Optimal Routing, Link Scheduling and Power Control in Multi-hop Wireless Networks

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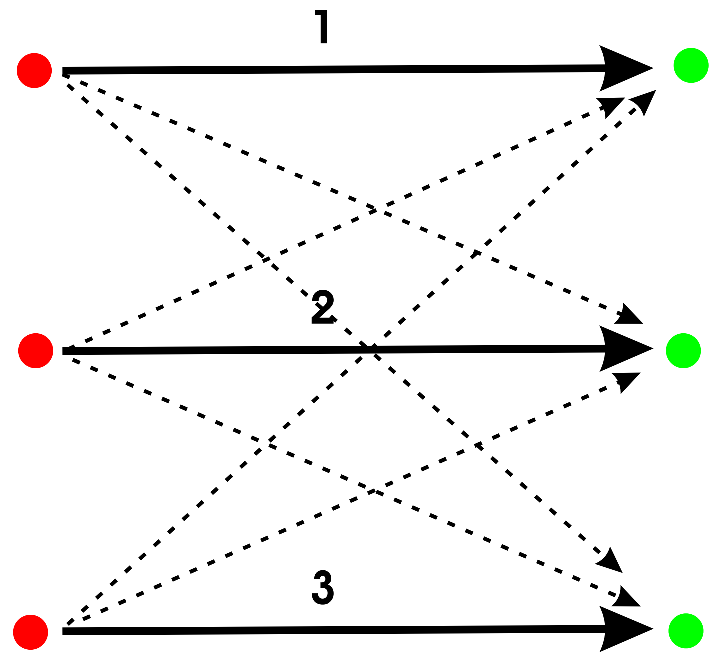
Overview

- The main emphasis of this paper is on link scheduling and power control in a wireless ad hoc network.
- The paper aims to find a power transmission strategy to minimize a cost function of the average power of each link, subject to average rate and peak power constraints on each link.
- The novel idea in this paper is to schedule links to decrease average power consumption by reducing interference, vs. the traditional approach to schedule (more generally, MAC) to maximize average throughput.
- The paper suffers from two main drawbacks:
  - low SINR assumption is made
  - the analysis lacks clarity

Link Scheduling - An Example



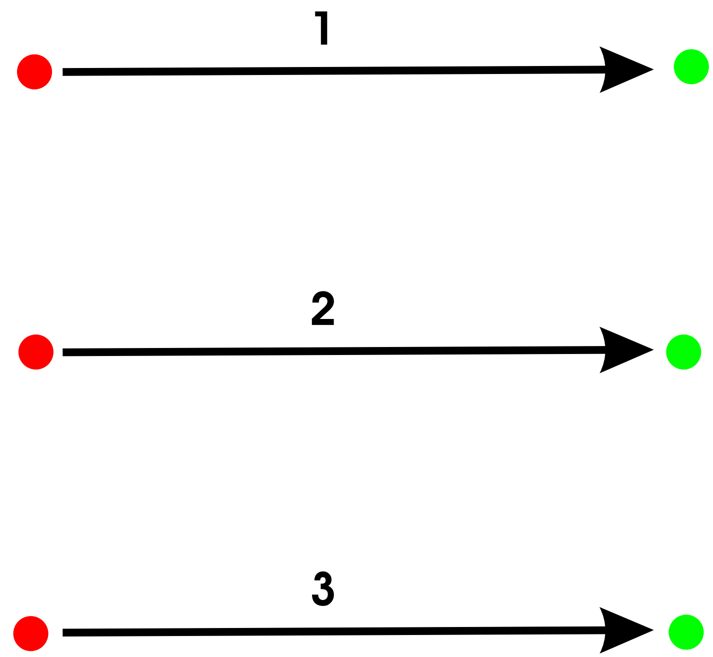
- Slotted time, slots $m = 1, 2, \ldots$
- Select power $P_m(l)$ for each link $l \in \{1, 2, 3\}$ over each slot m.
- Constraints
  - peak power - $P_m(l) \leq P^{\text{max}}$
  - avg. min. rate - $X^{\text{avg}}(l) \geq R^{\text{min}}$

Effect of scheduling

Let us consider the effect of link scheduling by considering some scheduling schemes. The system parameters used will be

