

# Combined Outage/Average $P_s$ . Doppler and ISI. Diversity.

## Lecture Outline

- Combined Outage and Average Probability of Error
- Doppler Effects on Performance
- Delay Spread (ISI) Effects on Performance
- Introduction to Diversity
- Realization of Independent Fading Paths
- Diversity Combining Techniques
- Performance of Diversity Combining in Fading
- MRC Diversity and its Performance

### 1. Combined outage and average error probability:

- Shadowing causes outage and flat-fading determines  $\bar{P}_s$  during nonoutage
- $\bar{P}_s$  obtained in small region where  $\bar{\gamma}_s$  approximately constant as  $\bar{P}_s = \int P_s(\gamma_s)p(\gamma_s|\bar{\gamma}_s)$ .
- A target  $\bar{\gamma}_s$  is needed to obtain a target  $\bar{P}_s$ .
- Outage occurs when shadowing causes  $\bar{\gamma}_s$  to fall below its target value.

### 2. Doppler Effects on Performance

- Doppler has little impact on coherent modulation (small bandwidth expansion).
- High doppler can cause channel phase to decorrelate between symbols.
- Leads to an irreducible error floor for differential modulation.
- Error floor approximated by  $P_{b_{\text{floor}}} \approx .5(\pi B_d T_b)^2$ .

### 3. Delay Spread (ISI) Effects on Performance.

- Delay spread exceeding a symbol time causes ISI (self-interference).
- ISI leads to an irreducible error floor.
- Without ISI compensation, avoid error floor by reducing data rate ( $T_s \gg T_m$ ).

### 4. Introduction to Diversity

- Basic concept is to send same information over independent fading paths.
- Paths are combined to mitigate the effects of fading.

### 5. Realization of Independent Fading Paths

- Space Diversity: Multiple antenna elements spaced apart by decorrelation distance.
- Polarization Diversity: Two antennas, one horizontally polarized and one vertically polarized.

- Frequency diversity: Multiple narrowband channels separated by channel coherence bandwidth.
- Time diversity: Multiple timeslots separated by channel coherence time.

## 6. Array and Diversity Gain

- Array gain is the gain in SNR from noise averaging over the multiple antennas. Gain in both AWGN and fading channels.
- Diversity gain is the change in slope of the probability of error due to diversity. Only applies to fading channels.

## 7. Techniques for Combining Independent Fading Paths

- Selection Combining: largest fading path chosen.
- Maximal Ratio Combining: all paths cophased and summed with optimal weighting to maximize SNR at combiner output.
- Equal Gain Combining: all paths cophased and summed with equal weighting.

## 8. Selection Combining and its Performance

- Combiner SNR  $\gamma_{\Sigma}$  is the maximum of the branch SNRs.
- This gives diminishing returns, in terms of power gain, as the number of antennas increases.
- CDF of  $\gamma_{\Sigma}$  easy to obtain, then pdf found by differentiating.
- Typically get 10-15 dB of gain for 2-3 antennas.

## 9. Maximal Ratio Combining (MRC)

- Branch weights optimized to maximize output SNR of combiner.
- Optimal weights are proportional to branch SNR.
- Resulting combiner SNR  $\gamma_{\Sigma}$  is sum of branch SNRs.
- Distribution obtained by characteristic function analysis (can be hard).

## 10. Performance of MRC with i.i.d. Rayleigh fading

- For  $M$  branch diversity with i.i.d. Rayleigh fading on each branch,  $\gamma_{\Sigma}$  is chi-squared with  $2M$  degrees of freedom.
- Can obtain  $P_{out}$  and  $\bar{P}_s$  from this distribution.
- For BPSK, get 15 dB gain at  $10^{-3}$  BER. Larger gains obtained at lower BERs.

## Main Points

- Doppler spread only impacts differential modulation, causing an irreducible error floor at low data rates.
- Delay spread causes irreducible error floor or imposes rate limits.
- Diversity is a powerful technique to overcome the effects of flat fading by combining multiple independent fading paths
- Diversity typically entails some penalty in terms of rate, bandwidth, complexity, or size.
- Both selection combining and MRC significantly reduce the impact of fading.
- The different combining techniques offer different levels of complexity and performance.