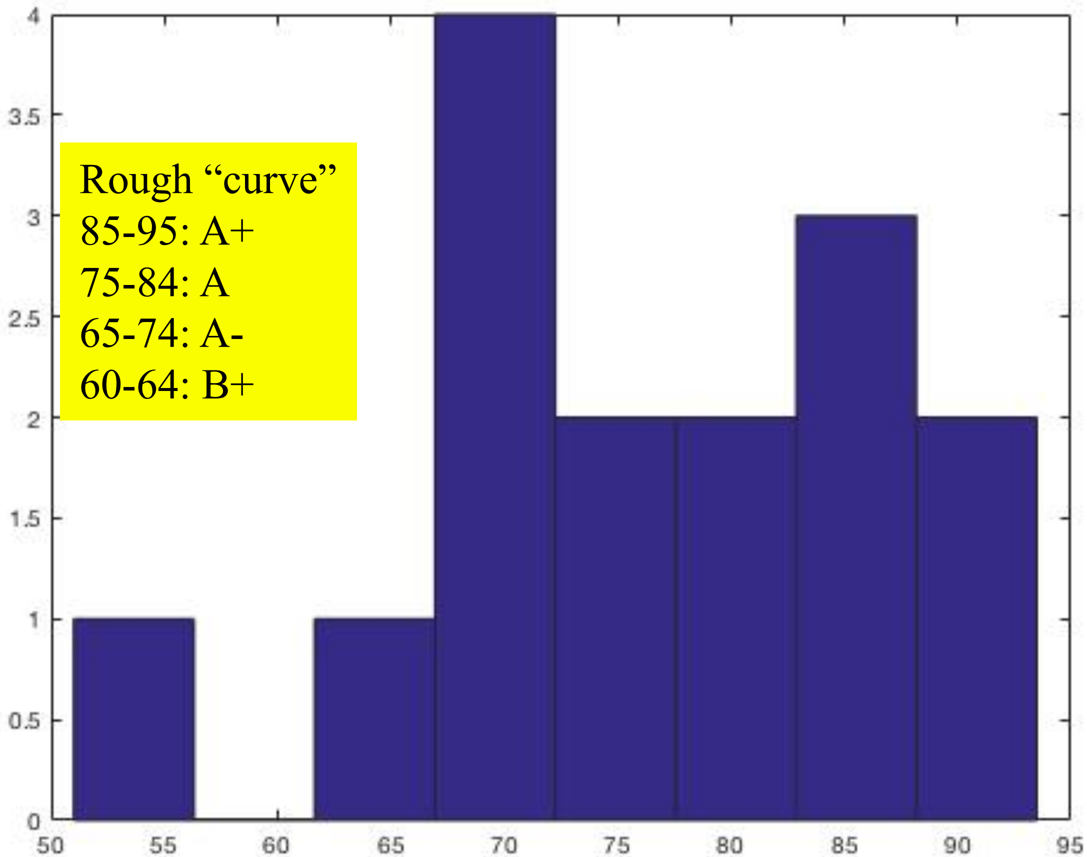
EE359 – Lecture 15 Outline

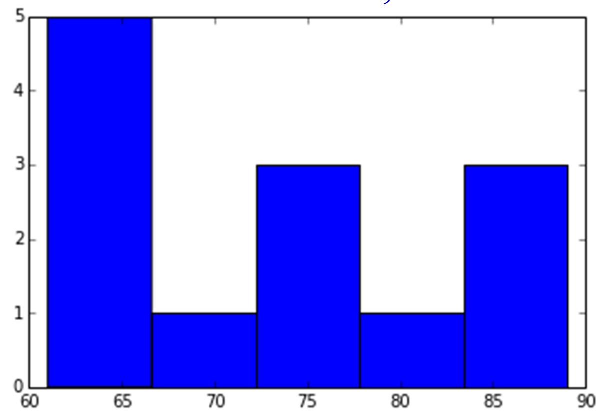
- Announcements:
  - HW posted, due Friday
  - MT exam grading done
    - Can pick up after class or from Julia
  - My OHs Thursday moved to 11-12 outside classroom
  - Project feedback by tomorrow
- MIMO Channel Capacity
- MIMO Beamforming
- Diversity/Multiplexing Tradeoffs
- MIMO Receiver Design