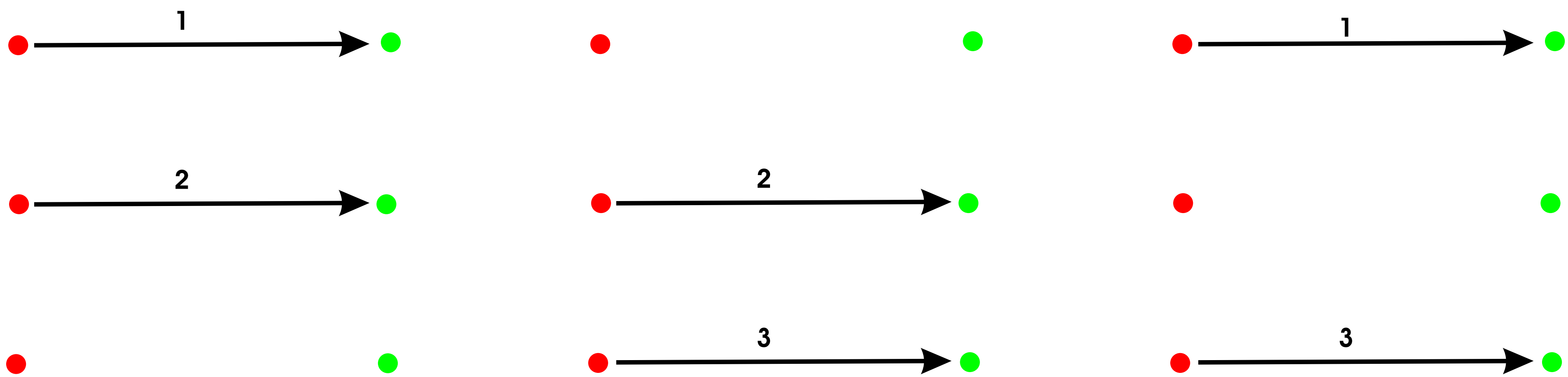
- Noise power - $N = 0.5$ mW.
- Gain matrix
$$G = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$
- SINR
$$\gamma(l) = \frac{G(l,l)P(l)}{\sum_{k \neq l} G(l,k)P(k) + N}$$
- $R^{\text{min}}(l)/B = 0.9$ for $l = 1, 2, 3$.

Scheme 1



- All links transmit all the time.
- Avg. power consumption per link is 3.23 mW.

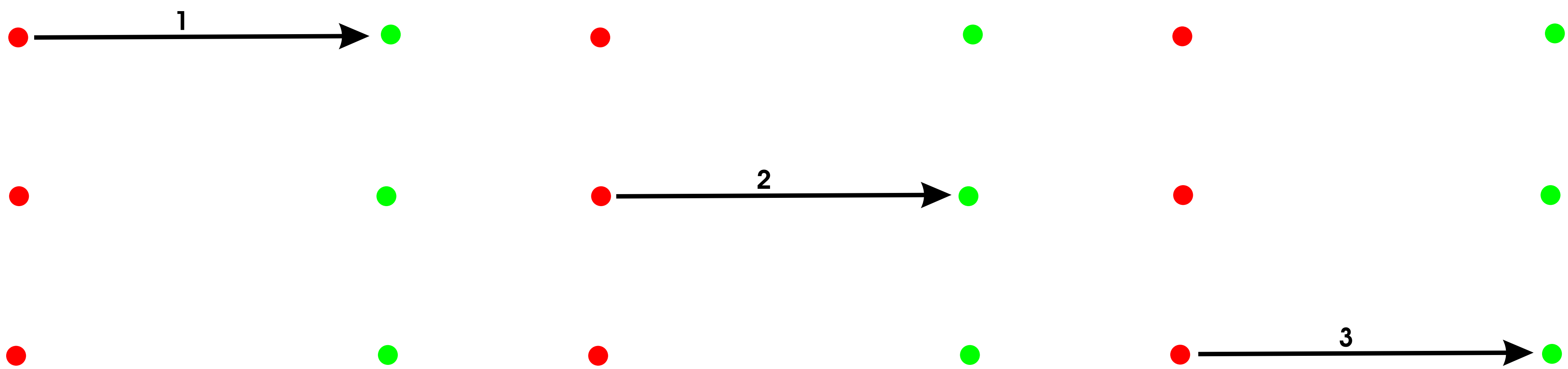
Scheme 2



(a) slot 1 (b) slot 2 (c) slot 3

- The schedule is a periodic schedule that repeats after every 3 time slots.
- Avg. power consumption per link is 2.29 mW.

Scheme 3



(a) slot 1 (b) slot 2 (c) slot 3

- The schedule is a periodic schedule that repeats after every 3 time slots.
- Avg. power consumption per link is 0.92 mW.

Joint Power Control and Scheduling - Difficult Problem

- Consider a network of L links.
- During each time slot, number of different combinations of active links is 2^L.
- Number of different possible schedules, periodic with period of M time slots is
$$2^L \times \ldots \text{ M times } = 2^{ML}$$
- This paper focuses on finding the optimal asymptotic schedule, i.e. the case where $M \to \infty$.

