

EE359 – Lecture 9 Outline

- Announcements
 - MT tentatively scheduled Wed. Nov. 4 8:45-10:45am.
- Capacity of Freq. Selective Fading Channels
- Linear Modulation Review
- Linear Modulation Performance in AWGN
- Modulation Performance in Fading
 - Outage Probability
 - Average P_s (P_b)

Review of Last Lecture

- Capacity of Flat-Fading Channels
 - Fading Statistics Known (few results)
 - Fading Known at RX (average capacity)

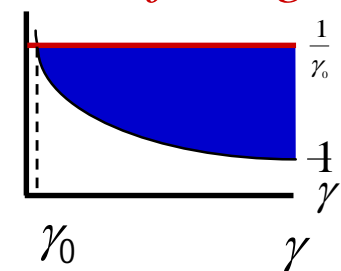
$$C = \int_0^{\infty} B \log_2(1 + \gamma) p(\gamma) d\gamma \leq B \log_2(1 + \bar{\gamma})$$

- Fading Known at TX and RX
 - Optimal Rate and Power Adaptation

$$C = \max_{S(\gamma) : E[S(\gamma)] = \bar{S}} \int_0^{\infty} B \log_2 \left(1 + \frac{\gamma S(\gamma)}{\bar{S}} \right) p(\gamma) d\gamma$$

$$\frac{C}{B} = \int_{\gamma_0}^{\infty} \log_2 \left(\frac{\gamma}{\gamma_0} \right) p(\gamma) d\gamma.$$

