Capacity of F.S. Fading Channels, Linear Modulation and its Performance in AWGN and Fading

Lecture Outline

- **Capacity of Frequency-Selective Fading Channels**
- **Linear Modulation**
- **Performance of Linear Modulation in AWGN**
- **Performance Metrics in Flat Fading**
- **Outage Probability**
- **Average Probability of Error**

1. **Capacity of Frequency Selective Fading Channels**

   - Capacity for time-invariant frequency-selective fading channels is a "water-filling" of power over frequency.
   - For time-varying ISI channels, capacity is unknown in general. Approximate by dividing up the bandwidth subbands of width equal to the coherence bandwidth (same premise as multicarrier modulation) with independent fading in each subband.
   - Capacity in each subband obtained from flat-fading analysis. Power is optimized over both frequency and time.

2. **Linear versus Nonlinear Modulation:**

   - Modulation Tradeoffs: High bits rates; High spectral efficiency (bps/Hz); High power efficiency (minimum SNR for target P_b); Robustness to channel and implementation impairments; Low cost implementation;
   - Linear modulation: bits encoded in amplitude (PAM), phase (PSK) or both (MQAM).
   - Nonlinear modulation: bits encoded in frequency (FSK).
   - Linear modulation more spectrally efficient. Nonlinear modulation has constant envelope, so less susceptible to amplitude and phase nonlinearities introduced by the channel and/or hardware. Focus in this class is on linear modulation.

3. **Linear Modulation**

   - Over the ith symbol period, bits are encoded in carrier amplitude or phase $s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$, where $\phi_1(t) = g(t)\cos(2\pi f_c t + \phi_0)$ and $\phi_2(t) = g(t)\sin(2\pi f_c t + \phi_0)$ for initial phase offset ϕ_0.
   - Pulse shape $g(t)$ determines signal bandwidth, and is typically Nyquist.
   - Baseband representation is $s(t) = \Re\{x(t)e^{j\phi_0}e^{j2\pi f_c t}\}$ for $x(t) = (s_{i1} + js_{i2})g(t)$.
   - The constellation point (s_{i1}, s_{i2}) has M possible values, hence there are $\log_2 M$ bits per symbol.

4. **Performance of Linear Modulation in AWGN:**

   - ML detection corresponds to decision regions.

- For coherent modulation, probability of symbol error P_s depends on the number of nearest neighbors α_M, and the ratio of their distance d_{min} to the square root $\sqrt{N_0}$ of the noise power spectral density (this ratio is a function of the SNR γ_s).
- P_s approximated by $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$, where β_M depends on the modulation.
- Alternate Q function representation $Q(z) = \frac{1}{\pi}\int_0^{.5\pi} \exp[-z^2/(2\sin^2\phi]d\phi$ leads to simpler calculations.

5. **Differential Modulation:**
   - Encodes information bits in phase difference between successive symbols, eliminating the need for a coherent phase reference at receiver.
   - Removes the effect of slow phase drift, but causes irreducible error floor at high Doppler (phase decorrelated between symbols).

6. **Performance of Linear Modulation in Fading:**
   - In fading γ_s and therefore P_s are random variables.
   - Three performance metrics to characterize the random P_s.
   - Outage: $p(P_s > P_{target} = p(\gamma < \gamma_{target})$
   - Average P_s ($\overline{P}_s = \int P_s(\gamma)p(\gamma)d\gamma$).
   - Combined outage and average P_s.

7. **Outage Probability:** $p(P_s < P_{s_{\text{target}}}) = p(\gamma_s < \gamma_{s_{\text{target}}})$.
   - Outage probability used when fade duration long compared to a symbol time.
   - Obtained directly from fading distribution and target γ_s.
   - Can obtain simple formulas for outage in log-normal shadowing or in Rayleigh fading.

8. **Average P_s:** $\overline{P}_s = \int P_s(\gamma_s)p(\gamma_s)d\gamma_s$.
   - Rarely leads to close form expressions for general $p(\gamma_s)$ distributions, and can be hard to evaluate numerically.
   - Can obtain closed form expressions for general linear modulation in Rayleigh fading (using approximation $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$ in AWGN).
   - Using alternate Q function representation, the average P_s becomes the moment generating function of the distribution: easy to calculate for any modulation and any fading distribution using standard Laplace transforms.

Main Points

- Capacity of frequency-selective fading channels obtained by breaking up wideband channel into subbands (similar to multicarrier).
- Linear modulation more spectrally efficient with higher rates than nonlinear, but less robust to channel errors.
- Can approximate symbol error probability P_s of MPSK and MQAM in AWGN using simple formula: $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$.
- Alternate Q function representation greatly simplifies calculations of P_s and $\overline{P}_s$.
- In fading, P_s is a random variable, characterized by average value, outage, or combined outage and average.
- Outage probability based on target SNR in AWGN.
- Fading greatly increases average P_s. Easy to compute using alternate Q function.