Midterm Grade Distribution

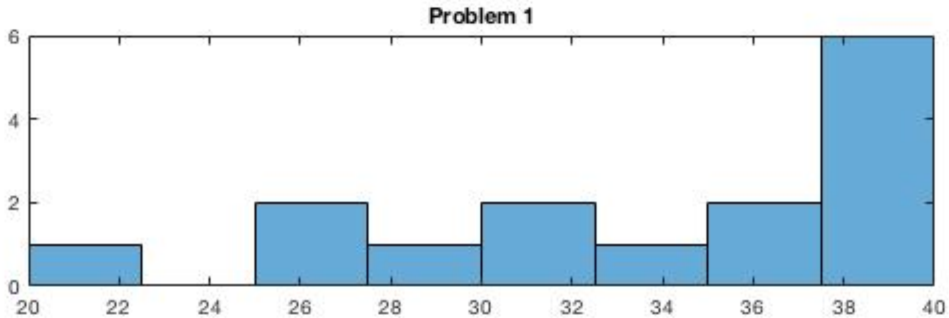
Mean: 76, STD: 11



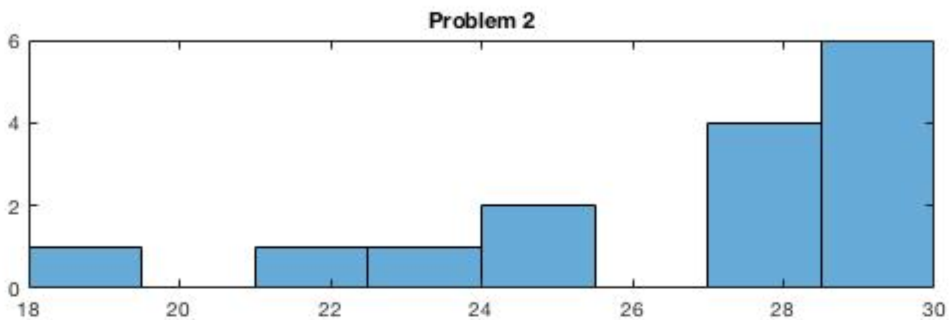
2016: Mean: 73.08, STD:10.4.



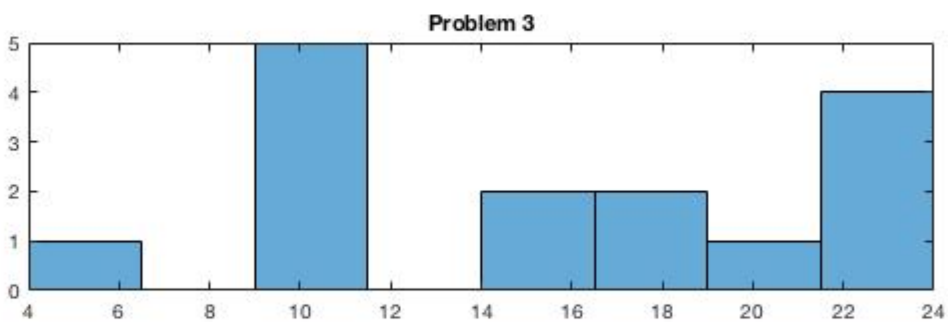
Grade breakdown by problem



Capacity of flat-fading channels



Performance in shadowing and fading w/wout diversity

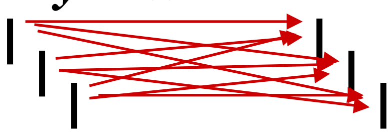


Time-varying channel characterization

Review of Last Lecture

- MIMO systems have multiple TX and RX antennas
  - System model defined via matrices and vectors
  - Channel decomposition: TX precoding, RX shaping

$\boldsymbol{y=Hx+n}$



$\mathrm{H=U\Sigma V^H}$

$\tilde{\boldsymbol{y}}=\boldsymbol{\Sigma}\tilde{\boldsymbol{x}}+\tilde{\boldsymbol{n}}$

$\tilde{y}_i=\sigma_i\tilde{x}+\tilde{n}_i$

- Capacity of MIMO Systems
  - Depends on what is known at TX/RX and if channel is static or fading
  - For static channel with perfect CSI at TX and RX, power water-filling over space is optimal:
  - Without transmitter channel knowledge, capacity metric is based on an outage probability
    - P_{out} is the probability that the channel capacity given the channel realization is below the transmission rate.
  - Massive MIMO: in high SNR, singular values converge to a constant: $C=\min(M_t,M_r)B\log(1+\rho)$: *will revisit after fading analysis*