Waterfilling

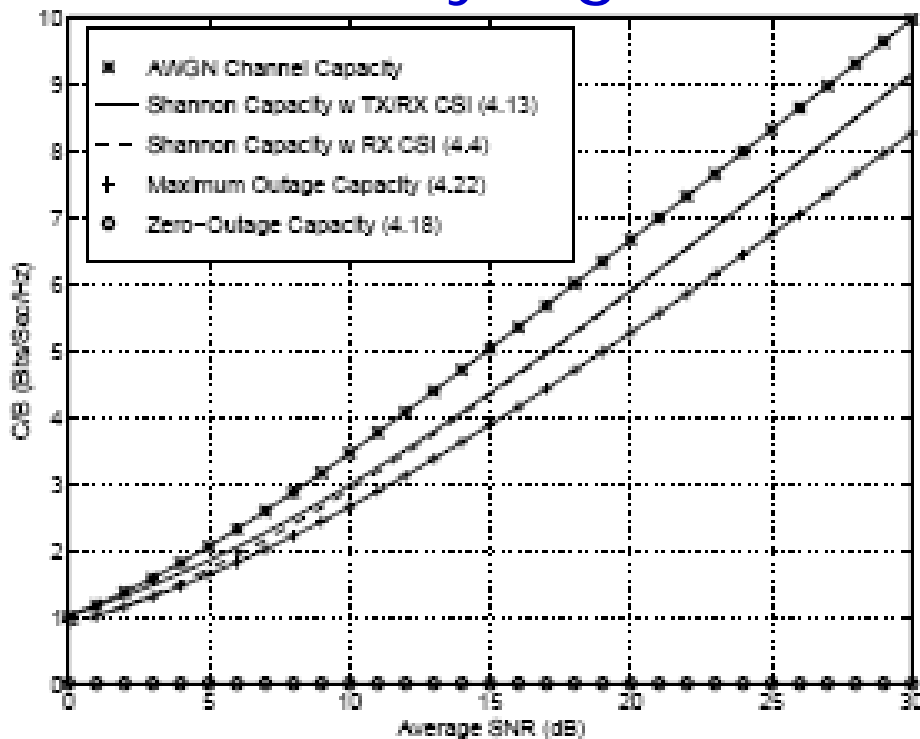


Review Continued: Channel Inversion

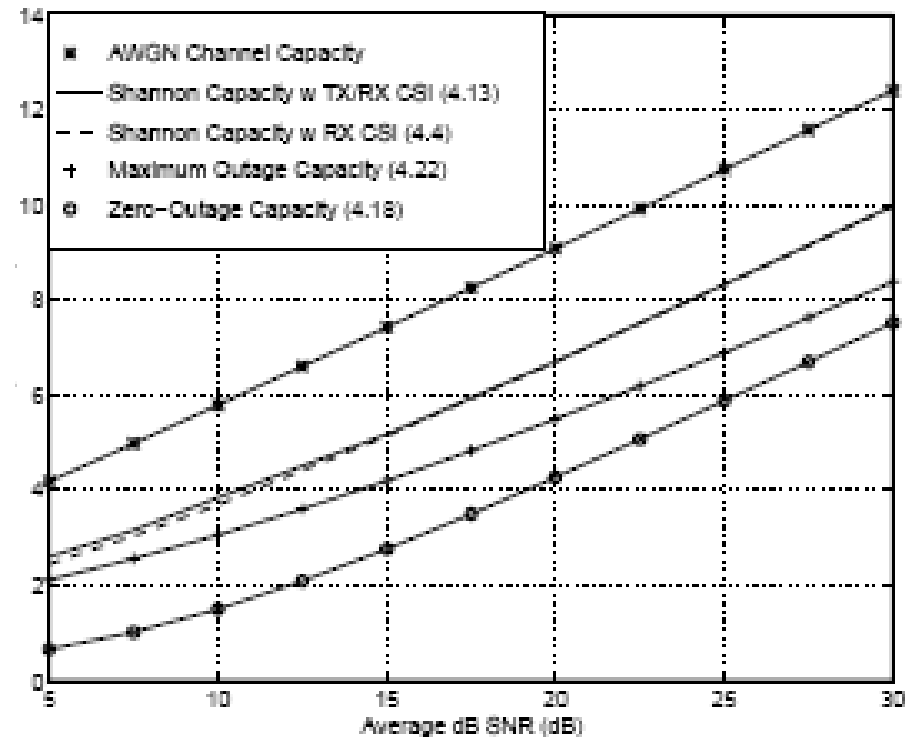
- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
 - Capacity is zero in Rayleigh fading
- Truncated inversion
 - Invert channel above cutoff fade depth
 - Constant SNR (fixed rate) above cutoff
 - Cutoff greatly increases capacity
 - Close to optimal

Capacity in Flat-Fading

Rayleigh

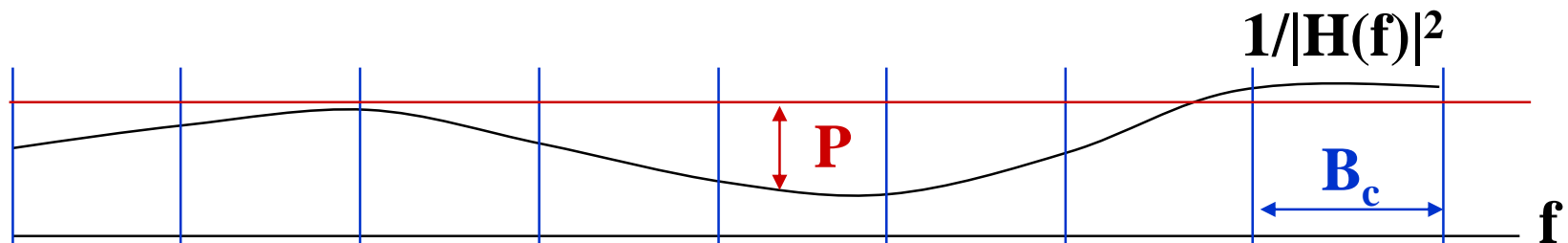


Log-Normal




Frequency Selective Fading Channels

- For TI channels, capacity achieved by water-filling in frequency
- Capacity of time-varying channel unknown
- Approximate by dividing into subbands
 - Each subband has width B_c (like MCM).
 - Independent fading in each subband
 - Capacity is the sum of subband capacities



