

# Shadowing, Combined Path Loss/Shadowing, Cell Coverage Area, Model Parameters.

## Lecture Outline

- Log Normal Shadowing
- Combined Path Loss and Shadowing
- Outage Probability
- Cell Coverage Area
- Model Parameters from Empirical Data

### 1. Log-normal shadowing:

- Statistical model for variations in the received signal amplitude due to blockage.
- The received signal power with the combined effect of path loss (power falloff model) and shadowing is, in dB, given by

$$P_r(\text{dB}) = P_u(\text{dB}) + 10 \log_{10} K - 10\gamma \log_{10}(d/d_0) + \psi(\text{dB}).$$

- Empirical measurements support the log-normal distribution for  $\psi$ :

$$p(\psi_{\text{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{\text{dB}}}} \exp \left[ -\frac{(\psi_{\text{dB}} - \mu_{\psi_{\text{dB}}})^2}{2\sigma_{\psi_{\text{dB}}}^2} \right].$$

- This empirical distribution can be justified by a LLN argument.
- The autocorrelation based on measurements follows an autoregressive model:

$$A_{\psi}(\delta) = \sigma_{\psi_{\text{dB}}}^2 e^{-\delta/X_c} = \sigma_{\psi_{\text{dB}}}^2 e^{-v\tau/X_c},$$

where  $X_c$  is the decorrelation distance, which depends on the environment.

### 2. Combined Path Loss and Shadowing

- Linear Model:

$$\frac{P_r}{P_t} = K \left( \frac{d_0}{d} \right)^{\gamma} \psi.$$

- dB Model:

$$\frac{P_r}{P_t}(\text{dB}) = 10 \log_{10} K - 10\gamma \log_{10}(d_0/d) + \psi_{\text{dB}}.$$

### 3. Outage Probability under Path Loss and Shadowing

- With path loss and shadowing, the received power at any given distance between transmitter and receiver is random.
- Outage probability  $p_{\text{out}}(P_{\text{min}}, d)$  is defined as the probability that the received power at a given distance  $d$ ,  $P_r(d)$ , is below a target  $P_{\text{min}}$ :  $p_{\text{out}}(P_{\text{min}}, d) = p(P_r(d) < P_{\text{min}})$ .

- For the simplified path loss model and log normal shadowing this becomes

$$p(P_r(d) \leq P_{min}) = 1 - Q\left(\frac{P_{min} - (P_t + 10 \log_{10} K - 10\gamma \log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right).$$

#### 4. Cell Coverage Area:

- Cellular systems designed for a given average received power  $\bar{P}_r(R)$  at cell boundary.
- Cell coverage area dictates the percentage of locations within the cell with  $P_r \geq \bar{P}_r(R)$ .
- Analysis yields:

$$C = Q(a) + \exp\left(\frac{2 - 2ab}{b^2}\right) Q\left(\frac{2 - ab}{b}\right), \quad a = \frac{P_{min} - \bar{P}_r(R)}{\sigma_{\psi_{dB}}}, \quad b = \frac{10\gamma \log_{10}(e)}{\sigma_{\psi_{dB}}}.$$

- When the minimum power  $P_{min}$  equals the average power at the cell boundary  $\bar{P}_r(R)$ ,  $a = 0$ , and

$$C = \frac{1}{2} + \exp\left(\frac{2}{b^2}\right) Q\left(\frac{2}{b}\right).$$

- Coverage area increases as  $\sigma_{\psi}$  decreases.
- Making  $\bar{P}_r$  much greater than required received power increases coverage area but causes more interference between cells.

#### 5. Model Parameters from Empirical Data:

- Constant  $K$  obtained from measurement at distance  $d_0$ .
- Power falloff exponent  $\gamma$  obtained by minimizing the MSE of the predicted model versus the data.
- The resulting path loss model will include average attenuation, so  $\mu_{\psi_{dB}} = 0$ .
- The shadowing variance  $\sigma_{\psi_{dB}}^2$  obtained by determining MSE of the data versus the empirical path loss model with the optimizing  $\gamma$ .

### Main Points

- Random attenuation due to shadowing modeled as log-normal (empirical parameters).
- Shadowing decorrelates over decorrelation distance.
- Combined path loss and shadowing leads to outage and amoeba-like cell shapes.
- Shadowing affects cell coverage area, defined as the percentage of locations within a cell with acceptable received power.
- Path loss and shadowing parameters are obtained from empirical measurements.