EE359 – Lecture 9 Outline

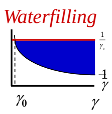
- Announcements:
  - Project proposals due this Friday at 5pm
  - Midterm will likely be @ Nov. 10, 6-8pm.
  - HWs will be due Friday 5pm SHARP going forward
- Linear Modulation Review
- Linear Modulation Performance in AWGN
- Q-Function representations
- Probability of error in fading
- Outage probability
- Average P_s (P_b)

Review of Last Lecture

- Capacity in Flat-Fading: γ known at TX/RX
  - Optimal Rate and Power Adaptation

$$C = \max_{S(\gamma):E[S(\gamma)]=\bar{S}} \int_0^\infty B\log_2\left(1+\frac{\gamma S(\gamma)}{\bar{S}}\right)p(\gamma)d\gamma$$

$$\frac{C}{B} = \int_{\gamma_0}^\infty \log_2\left(\frac{\gamma}{\gamma_0}\right)p(\gamma)d\gamma.$$



  - Channel Inversion and Truncated Inversion
    - Received SNR constant; Capacity is $B\log_2(1+\sigma)$ above an outage level associated with truncation
- Capacity of ISI channels
  - Water-filling of power over freq; or time and freq.

Passband Modulation Tradeoffs

- Want high rates, high spectral efficiency, high power efficiency, robust to channel, cheap. **Our focus**
- Amplitude/Phase Modulation (MPSK,MQAM)
  - Information encoded in amplitude/phase
  - More spectrally efficient than frequency modulation
  - Issues: differential encoding, pulse shaping, bit mapping.
- Frequency Modulation (FSK)
  - Information encoded in frequency
  - Continuous phase (CPFSK) special case of FM
  - Bandwidth determined by Carson's rule (pulse shaping)
  - More robust to channel and amplifier nonlinearities

Amplitude/Phase Modulation

- Signal over ith symbol period:

$$s(t) = s_{i1}g(t)\cos(2\pi f_c t+\phi_0) - s_{i2}g(t)\sin(2\pi f_c t+\phi_0)$$

  - Pulse shape g(t) typically Nyquist
  - Signal constellation defined by (s_{i1},s_{i2}) pairs
  - Can be differentially encoded
  - M values for $(s_{i1},s_{i2}) \Rightarrow \log_2 M$ bits per symbol
- P_s depends on
  - Minimum distance d_{min} *(depends on γ_s)*
  - # of nearest neighbors α_M
  - Approximate expression: $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$

Alternate Q Function Representation

- Traditional Q function representation

$$Q(z) = p(x>z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2}dx, \quad x \sim N(0,1)$$

  - Infinite integrand
  - Argument in integral limits
- New representation (Craig'93)

$$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/(\sin^2\varphi)}d\varphi$$

  - Leads to closed form solution for P_s in PSK
  - Very useful in fading and diversity analysis

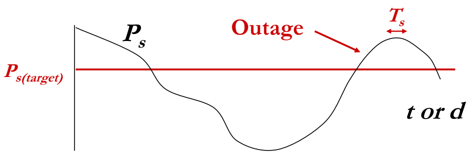
Linear Modulation in Fading

- In fading γ_s and therefore P_s random
- Performance metrics:
  - Outage probability: $p(P_s>P_{target})=p(\gamma<\gamma_{target})$
  - Average P_s , $\bar{P}_s$:

$$\bar{P}_s = \int_0^\infty P_s(\gamma)p(\gamma)d\gamma$$

  - Combined outage and average P_s (next lecture)

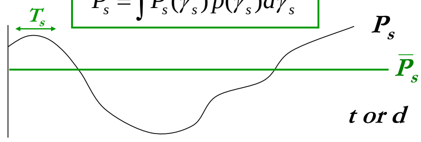
Outage Probability



- Probability that P_s is above target
- Equivalently, probability γ_s below target
- Used when $T_c >> T_s$

Average P_s

$$\bar{P}_s = \int P_s(\gamma_s) p(\gamma_s) d\gamma_s$$



- Expected value of random variable P_s
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone
- Alternate Q function approach: $Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/(\sin^2\varphi)} d\varphi$
  - Simplifies calculations (Get a Laplace Xfm)

Main Points

- Linear modulation more spectrally efficient but less robust than nonlinear modulation
- P_s approximation in AWGN: $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$
  - Alternate Q function representation simplifies calculations
- In fading P_s is a random variable, characterized by average value, outage, or combined outage/average
- Outage probability based on target SNR in AWGN.
- Fading greatly increases average P_s.