

Midterm

1. Wireless Channels [35 Points]

Consider a wideband channel characterized by the autocorrelation function

$$A_c(\tau, \Delta t) = \text{sinc}(W\Delta t) \frac{1}{\bar{T}_m} e^{-\frac{\tau}{\bar{T}_m}}, \quad \tau \geq 0$$

where $W = 50$ Hz and $\bar{T}_m = 20$ microseconds.

- Does the channel correspond to an indoor or an outdoor channel? Why?
- Is the fading distribution for this channel closer to Rayleigh or to Rician?
- For 8PSK modulation at a data rate of 50 Kbps sent over the channel, what is the appropriate performance metric for this system and why? What about for a sensor network with BPSK modulation and a data rate of 10 bps?
- For BPSK modulation at a data rate of 500 Kbps sent over the channel, what is the irreducible error floor due to ISI? How about a Doppler floor?
- Suppose you have a different channel from the wideband channel given above. This channel has shadowing plus Rayleigh fading. The distribution of the shadowing at the receiver is uniformly distributed between 0 and $2\bar{\gamma}$ (in linear units). What should $2\bar{\gamma}$ be such that the outage probability for DPSK modulation with an average error probability of 10^{-3} is ten percent.

2. How to be a billionaire: a day in the life of a startup [35 points]

This problem illustrates tradeoffs in different types of fixed rate modulation, and that fixed-rate modulation with outage can come fairly close to channel capacity. This is significant since the fixed-rate modulation is easy to implement and requires no coding, which are both desirable from an implementation perspective.

Consider a discrete time-varying AWGN channel with four channel states. Assuming a fixed transmit power \bar{P} , the received SNR associated with each channel state is $\gamma_1 = 6\text{dB}$, $\gamma_2 = 11\text{dB}$, $\gamma_3 = 15\text{dB}$, and $\gamma_4 = 24\text{dB}$, respectively. Each state is equally likely with $p_i = .25$, $i = 1, 2, 3, 4$. Assume both transmitter and receiver have perfect instantaneous estimates of the channel.

- Find the optimal transmission strategy and associated Shannon capacity per unit Hertz of this channel assuming perfect transmitter and receiver knowledge of the instantaneous channel. Given a Doppler of 80 Hz, if the system estimates the channel state at the receiver and sends this state to the transmitter along a feedback path every channel coherence time, what is the data rate of the feedback path?
- Find the optimal transmission strategy and associated Shannon capacity per unit Hertz of this channel assuming perfect receiver knowledge only of the instantaneous channel.
- Armed with your newly acquired EE359 knowledge, you propose to start a company to build a communication system for this channel using a fixed rate modulation strategy. For ease of implementation you use square MQAM modulation, where $M = 2^x$ for any integer x . Given the average power constraint \bar{P} , what is the maximum square MQAM constellation size that can be sent over the channel under this power control policy while meeting a target BER of 10^{-3} (use the bound $\text{BER} \leq .2e^{-1.5\gamma/(M-1)}$ for $M \geq 4$ and $\text{BER} \leq 2e^{-1.5\gamma/(M-1)}$ for $M = 2$.) What is the corresponding spectral efficiency for the channel (bps/Hz), assuming the symbol time $T_s = 1/B$ or, equivalently, that the number of bits/symbol equals the bps/Hz.

- (d) A competing company proposes to increase the spectral efficiency over your system by using the same fixed rate square MQAM modulation and target BER of part (b), but allowing for some outage. What is the maximum spectral efficiency that can be obtained for this channel and what is the associated transmission strategy?
- (e) For what types of applications will your system work better despite its lower average rate, and what types of applications will favor your competitor's solution over yours?

3. Macrodiversity [30 Points]

In many cellular systems today a mobile is connected to multiple base stations, and the base stations combine their transmitted signals using some form of diversity (e.g. selection combining or MRC) to exploit the fact that the antennas associated with each base station are geographically dispersed so they are not affected by the same shadow fading. In this problem we investigate the performance gains associated with this macrodiversity. Suppose you have a cellular system with 3 different base stations communicating with the same mobile unit (If you need to compute the Q function in this problem and don't have it on your calculator, use the following approximation: $Q(z) = e^{-z^2/2}/(z\sqrt{2\pi})$, $z > 0$, $Q(z) = .5$, $z \approx 0$, $Q(z) = 0$, $z < -1$).

- (a) Suppose the base stations know the channel perfectly and pool their antennas to do transmit MRC to the mobile. Assume free space path loss with no shadowing or Rayleigh fading, that each base station is 100 m from the mobile, and that all antennas are omnidirectional, i.e. their antenna gain is 0 dB. For a transmit power of 100 mW at the first base station, 80 mW at the second base station, and 70 mW at the third base station, assume a carrier frequency of 2.4 GHz, a signal bandwidth of 200 KHz, and a noise power spectral density of $N_0 = 10^{-12}$ W/Hz. Find the approximate probability of bit error for 16QAM modulation?
- (b) Assume the mobile has a single antenna, and the propagation from the base station to the mobile's antenna experiences log-normal shadowing only with a standard deviation $\sigma = 5$ dB. For DPSK modulation with a target BER of 10^{-3} , find the outage probability assuming selection combining of the base station transmissions, and that the received SNRs for the base stations' transmissions are 10 dB, 8 dB, and 5 dB, respectively.
- (c) Assume now that the mobile has 3 antennas with i.i.d. fading on each antenna. Assume the mobile performs MRC on the received signal at these antennas. Assume also that there is no shadowing on the system, the 3 base station transmissions have received SNRs of 10 dB, 8 dB, and 5 dB, respectively due to path loss alone, and there is Rayleigh fading on top of the path loss. For the same DPSK modulation and target BER as in part (b), find the outage probability at the mobile assuming the base station with the largest SNR at the MRC diversity combiner output is used.