

Fading Distributions. Average Fade Duration. Wideband Fading Models

Lecture Outline

- Envelope Distributions: Rayleigh, Rician, Nakagami
- Level Crossing Rate and Average Fade Duration
- Markov Models
- Wideband Channel Models
- Scattering Function

1. Signal Envelope Distributions

- CLT approximation leads to Rayleigh distribution (in-phase and quadrature zero mean and jointly Gaussian): $p_Z(z) = \frac{2z}{\overline{P}_r} \exp[-z^2/\overline{P}_r] = \frac{z}{\sigma^2} \exp[-z^2/(2\sigma^2)]$, $z \geq 0$.
- A LOS component leads to a received signal with non-zero mean. The Rician distribution models signal envelope in this case, with K factor dictating the relative power of the LOS component: $p_Z(z) = \frac{z}{\sigma^2} \exp\left[-\frac{(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right)$, $z \geq 0$.
- Experimental results support a Nakagami distribution for some environments. Similar to Rician, but can model “worse than Rayleigh.” Model generally leads to closed-form expressions in BER and diversity analysis: $p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)P_r^m} \exp\left[-\frac{mz^2}{P_r}\right]$, $m \geq .5$.

2. Level Crossing Rate and Average Fade Duration

- Level crossing rate L_R is the rate at which a signal envelope crosses the threshold R .
- In Rayleigh fading the level crossing rate is $L_R = \sqrt{2\pi} f_D \rho e^{-\rho^2}$, where $\rho = R/\sqrt{\Omega_p}$.
- Average Fade Duration (AFD) measures how long a signal’s envelope or power stays below a given target threshold: derived from the level crossing rate.
- For Rayleigh fading $\bar{t}_r = e^{\rho^2-1}/(\rho f_D \sqrt{2\pi})$, where ρ is the ratio of target envelope level to average envelope level (or square root of ratio of target power level to average power level).

3. Markov Models

- Simple model for flat-fading dynamics that is useful in performance analysis.
- Chops range of γ values into regions $R_j = \{\gamma : A_j \leq \gamma < A_{j+1}\}$, where the A_j values and number of regions are parameters of model.
- Assumes fading process follows a Markov chain to go between regions R_j , and can only go between adjacent regions in any timeslot.
- Transition probabilities are $p_{j,j+1} = L_{j+1}T/\pi_j$, $p_{j,j-1} = L_jT/\pi_j$, and $p_{j,j} = 1 - p_{j,j+1} - p_{j,j-1}$ for L_j the level crossing rate at A_j and $\pi_j = p(\gamma \in R_j)$.

4. Wideband Channel Models

- In wideband multipath channels the individual multipath components can be resolved by the receiver. True if $T_m > 1/B$.
- If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).

5. Channel Scattering Function:

- For deterministic channels, the scattering function is defined as the Fourier transform of $c(\tau, t)$ with respect to t .
- Typically $c(\tau, t)$ is unknown, so it must be characterized statistically.
- Since underlying process $c(\tau, t)$ is Gaussian and WSS, only need to characterize its mean and correlation, which is independent of time. We assume $c(\tau, t)$ has mean zero.
- Autocorrelation of $c(\tau, t)$ is $A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t)\delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t)$ since we assume channel response associated with different scatterers is uncorrelated.
- Statistical scattering function defined as $S(\tau, \rho) = \mathcal{F}_{\Delta t}[A_c(\tau, \Delta t)]$. This function measures the average channel gain as a function of both delay τ and Doppler ρ .
- $S(\tau, \rho)$ easy to measure empirically and is used to get average delay spread T_M , rms delay spread σ_τ , and Doppler spread B_d for empirical channel measurements.

Main Points

- The signal envelope under narrowband fading with uniform AOA is Rayleigh. Other common distributions are Ricean (when a LOS component exists) and Nakagami.
- Average Fade Duration used to determine how long a user is in continuous outage (e.g. for coding design).
- Markov model a simple model for flat-fading dynamics; useful in performance analysis.
- Wideband models characterized by scattering function, which measures average channel gain relative to delay and Doppler.
- Scattering function used to obtain key channel characteristics of rms delay spread and Doppler spread, which are important for system design.