

# Capacity of Flat and Freq.-Selective Fading Channels

## Linear Digital Modulation Review

### Lecture Outline

- **Optimal Power/Rate Adaptation: Some Features**
- **Channel Inversion with Fixed Rate Transmission**
- **Capacity of Frequency Selective Fading Channels**
- **Review of Linear Digital Modulation**
- **Performance of Linear Modulation in AWGN**

#### 1. Optimal Power and Rate Adaptation: Some Features

- Optimal power adaptation is a “water-filling” in time: power  $P(\gamma) = \gamma_0^{-1} - \gamma^{-1}$  increases with channel quality  $\gamma$  above an optimized cutoff value  $\gamma_0$ .
- Rate adaptation relative to  $\gamma \geq \gamma_0$  is  $B \log_2(\gamma/\gamma_0)$ : also increases with  $\gamma$  above cutoff.
- Resulting capacity is  $C = \int_{\gamma_0}^{\infty} B \log_2(\gamma/\gamma_0) p(\gamma) d\gamma$ .
- Optimal instantaneous rate and power only depend on  $\gamma$  and  $\gamma_0$ . Depend on the fading distribution through  $\gamma_0$  whose value is determined by the average power constraint and fading distribution.
- Capacity with power and rate adaptation not much larger than when just receiver knows channel, but has lower complexity and yields more insight into practical schemes.
- Capacity in flat-fading can exceed the capacity in AWGN, typically at low SNRs.

#### 2. Channel Inversion

- Suboptimal transmission strategy where fading is inverted to maintain constant received SNR.
- Simplifies system design and is used in CDMA systems for power control.
- Capacity with channel inversion greatly reduced over that with optimal adaptation (capacity equals zero in Rayleigh fading).
- Truncated inversion: performance greatly improved by inverting above a cutoff  $\gamma_0$ .

#### 3. Capacity of Frequency Selective Fading Channels

- Capacity for time-invariant frequency-selective fading channels is a “water-filling” of power over frequency.
- For time-varying ISI channels, capacity is unknown in general. Approximate by dividing up the bandwidth subbands of width equal to the coherence bandwidth (same premise as multicarrier modulation) with independent fading in each subband.
- Capacity in each subband obtained from flat-fading analysis. Power is optimized over both frequency and time.

#### 4. Linear Digital Modulation

- Linear modulation typically used in high-rate systems due to its high spectral efficiency.
- Over the  $i$ th symbol period, bits are encoded in carrier amplitude or phase  $s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$ , where  $\phi_1(t) = g(t) \cos(2\pi f_c t + \phi_0)$  and  $\phi_2(t) = g(t) \sin(2\pi f_c t + \phi_0)$  for initial phase offset  $\phi_0$ .
- Pulse shape  $g(t)$  determines signal bandwidth, and is typically Nyquist.
- Baseband representation is  $s(t) = \Re\{x(t)e^{j\phi_0}e^{j2\pi f_c t}\}$  for  $x(t) = (s_{i1} + js_{i2})g(t)$ .
- The constellation point  $(s_{i1}, s_{i2})$  has  $M$  possible values, hence there are  $\log_2 M$  bits per symbol.

#### 5. Performance of Linear Modulation in AWGN:

- ML detection corresponds to decision regions.
- For coherent modulation, probability of symbol error  $P_s$  depends on the number of nearest neighbors  $\alpha_M$ , and the ratio of their distance  $d_{min}$  to the square root  $\sqrt{N_0}$  of the noise power spectral density (this ratio is a function of the SNR  $\gamma_s$ ).
- $P_s$  approximated by  $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$ , where  $\beta_M$  depends on the modulation.
- Alternate Q function representation  $Q(z) = \frac{1}{\pi} \int_0^{5\pi} \exp[-z^2/(2 \sin^2 \phi)] d\phi$  leads to simpler calculations.

#### Main Points

- Capacity-achieving transmission scheme uses variable-rate variable-power transmission with power water-filling in time.
- Power and rate adaptation does not significantly increase capacity, and rate adaptation alone yields no increase. These results may not carry over to practical schemes.
- Channel inversion practical but has poor performance. Performance improved by truncating.
- Capacity of frequency-selective fading channels obtained by breaking up wideband channel into subbands (similar to multicarrier).
- Linear modulation encodes information in transmitted signal amplitude and/or phase via its signal constellation.
- Can approximate symbol error probability  $P_s$  of MPSK and MQAM in AWGN using simple formula:  $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$ .
- Can use standard or alternate Q function representation for calculation: Alternate form greatly simplifies average probability of error calculations in fading and with diversity.