

# EE359 – Lecture 4 Outline

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- Announcements:
  - 1<sup>st</sup> HW due tomorrow.
  - Makeup lecture Friday, 12pm-1:10, 102 Hewlitt, wth pizza at 11:45 (also 10/9, these are makeups for Oct. 12-14)
  - Discussion T 4-4:50 in 380-380W (Math dept. basement)
- Review of Last Lecture
- Model Parameters from Empirical Data
- Random Multipath Model
- Time Varying Channel Impulse Response
- Narrowband Approximation for RX Signal
- In-phase and Quadrature RX Signal Components

# Review of Last Lecture

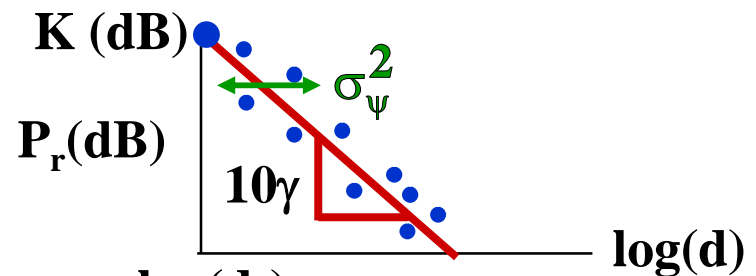
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- Log Normal Shadowing
- Combined Path Loss and Shadowing
- Cell Coverage Area

# Model Parameters from Empirical Measurements

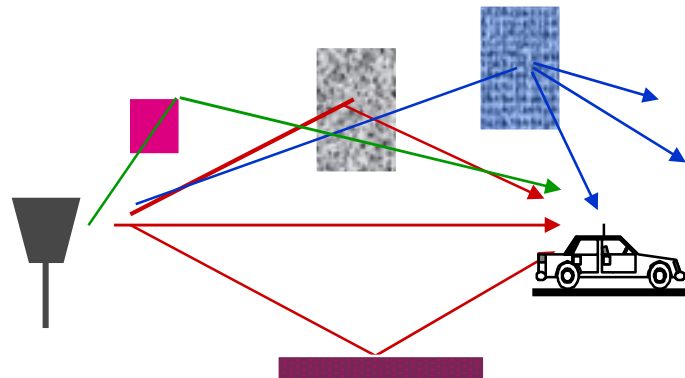
- Fit model to data
- Path loss ( $K, \gamma$ ),  $d_0$  known:
  - “Best fit” line through dB data
  - $K$  obtained from measurements at  $d_0$ .
  - Exponent is MMSE estimate based on data
  - Captures mean due to shadowing
- Shadowing variance
  - Variance of data relative to path loss model (straight line) with MMSE estimate for  $\gamma$



# Statistical Multipath Model

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- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response

# Time Varying Impulse Response

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- Response of channel at  $t$  to impulse at  $t-\tau$ :

$$\mathbf{c}(\tau, \mathbf{t}) = \sum_{n=1}^N \alpha_n(\mathbf{t}) e^{-j\phi_n(\mathbf{t})} \delta(\tau - \tau_n(\mathbf{t}))$$

- $t$  is time when impulse response is observed
- $t-\tau$  is time when impulse put into the channel
- $\tau$  is how long ago impulse was put into the channel for the current observation
  - path delay for MP component currently observed

# Received Signal Characteristics

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- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly
  - Constructive and destructive addition of signal components
  - Amplitude fading of received signal (both wideband and narrowband signals)

# Narrowband Model

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- Assume delay spread  $\max_{m,n} |\tau_n(t) - \tau_m(t)| \ll 1/B$
- Then  $u(t) \approx u(t - \tau)$ .
- Received signal given by

$$r(t) = \Re \left\{ u(t) e^{j2\pi f_c t} \left[ \sum_{n=0}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \right] \right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting  $u(t) = \delta(t)$

# In-Phase and Quadrature under CLT Approximation

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- In phase and quadrature signal components:

$$r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \cos(2\pi f_c t),$$

$$r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \sin(2\pi f_c t)$$

- For  $N(t)$  large,  $r_I(t)$  and  $r_Q(t)$  jointly Gaussian (sum of large # of random vars).
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- If  $\phi_n(t)$  uniform, the in-phase/quad components are mean zero, indep., and stationary.

# Main Points

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- Statistical multipath model leads to a time-varying channel impulse response
- Received signal has random amplitude fluctuations
- Narrowband model and CLT lead to in-phase/quad components that are stationary Gaussian processes
  - Processes completely characterized by their mean, autocorrelation, and cross correlation.
- Assuming uniform phase offsets, process is zero mean with joint expectation also zero.