Passband Modulation Tradeoffs

- Want high rates, high spectral efficiency, high power efficiency, robust to channel, cheap.
- Amplitude/Phase Modulation (MPSK, MQAM) **Our focus**
 - Information encoded in amplitude/phase
 - More spectrally efficient than frequency modulation
 - Issues: differential encoding, pulse shaping, bit mapping.
- Frequency Modulation (FSK)
 - Information encoded in frequency
 - Continuous phase (CPFSK) special case of FM
 - Bandwidth determined by Carson's rule (pulse shaping)
 - More robust to channel and amplifier nonlinearities

Amplitude/Phase Modulation

- Signal over i th symbol period:

$$s(t) = s_{i1}g(t)\cos(2\pi f_c t + \phi_0) - s_{i2}g(t)\sin(2\pi f_c t + \phi_0)$$

- Pulse shape $g(t)$ typically Nyquist
 - Signal constellation defined by (s_{i1}, s_{i2}) pairs
 - Can be differentially encoded
 - M values for $(s_{i1}, s_{i2}) \Rightarrow \log_2 M$ bits per symbol
- P_s depends on
 - Minimum distance d_{\min} (depends on γ_s)
 - # of nearest neighbors α_M
 - Approximate expression:

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$

Alternate Q Function Representation

- Traditional Q function representation

$$Q(z) = p(x > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad x \sim N(0,1)$$

- Infinite integrand
 - Argument in integral limits
- New representation (Craig'93)

$$Q(z) = \frac{1}{\pi} \int_0^{\pi/2} e^{-z^2 / (\sin^2 \varphi)} d\varphi$$

- Leads to closed form solution for P_s in PSK
 - Very useful in fading and diversity analysis

