

## EE359 – Lecture 10 Outline

- **Announcements:**
  - Project proposals due Friday (post; email link)
  - Midterm will be Nov. 4, 8:45-10:45am. Rm TBD.
- Probability of error in fading
- Outage probability
- Average  $P_s$  ( $P_b$ )
- Combined average and outage  $P_s$
- $P_s$  due to Doppler and ISI

## Review of Last Lecture

- Capacity of ISI Channels
- Modulation Tradeoffs
- Amplitude/Phase Modulation
  - Constellations, Differential Modulation, bit mappings
- Error Probability for Amp/Phase Modulation

$$P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$$

- Traditional Q function representation

$$Q(z) = p(x > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad x \sim N(0,1)$$

- Infinite integrand with argument in integral limits

## Alternate Q Function Representation

- New representation (Craig'93)

$$Q(z) = \frac{1}{\pi} \int_0^{\pi/2} e^{-z^2 / (\sin^2 \varphi)} d\varphi$$

- Leads to closed form solution for  $P_s$  in PSK
- Very useful in fading and diversity analysis

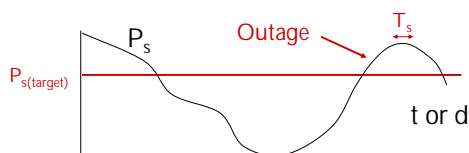
## Linear Modulation in Fading

- In fading  $\gamma_s$  and therefore  $P_s$  random
- Performance metrics:
  - Outage probability:  $p(P_s > P_{\text{target}}) = p(\gamma < \gamma_{\text{target}})$
  - Average  $P_s$ ,  $\bar{P}_s$ :

$$\bar{P}_s = \int_0^{\infty} P_s(\gamma) p(\gamma) d\gamma$$

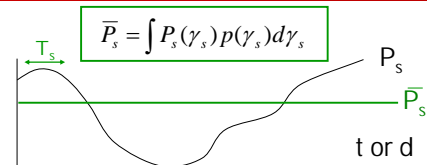
- Combined outage and average  $P_s$

## Outage Probability



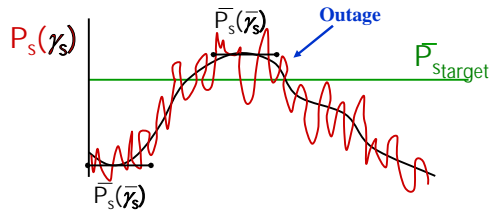
- Probability that  $P_s$  is above target
- Equivalently, probability  $\gamma_s$  below target
- Used when  $T_c \gg T_s$

## Average $P_s$



- Expected value of random variable  $P_s$
- Used when  $T_c \sim T_s$
- Error probability much higher than in AWGN alone
- Alternate Q function approach:  $Q(z) = \frac{1}{\pi} \int_0^{\pi/2} e^{-z^2 / (\sin^2 \varphi)} d\varphi$ 
  - Simplifies calculations (Get a Laplace Xfm)

## Combined outage and average $P_s$



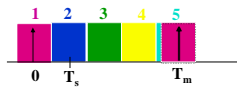
- Used in combined shadowing and flat-fading
- $\bar{P}_s$  varies slowly, locally determined by flat fading
- Declare outage when  $\bar{P}_s$  above target value

## Doppler Effects

- High doppler causes channel phase to decorrelate between symbols
- Leads to an irreducible error floor for differential modulation
  - Increasing power does not reduce error
- Error floor depends on  $B_d T_s$

## ISI Effects

- Delay spread exceeding a symbol time causes ISI (self interference).



- ISI leads to irreducible error floor
  - Increasing signal power increases ISI power
- ISI requires that  $T_s \gg T_m$  ( $R_s \ll B_c$ )

## Main Points

- In fading  $P_s$  is a random variable, characterized by average value, outage, or combined outage/average
  - Outage probability based on target SNR in AWGN.
  - Fading greatly increases average  $P_s$ .
  - Alternate Q function approach simplifies  $P_s$  calculation, especially its average value in fading (Laplace Xfm).
- Doppler spread only impacts differential modulation causing an irreducible error floor at low data rates
- Delay spread causes irreducible error floor or imposes rate limits
- Need to combat flat and frequency-selective fading
  - Focus of the remainder of the course