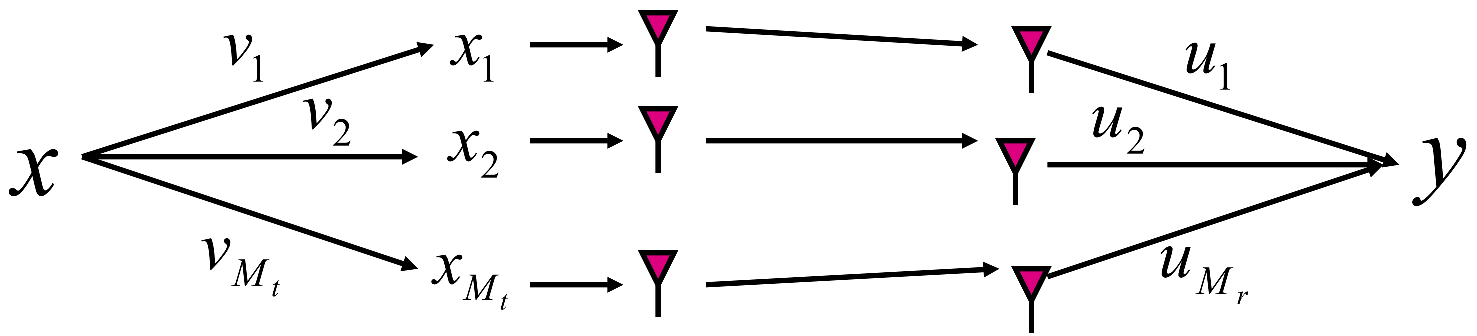
MIMO Fading Channel Capacity

- If channel H known, waterfill over space (fixed power at each time instant) or space-time
- Without transmitter channel knowledge, capacity is based on an outage probability
  - P_{out} is the probability that the channel capacity given the channel realization is below the transmission rate.

$$P_{out} = p\left(\mathbf{H} : B\log_2 \det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] > C\right).$$

Beamforming

- Scalar codes with transmit precoding

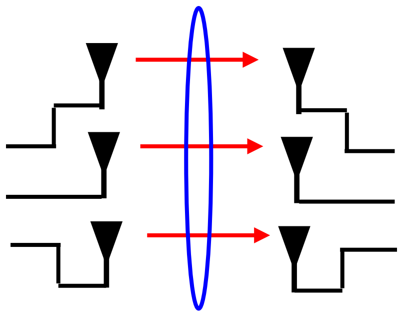


$$\boxed{y = \boldsymbol{u}^H \boldsymbol{H} \boldsymbol{v} x + \boldsymbol{u}^H \boldsymbol{n}}$$

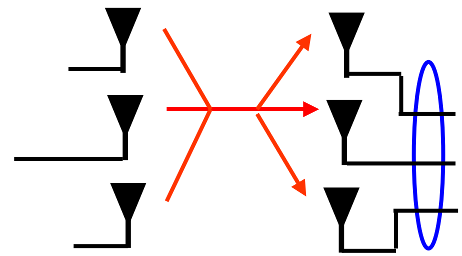
- Transforms system into a SISO system with diversity.
  - Array and diversity gain
  - Greatly simplifies encoding and decoding.
  - Channel indicates the best direction to beamform
  - Need “sufficient” knowledge for optimality of beamforming

Diversity vs. Multiplexing

- Use antennas for multiplexing or diversity



Error Prone



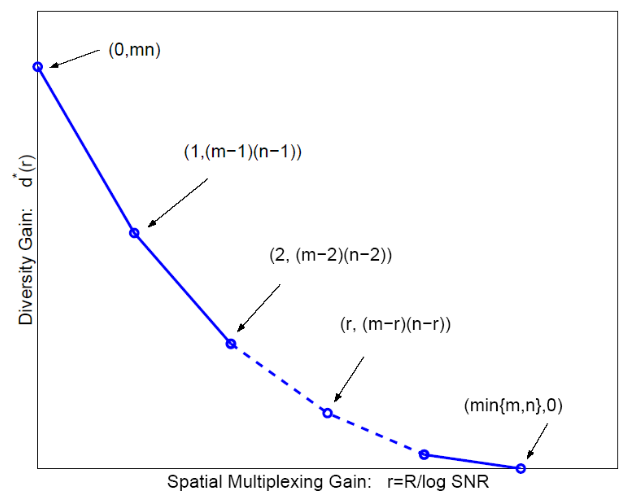
Low P_e

- Diversity/Multiplexing tradeoffs (Zheng/Tse)