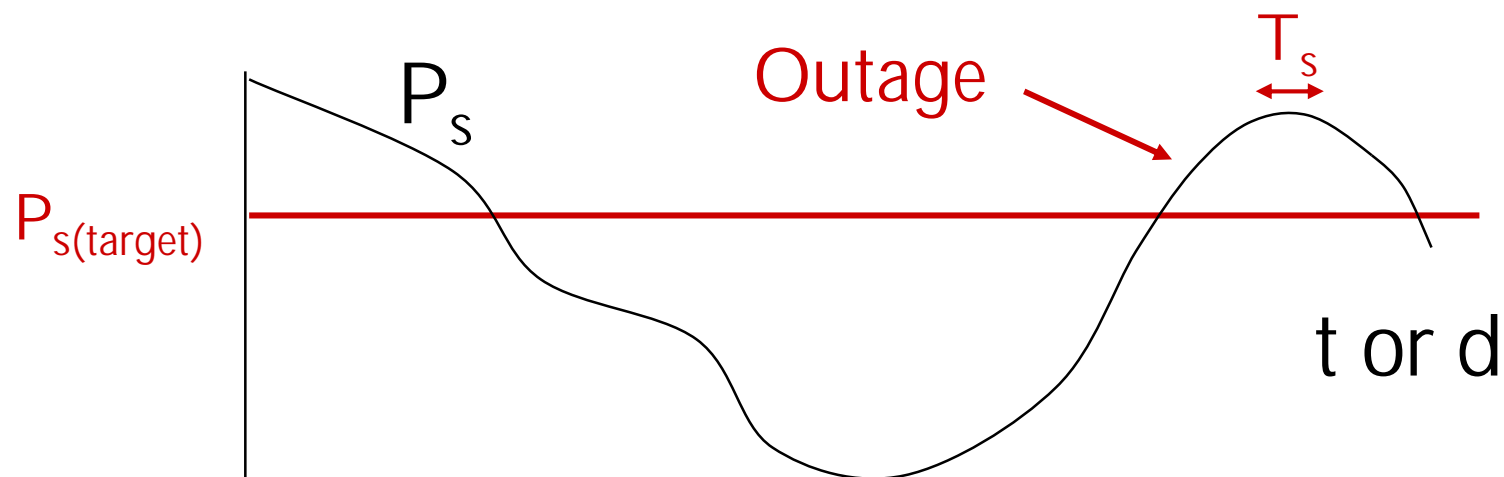
Linear Modulation in Fading

- In fading γ_s and therefore P_s random
- Performance metrics:
 - Outage probability: $p(P_s > P_{\text{target}}) = p(\gamma < \gamma_{\text{target}})$
 - Average P_s , $\overline{P_s}$:

$$\overline{P_s} = \int_0^{\infty} P_s(\gamma) p(\gamma) d\gamma$$

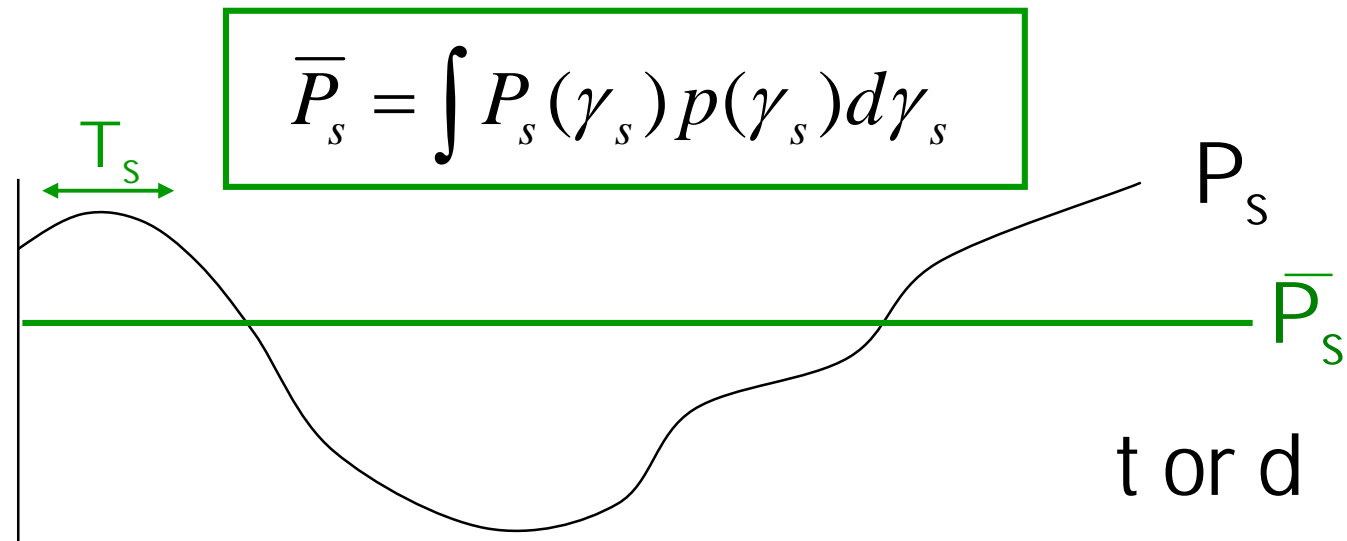
- Combined outage and average P_s (next lecture)

Outage Probability



- Probability that P_s is above target
- Equivalently, probability γ_s below target
- Used when $T_c \gg T_s$

Average P_s



- Expected value of random variable P_s
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone

Main Points

- Capacity of frequency-selective fading channels obtained by breaking it into subchannels
- Linear modulation more spectrally efficient but less robust than nonlinear modulation
- P_s approximation in AWGN: $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$
 - Alternate Q function representation simplifies calculations
- In fading P_s is a random variable, characterized by average value, outage, or combined outage/average
- Outage probability based on target SNR in AWGN.
- Fading greatly increases average P_s .