

EE359 – Lecture 10 Outline

- **Announcements:**
 - Project proposals due Friday (post; email link)
 - Midterm will be Nov. 4, 8:45-10:45am. Rm TBD.
- Probability of error in fading
- Outage probability
- Average P_s (P_b)
- Combined average and outage P_s
- P_s due to Doppler and ISI

Review of Last Lecture

- Capacity of ISI Channels
- Modulation Tradeoffs
- Amplitude/Phase Modulation
 - Constellations, Differential Modulation, bit mappings
- Error Probability for Amp/Phase Modulation

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$

- Traditional Q function representation

$$Q(z) = p(\mathbf{x} > \mathbf{z}) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad \mathbf{x} \sim N(0,1)$$

- Infinite integrand with argument in integral limits

Alternate Q Function Representation

- New representation (Craig'93)

$$Q(z) = \frac{1}{\pi} \int_0^{\pi/2} e^{-z^2 / (\sin^2 \varphi)} d\varphi$$

- Leads to closed form solution for P_s in PSK
- Very useful in fading and diversity analysis

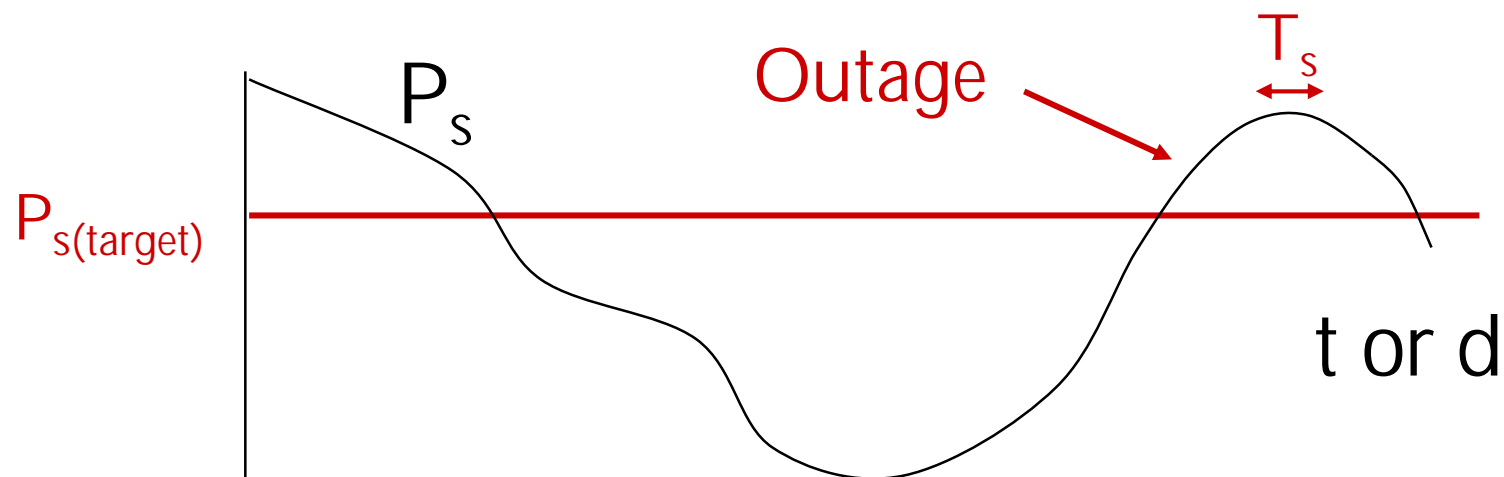
Linear Modulation in Fading

- In fading γ_s and therefore P_s random
- Performance metrics:
 - Outage probability: $p(P_s > P_{\text{target}}) = p(\gamma < \gamma_{\text{target}})$
 - Average P_s , \bar{P}_s :