$$\lim_{SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR} = -d$$

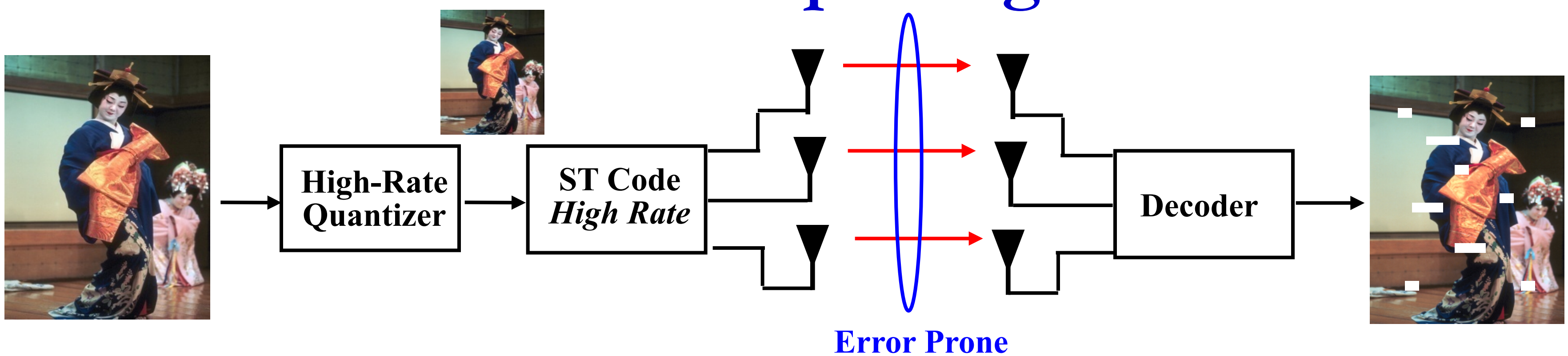
$$\lim_{SNR\to\infty} \frac{R(SNR)}{\log SNR} = r$$

$$\boxed{d^*(r) = (M_t - r)(M_r - r)}$$

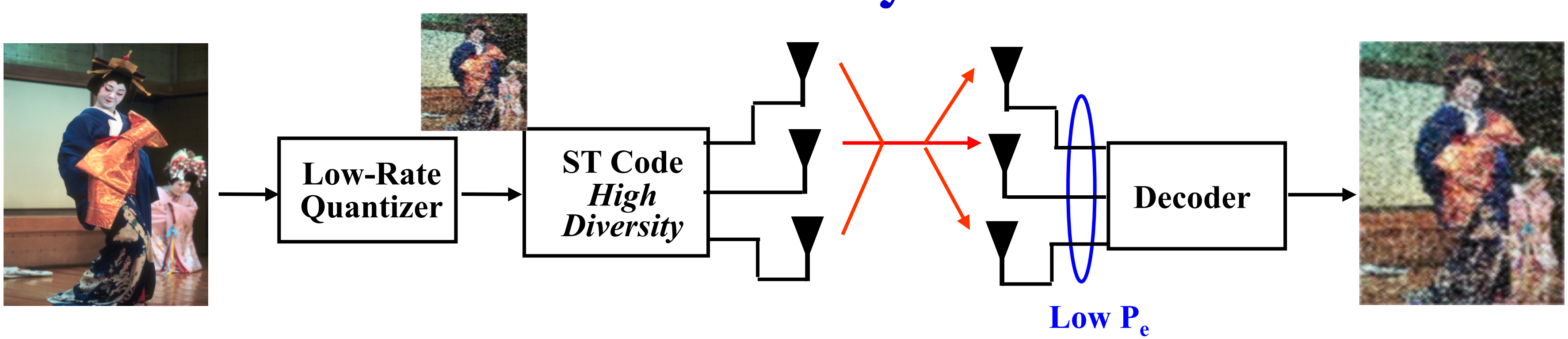


How should antennas be used?

- Use antennas for multiplexing:



- Use antennas for diversity



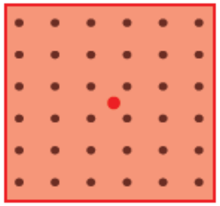
Depends on end-to-end metric: *Solve by optimizing app. metric*

MIMO Receiver Design

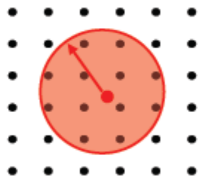
- Optimal Receiver:
  - Maximum likelihood: finds input symbol most likely to have resulted in received vector
  - Exponentially complex # of streams and constellation size
- Linear Receivers
  - Zero-Forcing: forces off-diagonal elements to zero, enhances noise
  - Minimum Mean Square Error: Balances zero forcing against noise enhancement
- Sphere Decoder:
  - Only considers possibilities within a sphere of received symbol.
    - If minimum distance symbol is within sphere, optimal, otherwise null is returned

$$\hat{x} = \arg\min_{x} |y - Hx|^2$$

ML Decoding



Sphere Decoding



$$\hat{x} = \arg\min_{x:|Q^H y - Rx| < r} |Q^H y - Rx|^2$$

Main Points

- Capacity of fading MIMO systems
  - With TX and RX channel knowledge, water-fill power over space or space-time to achieve capacity
  - Without TX CSI, outage is the capacity metric
  - For asymptotically large arrays, at high SNR, capacity is constant
- Beamforming transforms MIMO system into a SISO system with TX and RX diversity.
  - Beamform along direction of maximum singular value
- MIMO introduces diversity/multiplexing tradeoff
  - Optimal use of antennas depends on application
- MIMO RX design trades complexity for performance
  - ML detector optimal - exponentially complex
  - Linear receivers balance noise enhancement against stream interference
  - Sphere decoding provides near ML performance with linear complexity