EE 359 - Wireless Communications - Fall 2006

Midterm: Total 100 Points, Duration: 11am to 1pm

This exam is open book and notes, and calculators are needed. Please state all assumptions used in your calculations. You may use any derivations or statements from the book as long as you cite where they come from.

1. **[35 points]**

   In this problem the three types of variations in the channel, i.e., large-scale path losses, slow fading (shadowing), and fast fading (multipath) for an outdoor macrocellular mobile radio application are explored.

   To calculate the Q function in this problem, you can use your calculator's Q-function if available, or the approximation

   $$Q(x) \approx e^{-\left(\frac{x^2}{2}\right)}$$

   (a) **[10 points]**
   Using the simplified path loss model (Lee's Model) find cell radius if an average power of -100dBm is to be maintained at cell boundary. Base-transmitted power is 10 Watts, carrier frequency is 900Mhz, path loss exponent is 3.7, and K (path gain at reference distance of d_0= 1Km) is 18 dB below K for free-space. (Lee's Model was developed by W.C.Y. Lee and used in the design of first generation analog cellular system AMPS. The numerical values given in this problem are based on the Philadelphia propagation measurements, which were also carried out in support of AMPS.)

   (b) **[10 points]**
   Find cell coverage area (percentage coverage throughout the cell). Assume lognormal shadowing with a standard deviation of 7 dB. Minimum acceptable power level is 10dB below average power at cell boundary.

   (c) **[15 points]**
   Find: (i) probability of being in a deep fade, (ii) Level Crossing Rate (LCR), and (iii) Average Duration of Fades (ADF) for an automobile traveling at a speed of 36km/hr at a distance d = 4km from the base. Path loss due to shadowing alone is 10dB. Deep fade is defined as a power threshold level 15dB below mean. Short-term fading is assumed to be Rayleigh.

2. **[40 points]**

   This problem demonstrates that for wideband channels, transmission by modulation in frequency domain can give higher data rates that doing it in time-domain. This is one of the reasons that techniques like OFDM (a form of transmission in frequency-domain) are extremely popular.

   (a) **[2 points]**
   Assume the power delay profile for the wideband channel is given as

   $$A_c(\tau) = \frac{1}{\bar{T}_m} e^{-\frac{\tau}{\bar{T}_m}} \text{ mW}, \quad \tau \geq 0$$

   where $\bar{T}_m$ = 1ms. What are the values for mean μ_{T_m} and rms σ_{T_m} delay spread. What is the coherence bandwidth B_c.

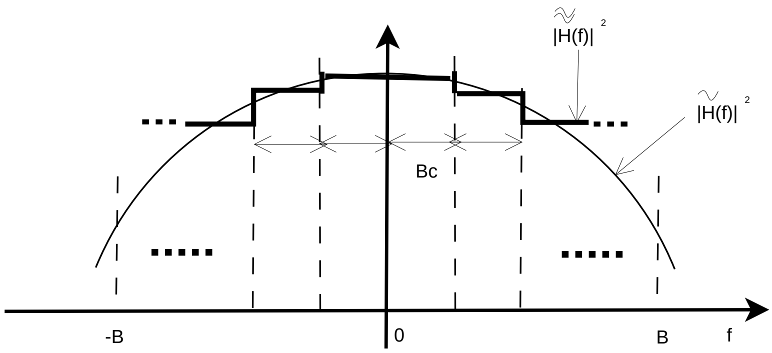

Figure 1: Problem 1, Part d)

(b) **[4 points]**

Suppose that we want to approximate the channel using a 2-tap model i.e.

$$\tilde{h}(\tau) = c(\tau, t) = a_0\delta(\tau) + a_1\delta(\tau - T_s), \quad \text{for all } t$$

where T_s=0.5 ms. Find the a_i's, $i = 0, 1$. Is the channel time-varying?

[Hint: $a_i = \sqrt{A_c(iT_s)}$]

(c) **[10 points]**

Using transmit power P_T=10 mW and $\frac{N_0}{2} = 10^{-9}$ mW/Hz, find the capacity of the channel characterized by $\tilde{h}(\tau)$ for transmission in the time-domain. What is the capacity when $P_T = \infty$? You can treat ISI, if any, as additional Gaussian noise [Equation (6.94) in Textbook].

[Hint: $B = \frac{1}{T_s}$, $P_r = |a_0|^2 P_T$, $I = |a_1|^2 P_T$]

(d) **[8 points]**

Find the channel response $\tilde{H}(f)$. Also evaluate $|\tilde{H}(f)|^2$.

