EE359 – Lecture 5 Outline

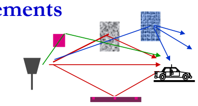
- Announcements:
  - No lecture Mon 10/17
  - Lecture Wed. 10/19 moved to 6pm (w/pizza)
  - Makeup lecture 10/21 9:30-10:45am (w/donuts)

Correction: CLT (not LLN) means: $\lim_{n\to\infty}\sum_{i=1}^{n} x_i \Rightarrow Gaussian\ for\ x_i\ iid$

- Review of Last Lecture
- Narrowband Fading Model
- In-Phase and Quad Signal Components
- Crosscorrelation of RX Signal in NB Fading
- Correlation and PSD in uniform scattering
- Signal Envelope Distributions

Review of Last Lecture

- Model Parameters from Measurements
- Random Multipath Model
- Channel Impulse Response



$$c(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\varphi_n(t)}\delta(\tau-\tau_n(t))$$

- Received signal characteristics
  - Many multipath components
  - Amplitudes change slowly
  - Phases change rapidly

Narrowband Model

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)|<<1/B$
- Then $u(t)\approx u(t-\tau)$.
- Received signal given by

$$r(t)=\Re\left\{u(t)e^{j2\pi f_c t}\left[\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\right]\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=e^{j\phi_0}$

In-Phase and Quadrature under CLT Approximation

- In phase and quadrature signal components:

$$r_I(t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\cos(2\pi f_c t),$$

$$r_Q(t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\sin(2\pi f_c t)$$

- For $N(t)$ large, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT (sum of large # of random vars).
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- If $\varphi_n(t)$ uniform, the in-phase/quad components are mean zero, indep., and stationary.

Auto and Cross Correlation

$r_I(t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\cos(2\pi f_c t),\ \ r_Q(t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\sin(2\pi f_c t),\ \ \phi_n\sim U[0,2\pi]$

- Recall that θ_n is the multipath arrival angle
- Autocorrelation of inphase/quad signal is

$$A_{r_I}(\tau)=A_{r_Q}(\tau)=PE_{\theta_n}[\cos 2\pi f_{D_n}\tau],\quad f_{D_n}=v\cos\theta_n/\lambda$$

- Cross Correlation of inphase/quad signal is

$$A_{r_I,r_Q}(\tau)=PE_{\theta_n}[\sin 2\pi f_{D_n}\tau]=-A_{r_I,r_Q}(\tau)$$

- Autocorrelation of received signal is

$$A_r(\tau)=A_{r_I}(\tau)\cos(2\pi f_c\tau)-A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

Uniform AOAs

- Under uniform scattering, in phase and quad comps have no cross correlation and autocorrelation is

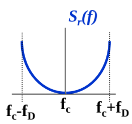
$$A_{r_I}(\tau)=A_{r_Q}(\tau)=PJ_0(2\pi f_D\tau)$$

Decorrelates over roughly half a wavelength

- The PSD of received signal is

$$S_r(f)=.25[S_{r_I}(f-f_c)+S_{r_I}(f+f_c)]$$

$$S_{r_I}(f)=\mathcal{F}[PJ_0(2\pi f_D\tau)]$$



Used to generate simulation values

Signal Envelope Distribution

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models "worse than Rayleigh"
  - Lends itself better to closed form BER expressions

Main Points

- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath
- Uniform scattering makes autocorrelation of inphase and quad comps of RX signal follow Bessel function
  - Signal components decorrelate over half wavelength
  - The PSD has a bowel shape centered at carrier frequency
- Fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common