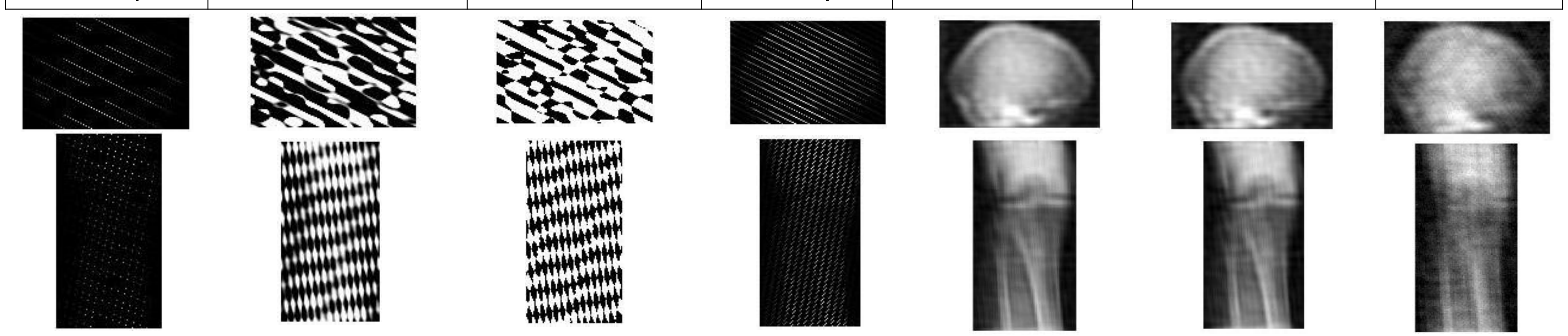
$$\implies a = (E_I F^{-1} E_J^T)^{-1} y$$

$$\implies x = F^{-1} E_J^T (E_I F^{-1} E_J^T)^{-1} y$$

- If we know the support **J**, and the submatrix of the inverse of **F** with rows **I** and columns **J** is invertible, we can reconstruct **x[n]**. A universal sampling set is a index set **I** such that no matter what **J** is, the resulting matrix is invertible.
- IHT and CoSaMP are iterative algorithms which work essentially by gradient descent. See [2].

Experimental Results

M/k=1 M/N = 0.03			M/k=5 M/N = 0.15			
USS subsamples	reconstruction with AA	reconstruction without AA	USS subsamples	reconstruction with AA	reconstruction without AA	CoSaMP



Conclusions

- In the case that **k=M**, even though there is a theoretical guarantee that the DFT submatrix will be invertible, the conditioning can be so poor that the reconstruction fails due to noise and numerical precision issues.
- When **M>k**, the reconstruction succeeds by using a pseudoinverse. In this case the USS method visually outperforms the compressed sensing methods.
- The tradeoff is that the support **J** does not need to be known for the compressed sensing algorithms.

[1] Brad Osgood, Aditya Siripuram, and William Wu, Discrete Sampling and Interpolation: Universal Sampling Sets for Discrete Bandlimited Spaces, IEEE Transactions on Information Theory 58 (2012), 4176-4200.
[2] D. Needell and J. A. Tropp, CoSaMP: Iterative Signal Recovery from Incomplete and Inaccurate Samples (Accessed 4/7/2014), available at http://arxiv.org/pdf/0803.2392v2.pdf.