

EE359 – Lecture 6 Outline

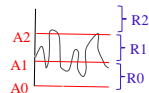
- **Announcements:**
 - No lecture Mon, lectures 10/19 6pm, 10/21 9:30am
 - HW this week deadline extended to Friday 5pm
 - Next HW posted today, due Friday 10/21@5pm
- Review of Last Lecture
- Signal Envelope Distributions
- Average Fade Duration
- Markov Models
- Wideband Multipath Channels
- Scattering Function
- Multipath Intensity Profile
- Doppler Power Spectrum

Signal Envelope Distribution

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
 - Similar to Ricean, but models “worse than Rayleigh”
 - Lends itself better to closed form BER expressions

Markov Models for Fading

- Model for fading dynamics
 - Simplifies performance analysis



- Divides range of fading power into discrete regions $R_j = \{\gamma: A_j \leq \gamma < A_{j+1}\}$
 - A_j s and # of regions are functions of model
- Transition probabilities (L_j is LCR at A_j):

$$p_{j,j+1} = \frac{L_{j+1}T}{\pi_j}, p_{j,j-1} = \frac{L_j T}{\pi_j}, p_{j,j} = 1 - p_{j,j+1} - p_{j,j-1}$$

Review of Last Lecture

- For $\phi_n \sim U[0, 2\pi]$, $r_I(t), r_Q(t)$ zero mean, WSS, with

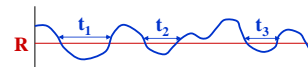
$$A_n(\tau) = \overline{P_r} E_{\theta_n} [\cos 2\pi f_{D_n} \tau] = A_{r_Q}(\tau), f_{D_n} = v \cos \theta_n / \lambda$$

$$A_{r_I, r_Q}(\tau) = \overline{P_r} E_{\theta_n} [\sin 2\pi f_{D_n} \tau] = -A_{r_I, r_Q}(\tau)$$
- Uniform AoAs in Narrowband Model
 - In-phase/quad comps have zero cross correlation and

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = PJ_0(2\pi f_D \tau)$$
Decorrelates over roughly half a wavelength
 - PSD is maximum at the maximum Doppler frequency
 - PSD used to generate simulation values

Level crossing rate and Average Fade Duration

- LCR: rate at which the signal crosses a fade value
- AFD: How long a signal stays below target R/SNR
 - Derived from LCR

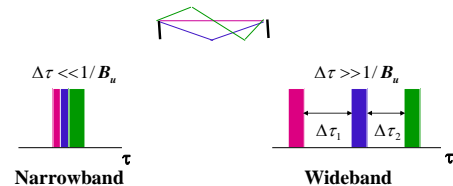


- For Rayleigh fading

$$\bar{t}_R = (e^{\rho^2} - 1) / (\rho f_D \sqrt{2\pi})$$
 - Depends on ratio of target to average level (ρ)
 - Inversely proportional to Doppler frequency

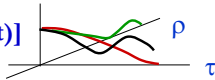
Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth



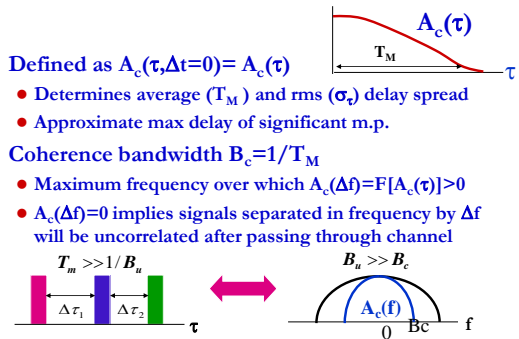
Scattering Function

- Fourier transform of $c(\tau, t)$ relative to t
- Typically characterize its statistics, since $c(\tau, t)$ is different in different environments
- Underlying process WSS and Gaussian, so only characterize mean (0) and correlation
- Autocorrelation is $A_c(\tau_1, \tau_2, \Delta t) = A_c(\tau, \Delta t)$
- Statistical scattering function:

$$s(\tau, \rho) = \mathcal{F}_{\Delta t}[A_c(\tau, \Delta t)]$$


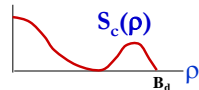
Multipath Intensity Profile

- Defined as $A_c(\tau, \Delta t=0) = A_c(\tau)$
 - Determines average (T_M) and rms (σ_τ) delay spread
 - Approximate max delay of significant m.p.
- Coherence bandwidth $B_c = 1/T_M$
 - Maximum frequency over which $A_c(\Delta f) = \mathcal{F}[A_c(\tau)] > 0$
 - $A_c(\Delta f) = 0$ implies signals separated in frequency by Δf will be uncorrelated after passing through channel



Doppler Power Spectrum

- $S_c(\rho) = \mathcal{F}[A_c(\tau=0, \Delta t)] = \mathcal{F}[A_c(\Delta t)]$
- Doppler spread B_d is maximum doppler for which $S_c(\rho) > 0$.
- Coherence time $T_c = 1/B_d$
 - Maximum time over which $A_c(\Delta t) > 0$
 - $A_c(\Delta t) = 0$ implies signals separated in time by Δt will be uncorrelated after passing through channel



Main Points

- Fading distribution depends on environment
 - Rayleigh, Rician, and Nakagami all common
- Average fade duration determines how long a user is in continuous outage (e.g. for coding design)
- Markov model approximates fading dynamics.
- Scattering function characterizes rms delay and Doppler spread. Key parameters for system design.
- Delay spread defines maximum delay of significant multipath components. Inverse is coherence BW
- Doppler spread defines maximum nonzero doppler, its inverse is coherence time