# Scaling Laws for Sensor Networks

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**Abstract**

In the papers summarized in this write-up $B_s$ sources are simultaneously observed by $M$ sensors. All these observations have to be communicated in a suitable form to a central data collection point to learn about the $B_s$ sources with highest fidelity possible. Results are for Gaussian sources. Best achievable distortion scales like $1/M$.

## I. Main Ideas

It is shown that separation based transmission results in sub-optimal lerformance in Gaussian sensor networks. In this case achievable distortion decays as $1/\log M$, where $M$ is the number of the sensors. A simple joint source channel coding is proposed which gives distortion decaying as $1/M$. This outperfoms the separation based approach. Later it is shown that as $M$ tends to infinity, this strategy the smallest possible distortion. These results are extended to a general joint source channel coding paradigm using the arguments giving the conditions for optimality of joint source channel coding.

## II. Assumptions

In the first paper, authors have assumed multiple sources, but the second one have assumed single source, we will include general assumptions. There are $B_s$ sources each producing samples $S_i[n]$ for different time slots $n$. The random vector $S[n]$ is not directly observed by the sensors. Sensor $m$ receives a sequence $U_m^n$. Based on these pbservations sensor $m$ sends a signal $X_m^n = F_m(U_m^n)$. The transmitted signals satisfy the constraint $E\rho(X_1^n, X_2^n, \ldots, X_M^n) \leq \Gamma$. The final destination constructs estimates $(\hat{S}_1^n, \hat{S}_2^n, \ldots, \hat{S}_{B_s}^n)$. We define achieved distortion as $\Delta = Ed(S^n, \hat{S}^n)$. Referred papers demonstrate the best distortion $\Delta$, cost $\Gamma$ possible and propose scaling laws for distortion.

## III. Single Source Gaussian Sensor Networks

In this case $B_s = 1$ and $U_m[n] = \alpha_m S[n] + W_m[n]$, $S[n]$ is a sequence of independent and identically distributed circularly complex Gaussian Random variables of variance $\sigma_s^2$ and $W_m[n]$ is a sequence of iid circularly complex Gaussian random variables of mean zero and variance $\sigma_W^2$ and different noise sources are independent. Channel coefficients are assumed fixed and known throughout the network.

$$\sum_{m=1}^{M} E|X_m|^2 \leq P_{tot}(M)$$

The final destination receives $Y[n] = \sum_{m=1}^{M} \delta_m X_m[n] + Z[n]$. Distortion measure is the mean squared error. The goal of analysis in these two papers is to determine the best trade-offs between the total sensor power $P_{tot}(M)$ and the incurred end to end distortion $D$.

### A. *Scaling law for separate source channel coding*

In separate source and channel coding case, the authors have shown the logarithmic decrement of distortion by $M$ number of sensors. It was shown that :

$$D_{sep}(M, P_{tot}(M)) \geq \frac{\sigma_W^2/\sigma_S^2}{\log(1 + P_{tot}(M)\sum_{m=1}^{M}|\sigma_m|^2/\sigma_Z^2)}$$

Where $\sigma_S$ is the variance of the underlying source, $\sigma_W^2$ is the variance of the observation noise, $\sigma_Z^2$ is the variance of the noise in the multi-access channel, and $P_{tot}(M)$ is the total sensor transmit power for the $M$ involved sensors.

### B. *Improved Achievable Performance*

The analysis of a simple joint source channel coding scheme provides an improved achievable performance, by doing MSE in the receiver the following distortion is achievable:

$$D_{ach}(M, P_{tot}(M)) = \frac{\sigma_s^2 \sigma_W^2}{\sigma_W^2 + \sum_{m=1}^{M}|\alpha_m|^2 \sigma_S^2}\left(1 + \frac{(\sigma_S^2\sigma_Z^2/\sigma_W^2)\sum_{m=1}^{M}|\alpha_m|^2}{\sigma_Z^2 + P_{tot}(M)b(M)}\right)$$

In which $b(M)$ is a linear function of $M$. This distortion can be lower bounded by

$$D_{lower}(M, P_{tot}(M)) = \frac{\sigma_S^2 \sigma_W^2}{\sigma_W^2 + \sum_{m=1}^{M}|\alpha_m|^2 \sigma_S^2}\left(1 + \frac{(\sigma_S^2\sigma_Z^2/\sigma_W^2)\sum_{m=1}^{M}|\alpha_m|^2}{\sigma_Z^2 + P_{tot}(M)\sum_{m=1}^{M}|\delta_m|^2}\right)$$

It can be shown that the optimal scaling law for the single source Gaussian sensor network is given by:

$$D(M, P_{tot}(M)) = \frac{\sigma_s^2 \sigma_W^2}{\sigma_W^2 + \sum_{m=1}^{M}|\alpha_m|^2 \sigma_S^2}\left(1 + \frac{(\sigma_S^2\sigma_Z^2/\sigma_W^2)\sum_{m=1}^{M}|\alpha_m|^2}{\sigma_Z^2 + P_{tot}(M)\sum_{m=1}^{M}|\delta_m|^2}\right)$$

### C. Discussion

The authors show that clearly no matter how high a total power is chosen, the sclaing behavior at best is like $1/M$. This linear decrease is much better than logarithmic decrease of separation based system.

## IV. Multi-Source Gaussian Sensor Network

Authors have studied the case of sensor networks with multiple sources. In this set sensor $m$ receives the following signal:

$$U_m[n] = \sum_{l=1}^{B_s} \alpha_{ml} S_l[n] + W_m[n]$$

The assumptions for each source and distortions are exactly the same as the single source case. Following the same guidelines as the single source case above, authors have shown that in separated based method, distortion scales with $1/\log M$. Later using their joint source channel coding method they approve a linear decrease in distortion under some certain conditions which imposes a trade off between Bandwidth and trade-off in this case. They have answered to the question : which power-bandwidth pairs give permit to achieve the $1/M$ scaling law? Under some additional restrictions the total power required to sustain a scaling law of $D(M, P_{tot}(M))$ $\frac{1}{M}$ satisfies $P_{tot}(M) \geq P_{tot,lower}(M)$, where $P_{tot,lower}(M)$ $B_c M^{\frac{B_S}{B_C}-1}$, where $B_S$ is the number of sources (or equivalently, the source bandwidth), and $B_C$ is the number of channel uses per source sample ( or equivalently, the channel bandwidth ). Interestingly it can be seen that :

1) If $B_c < B_s$, the total power must increase with $M$.
2) If $B_c = B_s$, then a constant total power is sufficient to achieve the optimum distortion scaling law.
3) If $B_c > B_s$ authors have claimed that a total power that decreases with $M$ may be sufficient, In particular they believe that simple joint source channel coding feedback schemes are able to have the gains offered by the additional bandwidth.

## V. Possible extensions

Further extensions of this work includes the following topics: first extending results to other statistics and distortion measures than Gaussian. Second the work can be generalized to other topologies of sensor networks like the case of multiple data collection points and of feedback links.

# REFERENCES

[1] M. Gastpar, M. Vetterli, “Scaling laws for homogeneous sensor networks”, 41st Annual Allerton Conf. on Commun., Control and Comp., Monticello, IL, Oct. 1-3, 2003.

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