# EE359 – Lecture 4 Outline

- Announcements:
  - 1st HW due tomorrow.
  - Lecture changes (free food ☺)
    - No lecture Mon 10/17
    - Lecture Wed. 10/19 moved to 11:55-1:05? 3-4:15? 4-5:15?
    - Makeup lecture 10/21 @ 9:30-10:45? 11:55-1:05? 3-4:15?
- Review of Last Lecture
- Model Parameters from Empirical Measurements
- Random Multipath Model
- Time Varying Channel Impulse Response
- Narrowband Fading Model
- In-Phase and Quad Signal Components
- Auto and Crosscorrelation of received signal

# Review of Last Lecture

- Shadowing
  - Log-normal random variable based on LLN applied to many attenuating objects
- Combined Path Loss and Shadowing

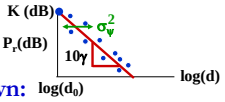
$$\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma \log_{10}\left(\frac{d}{d_0}\right) + \psi_{dB},\ \psi_{dB} \sim N(\mu_\psi, \sigma_\psi^2)$$

*$\mu_\psi$=0 when average shadowing incorporated into K and $\gamma$, else $\mu_\psi$<0*

- Cell Coverage Area
  - % of locations in a cell meeting a min. SNR criterion
  - Correction:

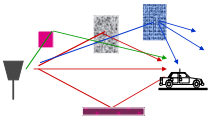
$$C = \frac{1}{\pi R^2}\int_{cell\ area} P_A dA = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R P_A r dr d\theta$$

# Model Parameters from Empirical Measurements




- Fit model to data
- Path loss (K,γ), $d_0$ known:
  - "Best fit" line through dB data
  - K obtained from measurements at $d_0$.
  - Exponent is MMSE estimate based on data
  - Captures mean due to shadowing
- Shadowing variance
  - Variance of data relative to path loss model (straight line) with MMSE estimate for γ

# Statistical Multipath Model



- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response

# Time Varying Impulse Response

- Response of channel at t to impulse at t-τ:

$$c(\tau, t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\varphi_n(t)} \delta(\tau - \tau_n(t))$$

  - t is time when impulse response is observed
  - t-τ is time when impulse put into the channel
  - τ is how long ago impulse was put into the channel for the current observation
    - path delay for MP component currently observed

# Received Signal Characteristics

- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly
  - Constructive and destructive addition of signal components
  - Amplitude fading of received signal (both wideband and narrowband signals)

## Narrowband Model

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)| << 1/B$
- Then $u(t) \approx u(t-\tau)$.
- Received signal given by

$$r(t) = \Re\left\{u(t)e^{j2\pi f_c t}\left[\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\right]\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=e^{j\phi_0}$

## In-Phase and Quadrature under CLT Approximation

- In phase and quadrature signal components:

$$r_I(t) = \sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\cos(2\pi f_c t),$$

$$r_Q(t) = \sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\sin(2\pi f_c t)$$

- For $N(t)$ large, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT (sum of large # of random vars).
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- If $\varphi_n(t)$ uniform, the in-phase/quad components are mean zero, indep., and stationary.

## Auto and Cross Correlation

- Assume $\phi_n \sim U[0,2\pi]$
- Recall that $\theta_n$ is the multipath arrival angle
- Autocorrelation of inphase/quad signal is

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = PE_{\theta_n}[\cos 2\pi f_{D_n}\tau], \quad f_{D_n} = v\cos\theta_n/\lambda$$

- Cross Correlation of inphase/quad signal is

$$A_{r_I,r_Q}(\tau) = PE_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau)$$

- Autocorrelation of received signal is

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

## Main Points

- Path loss and shadowing parameters are obtained from empirical measurements
- Statistical multipath model leads to a time-varying channel impulse response
- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath