

Statistical Fading Models

Lecture Outline

- Model Parameters from Empirical Data.
- Random Multipath Model.
- Time Varying Channel Impulse Response.
- Narrowband Approximation.
- In-Phase and Quad Signal Components under CLT.
- Mean and Correlation of I and Q Signals.

1. Model Parameters from Empirical Data:

- Constant K obtained from measurement at distance d_0 .
- Power falloff exponent γ obtained by minimizing the MSE of the predicted model versus the data.
- The resulting path loss model will include average attenuation, so $\mu_{\psi_{dB}} = 0$.
- The shadowing variance $\sigma_{\psi_{dB}}^2$ obtained by determining MSE of the data versus the empirical path loss model with the optimizing γ .

2. Statistical Multipath Model:

- At each time instant there are a random number $N(t)$ of multipath signal components.
- Each component has a random amplitude α_n , phase θ_n , Doppler shift $f_{Dn} = \frac{v}{\lambda} \cos \theta_n$, and path delay $\tau_n = (d_n - d)/c$.
- Thus, received signal is

$$r(t) = \Re \left\{ \left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t - \tau_n(t)) \right] e^{j2\pi f_c t} \right\}.$$

3. Time-Varying Channel Impulse Response:

- The received signal $r(t)$ is the convolution of the input signal with the equivalent low-pass channel impulse response $c(\tau, t)$:

$$r(t) = \Re \left\{ \left(\int_{-\infty}^{\infty} c(\tau, t) u(t - \tau) d\tau \right) e^{j2\pi f_c t} \right\}.$$

- The channel can thus be modeled as a time-varying linear filter:

$$c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t)),$$

where $c(\tau, t)$ is the channel response at time t to an impulse at time $t - \tau$.

- Note there are two time parameters in this expression. The parameter t denotes the time when the impulse response is observed. The parameter $t - \tau$ denotes the time when the impulse was put into the channel relative to the observation time t .

- In other words, τ denotes how long ago the impulse was put into the channel for the current observation.

4. Properties of Received Signal:

- The received signal consists of N multipath components scaled by amplitude $\alpha_n(t)$ and phase $\phi_n(t)$.
- The amplitude term $\alpha_n(t)$ varies slowly (due to shadowing).
- The phase $\phi_n(t)$ varies rapidly due to small shifts in the signal component path delays.
- The phase variation causes rapid variation in the received signal amplitude due to constructive and destructive interference of the multipath components.

5. Narrowband Approximation:

- Define the multipath delay spread as $T_m(t) = \max_n \tau_n(t) - \min_n \tau_n(t)$. For random multipath the delay spread is defined relative to its mean or standard deviation.
- Assume $T_m(t) \ll 1/B$ for all t (or the equivalent for random multipath). Then $u(t) \approx u(t - \tau_n(t))$ for all n and t .
- Received signal simplifies to

$$r(t) = \Re \left\{ u(t) e^{j2\pi f_c t} \left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \right] \right\}$$

- No signal distortion: multipath only affects complex scale factor in brackets.
- Characterize scale factor by assuming $u(t) = 1$

6. In-Phase and Quad Signal Components under CLT

- Received signal can be written in terms of in-phase and quadrature components as $r(t) = r_I(t) \cos(2\pi f_c t) + r_Q(t) \sin(2\pi f_c t)$ where $r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t) \cos(\phi_n(t))$ and $r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t) \sin(\phi_n(t))$.
- If $N(t)$ large then in-phase and quadrature signal components are jointly Gaussian (amplitude of scale factor is Rayleigh).
- Thus, received signal characterized by mean, autocorrelation, and cross correlation.
- Assuming $\phi_n(t)$ uniform, $E[r_I(t)] = E[r_Q(t)] = 0$ and $E[r_I(t)r_Q(t)] = 0$.

Main Points

- Statistical multipath model leads to a time varying channel impulse response
- The resulting received signal has rapidly varying amplitude due to constructive and destructive multipath combining
- Narrowband model and CLT lead to inphase, quadrature, and received signals that are stationary Gaussian processes with zero mean.