$$\bar{P}_s = \int_0^{\infty} P_s(\gamma) p(\gamma) d\gamma$$

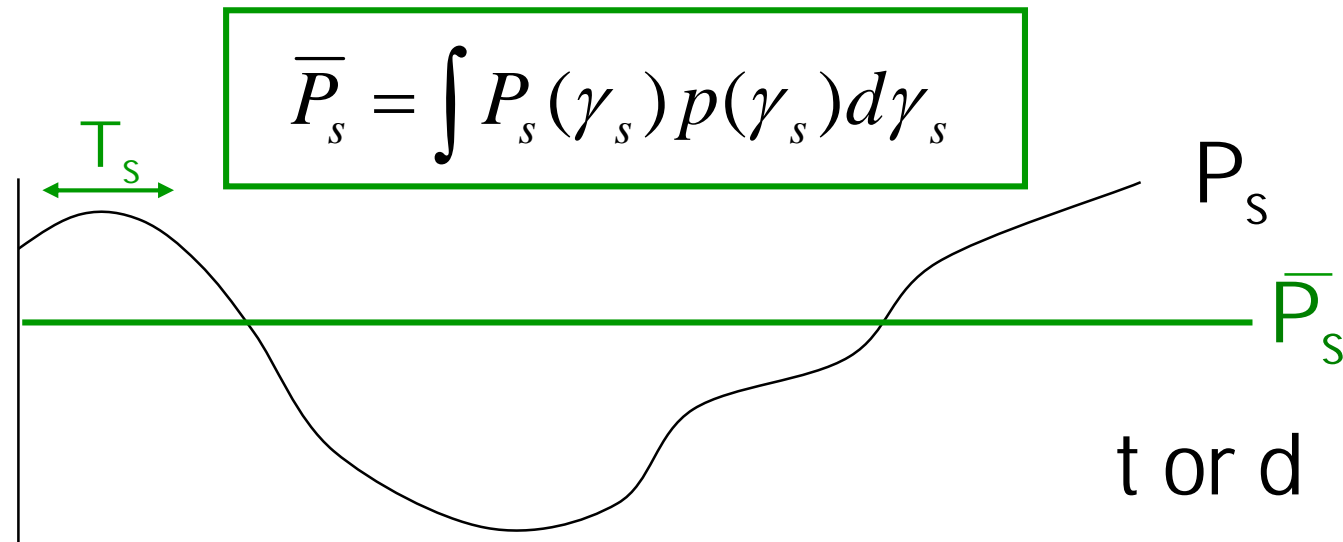
- Combined outage and average P_s

Outage Probability



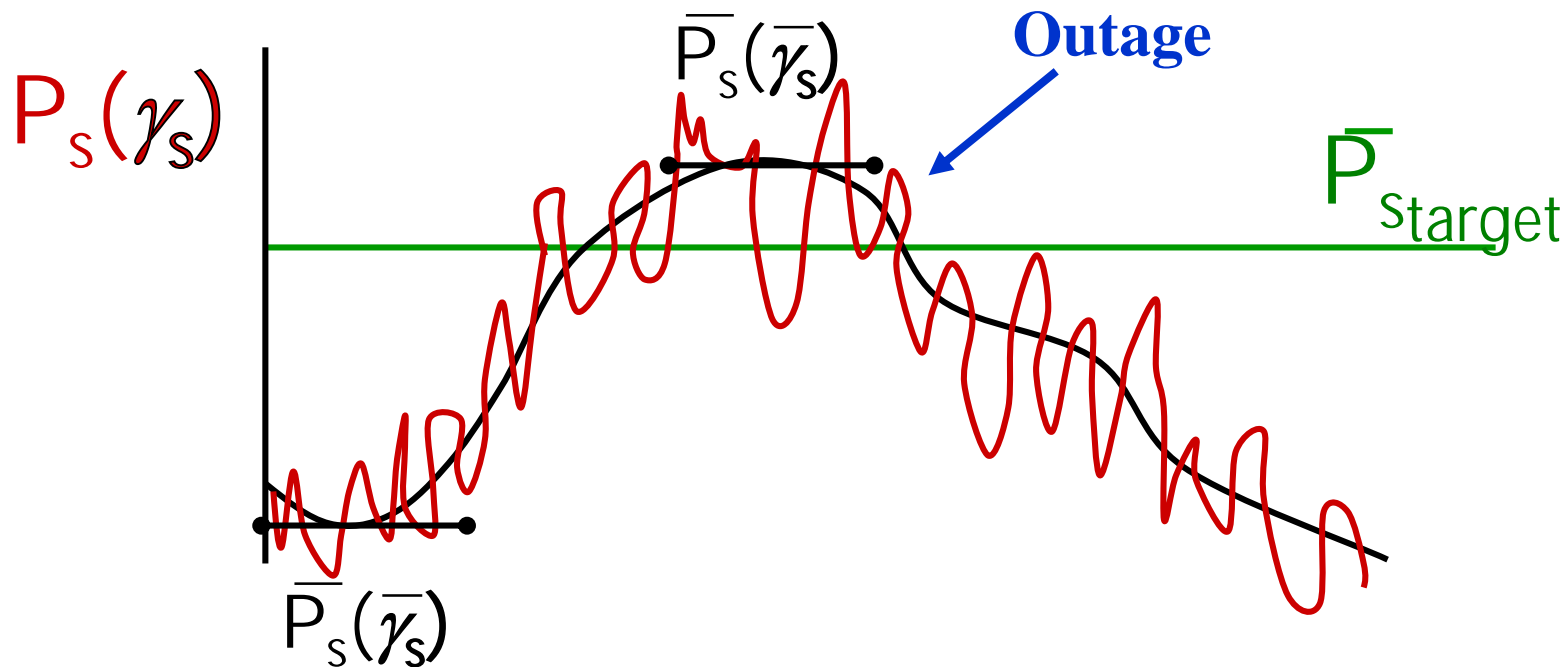
- Probability that P_s is above target
- Equivalently, probability γ_s below target
- Used when $T_c \gg T_s$

Average P_s



- Expected value of random variable P_s
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone
- Alternate Q function approach: $Q(z) = \frac{1}{\pi} \int_0^{\pi/2} e^{-z^2 / (\sin^2 \varphi)} d\varphi$
 - Simplifies calculations (Get a Laplace Xfm)

Combined outage and average P_s



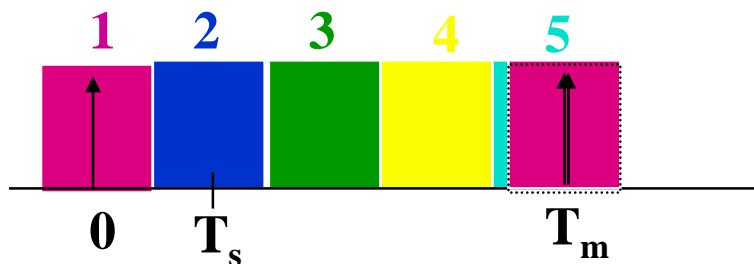
- Used in combined shadowing and flat-fading
- \bar{P}_s varies slowly, locally determined by flat fading
- Declare outage when \bar{P}_s above target value

Doppler Effects

- High doppler causes channel phase to decorrelate between symbols
- Leads to an irreducible error floor for differential modulation
 - Increasing power does not reduce error
- Error floor depends on $B_d T_s$

ISI Effects

- Delay spread exceeding a symbol time causes ISI (self interference).



- ISI leads to irreducible error floor
 - Increasing signal power increases ISI power
- ISI requires that $T_s \gg T_m$ ($R_s \ll B_c$)

Main Points

- In fading P_s is a random variable, characterized by average value, outage, or combined outage/average
 - Outage probability based on target SNR in AWGN.
 - Fading greatly increases average P_s .
 - Alternate Q function approach simplifies P_s calculation, especially its average value in fading (Laplace Xfm).
- Doppler spread only impacts differential modulation causing an irreducible error floor at low data rates
- Delay spread causes irreducible error floor or imposes rate limits
- Need to combat flat and frequency-selective fading
 - Focus of the remainder of the course