[Hint: Using $H(f) = \int h(\tau)e^{-j2\pi f\tau}d\tau$, for $h(\tau) = a_0\delta(\tau) + a_1\delta(\tau - T_s)$, we get $H(f) = a_0 + a_1 e^{-j2\pi f T_s}$]

(e) **[6 points]**

Now suppose the transmitter alternatively wants to transmit in the frequency domain. For this, it splits the total available bandwidth into bands of the size of coherence bandwidth. It further assumes that each band now is an independent narrowband channel, where the entire band has the same channel gain value equal to the value in the center of that band. The picture of the channel assumed by the transmitter is depicted in Fig. 1.

Using the coherence bandwidth calculated in part (a), find $|\tilde{\tilde{H}}(f)|^2$ for $-B \leq f \leq B$ which is an approximation of $|\tilde{H}(f)|^2$.

(f) **[10 points]**

Find the capacity of this channel characterized by $|\tilde{\tilde{H}}(f)|^2$ for the same P_T and $\frac{N_0}{2}$ values. What is the capacity when $P_T = \infty$?

(g) **[Extra Credit Problem: 15 points]**

Suppose that we now add the time-varying part. What is the capacity of the channel when there is independent fading in each band given by

$$H_{\text{fading}}(f) = \begin{cases} \tilde{\tilde{H}}(f) & \text{w.p. } \frac{3}{4} \\ 0.5\tilde{\tilde{H}}(f) & \text{w.p. } \frac{1}{4} \end{cases}$$

There is perfect CSIT and CSIR.

3. **[25 points]**

This problem studies the loss in performance of diversity schemes due to practical non-ideality. We will compare a few performance measures like average probability of symbol error $\bar{P}_s$, array gain and diversity gain with and without non-ideality to demonstrate this. Array gain g and diversity gain d can be approximately defined as

$$g \approxeq \frac{\bar{\gamma}_\Sigma}{\bar{\gamma}}$$

$$d \approxeq -\frac{\log\left(\bar{P}_{s,\text{diverity}}\right)}{\log\left(\bar{\gamma}\right)}$$

In this problem, use transmit power P_T=1W, noise power σ_n^2=1W and assume DPSK transmission.

(a) **[5 points]**

Suppose SNR γ has a distribution given as

$$\eta = \begin{cases} 8 & \text{w.p. } 1/3 \\ 11 & \text{w.p. } 2/3 \end{cases}$$

and

$$\gamma = \frac{\eta P_T}{\sigma_n^2}$$

Notice that received power is ηP_T and so SNR is $\gamma = \frac{\eta P_T}{\sigma_n^2}$. Find $\bar{\gamma}$ and $\bar{P}_s$.

(b) **[8 points]**

Now suppose the receiver employs a 2-branch diversity combining using MRC. The SNR on each branch is i.i.d according to the distribution in part (a). Find $\bar{\gamma}_\Sigma$ and $\bar{P}_{s,\text{diverity}}$. Also find g and d.

(c) **[12 points]**

Now suppose the paths are not independent. Specifically, $\gamma_1 = r_1 + \xi$ and $\gamma_2 = r_2 + \xi$, where the shadowing term ξ has the following distribution

$$\xi = \begin{cases} 7 & \text{w.p. } 1/3 \\ 10 & \text{w.p. } 2/3 \end{cases}$$

ξ is independent of r_1 and r_2, which are i.i.d with the distribution

$$r_i = \begin{cases} .6 & \text{w.p. } 1/3 \\ 1.2 & \text{w.p. } 2/3 \end{cases}$$

$i = 1, 2$. Find $\bar{\gamma}_\Sigma$ and $\bar{P}_{s,\text{diverity}}$. Find g and d. Compare with part (b).

(d) **[Extra Credit Problem: 15 points]**

Now suppose there is channel estimation error. Due to this the channel gain on the ith path is modified as $\tilde{\eta}_i = \eta_i + \epsilon_i, i = 1, 2$, where η_1 and η_2 are i.i.d. according to the distribution of η in part (a). The term ϵ_i is due to estimation error and ϵ_1 and ϵ_2 are i.i.d. $\mathcal{N}(0, 1)$. Find $\bar{\gamma}_\Sigma$ and $\bar{P}_{s,\text{diverity}}$. Find g and d. Compare with parts (b) and (c).

[Hint: For the ith branch, received power will be the same as before $\eta_i P_T$, however there will be additional noise power equal to $\sigma_{\epsilon_i}^2 P_T$. Hence SNR will be reduced.]