

## EE364a Homework 2 additional problems

1. *'Hello World' in CVX.* Use CVX to verify the optimal values you obtained (analytically) for exercise 4.1.
2. *Continued fraction function.* Show that the function

$$f(x) = \frac{1}{x_1 - \frac{1}{x_2 - \frac{1}{x_3 - \frac{1}{x_4}}}}$$

defined where every denominator is positive, is convex and decreasing. (There is nothing special about  $n = 4$  here; the same holds for any number of variables.)

3. *Dual of intersection of cones.* Let  $C$  and  $D$  be closed convex cones in  $\mathbf{R}^n$ . In this problem we will show that

$$(C \cap D)^* = C^* + D^*.$$

Here,  $+$  denotes set addition:  $C^* + D^*$  is the set  $\{u + v \mid u \in C^*, v \in D^*\}$ . In other words, the dual of the intersection of two closed convex cones is the sum of the dual cones.

- (a) Show that  $C \cap D$  and  $C^* + D^*$  are convex cones. (In fact,  $C \cap D$  and  $C^* + D^*$  are closed, but we won't ask you to show this.)
- (b) Show that  $(C \cap D)^* \supseteq C^* + D^*$ .
- (c) Now let's show  $(C \cap D)^* \subseteq C^* + D^*$ . You can do this by first showing

$$(C \cap D)^* \subseteq C^* + D^* \iff C \cap D \supseteq (C^* + D^*)^*.$$

You can use the following result:

If  $K$  is a closed convex cone, then  $K^{**} = K$ .

Next, show that  $C \cap D \supseteq (C^* + D^*)^*$  and conclude  $(C \cap D)^* = C^* + D^*$ .

- (d) Show that the dual of the polyhedral cone  $V = \{x \mid Ax \succeq 0\}$  can be expressed as

$$V^* = \{A^T v \mid v \succeq 0\}.$$