

EE359 – Lecture 6 Outline

- Announcements:
 - Makeup lecture Friday (with pizza): last one
 - Extra OHs for me: Friday after class and by appt.
- Review of Last Lecture
- Signal Envelope Distributions
- Level Crossing Rate
- Average Fade Duration
- Markov Models
- Wideband Multipath Channels
- Scattering Function

Review of Last Lecture

- For a narrowband fading model ($T_m \ll 1/B$), received signal has a random, complex amplitude gain
- Under CLT approximation, in-phase and quadrature signal components of received signal are Gaussian processes
 - For $\phi_n \sim \mathcal{U}[0, 2\pi]$, the processes are zero mean, WSS, with

$$A_{r_i}(\tau) = .5\Omega_p E_{\theta_i} [\cos 2\pi f_{D_n} \tau] = A_{r_0}(\tau), \quad f_{D_n} = v \cos \theta_n / \lambda$$

$$A_{r_{i,\theta_0}}(\tau) = .5\Omega_p E_{\theta_i} [\sin 2\pi f_{D_n} \tau] = -A_{r_{i,\theta_0}}(\tau) = 0$$

$$A_r(\tau) = A_{r_i}(\tau) \cos(2\pi f_c \tau) - A_{r_{i,\theta_0}}(\tau) \sin(2\pi f_c \tau)$$
- Auto and cross correlation depends on AOAs of multipath

Review of Last Lecture (Cont'd)

- Uniform AoAs in Narrowband Model
 - In-phase/quad comps have no cross correlation and

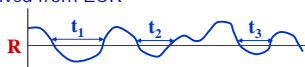
$$A_{r_i}(\tau) = A_{r_0}(\tau) = PJ_0(2\pi f_D \tau) \quad \text{Decorrelates over roughly half a wavelength}$$
 - PSD is maximum at the maximum Doppler frequency
 - PSD used to generate simulation values

Signal Envelope Distribution

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
 - Similar to Ricean, but models "worse than Rayleigh"
 - Lends itself better to closed form BER expressions

Level crossing rate and Average Fade Duration

- LCR: rate at which the signal crosses a fade value
- AFD: How long a signal stays below target R/SNR
 - Derived from LCR



- For Rayleigh fading

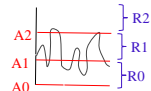
$$\bar{t}_R = (e^{\rho^2} - 1) / (\rho f_D \sqrt{2\pi})$$

- Depends on ratio of target to average level (ρ)
- Inversely proportional to Doppler frequency

Markov Models for Fading

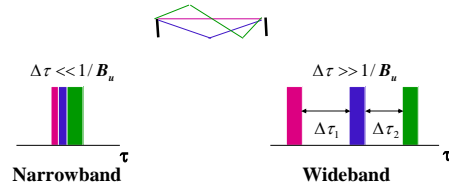
- Model for fading dynamics
 - Simplifies performance analysis
- Divides range of fading power into discrete regions $R_j = \{\gamma: A_j \leq \gamma < A_{j+1}\}$
 - A_j s and # of regions are functions of model
- Transition probabilities (L_j is LCR at A_j):

$$p_{j,j+1} = \frac{L_{j+1}T}{\pi_j}, \quad p_{j,j-1} = \frac{L_j T}{\pi_j}, \quad p_{j,j} = 1 - p_{j,j+1} - p_{j,j-1}$$



Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth



Scattering Function

- Fourier transform of $c(\tau, t)$ relative to t
- Typically characterize its statistics, since $c(\tau, t)$ is different in different environments
- Underlying process WSS and Gaussian, so only characterize mean (0) and correlation
- Autocorrelation is $A_c(\tau_1, \tau_2, \Delta t) = A_c(\tau, \Delta t)$
- Statistical scattering function:

$$s(\tau, \rho) = F_{\Delta t} [A_c(\tau, \Delta t)]$$

The graph shows the statistical scattering function $s(\tau, \rho)$ as a function of delay τ . The vertical axis is labeled ρ . The curve shows a peak at $\tau = 0$ and decays as τ increases. A small sketch of a signal path is shown to the right of the graph.

Main Points

- Narrowband fading distribution depends on environment
 - Rayleigh, Rician, and Nakagami all common
- Average fade duration determines how long a user is in continuous outage (e.g. for coding design)
- Markov model approximates fading dynamics.
- Scattering function characterizes rms delay and Doppler spread. Key parameters for system design.