Low SINR Assumption

- The SINR $\gamma(l)$ over a link is assumed to be low enough such that for a fixed BER of 10^{-q}, we can approximate

$$X(l) = K\gamma(l)$$

where

$$K = \frac{B}{2q \log_2 10}$$

- Assumption justified on the basis of the presence of many users in a CDMA system (?).

Optimization Problem

min. $h(P) = \sum_{l=1}^{3} \alpha(l) P^{\text{avg}}(l)$

s.t. $0 \leq P_m(l) \leq P^{\text{max}} \qquad l = 1, 2, 3$

$m = 1, 2, \ldots$

$$\liminf_{M\rightarrow\infty} \frac{1}{M} \sum_{m=1}^{M} \frac{KG(l,l)P_m(l)}{\sum_{k\neq l} G(l,k)P_m(k)+N} \geq R^{\text{min}} \qquad l = 1, 2, 3$$

where

$$P^{\text{avg}}(l) = \limsup_{M\longrightarrow\infty} \frac{1}{M} \sum_{m=1}^{M} P_m(l), \quad l = 1, 2, 3$$

- The constraint on minimum rates is *non-convex*.

Notation

- We will use P to represent a transmission schedule, i.e. the transmission powers $P_1(1), P_1(2), P_1(3), P_2(1), \ldots$.

- Let

$$X^{\mathsf{avg}}(P,l) = \liminf_{M \longrightarrow \infty} \frac{1}{M} \sum_{m=1}^{M} \frac{KG(l,l)P_m(l)}{\sum_{k \neq l} G(l,k)P_m(k) + N}, \quad l = 1,2,3$$

Lagrangian

Relax the constraint on minimum rates to form the Lagrangian

$$L(P, \lambda) = h(P) + \sum_{l=1}^{3} \lambda(l)\Big(R^{\mathsf{min}} - X^{\mathsf{avg}}(P, l)\Big)$$

Dual Problem

- The *Lagrange dual function* is

$$g(\lambda) = \inf_{P:\ 0 \leq P_m(l) \leq P^{\mathsf{max}}} L(P, \lambda)$$

- The dual problem is

$$\begin{array}{ll} \text{max.} & g(\lambda) \\ \text{s.t.} & \lambda \geq 0 \end{array}$$

Analysis

- The authors argue that the Lagrangian is linear is $P^{\mathsf{avg}}(l)$ and $X^{\mathsf{avg}}(P, l)$.
- Hence

$$g(\lambda) = \inf_{P:\ 0 \le P^{\mathsf{avg}}(l) \le P^{\mathsf{max}}} L(P^{\mathsf{avg}}, \lambda)$$

$$= \inf_{P_1:\ 0 \le P_1(l) \le P^{\mathsf{max}}} \left[h(P_1) + \sum_{l=1}^{3} \lambda(l) \Big(R^{\mathsf{min}} - X(P_1, l) \Big) \right]$$

- The optimal schedule needs the computation of an optimal time - sharing schedule between an exponential number of active link combinations.

Alternate Problem Formulation

- Consider a time schedule that consists of slots of length τ, and is periodic with a period of M time slots.
- Let us approximate the rate over the link l as

$$X(l) = \alpha \log\big(1 + K\gamma(l)\big)$$

- $X^{\text{avg}}(l)$ and $P^{\text{avg}}(l)$ are the average rate and power consumption on link l, averaged over M time slots.
- We would like to compute a schedule of transmission powers such that the average rate constraints and peak power constraints are satisfied.

Optimization Problem

min. $$h(P) = \sum_{l=1}^{3} \alpha(l) P^{\mathsf{avg}}(l)$$

s.t.
$$0 \leq P_m(l) \leq P^{\mathsf{max}} \quad l = 1,2,3, \quad m = 1,\ldots,M$$
$$\frac{1}{M}\sum_{m=1}^{M} \alpha \log\left(1 + K\frac{KG(l,l)P_m(l)}{\sum_{k\neq l} G(l,k)P_m(k)+N}\right) \geq R^{\mathsf{min}} \quad l = 1,2,3$$

where
$$P^{\mathsf{avg}}(l) = \frac{1}{M}\sum_{m=1}^{M} P_m(l), \quad l = 1,2,3$$

- The constraint on minimum rates is *non-convex*.
- With a change of variables, we can replace this constraint with a convex constraint in the high SINR regime - need scheduling!

Project Preview

- Joint routing, link scheduling and power control to
  - min. a linear cost function of average link powers
  - max. network lifetime
- Collabrative work with Shuguang Cui.